APPENDICES

TO VARIOUS

NAUTICAL ALMANACS

BETWEEN THE YEARS

1834 and 1854.

Great Britain

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CONTENTS.

ADVERTISEMENT.

The Papers contained in the present Volume consist of the Appendices which have appeared in various Nautical Almanacs between the years 1834 and 1854, and are now collected together for separate publication, as being not only a more convenient form for reference, but the means of giving to them a more permanent existence.

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Nautical Almanac Office,
3, *Verulam Buildings, Gray's Inn,*
London,
June 30, 1851.

NEW TABLES

FOR COMPUTING

THE OCCULTATIONS OF JUPITER'S SATELLITES BY JUPITER,

THE TRANSITS OF THE SATELLITES AND THEIR SHADOWS OVER THE DISC OF THE PLANET,

AND THE

POSITIONS OF THE SATELLITES WITH RESPECT TO JUPITER AT ANY TIME.

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CONCISENESS and simplicity are considerations which ought principally to be looked to in determining on any particular mode of calculation, and most especially when an extensive series of the same quantity has to be computed, as in the determinations which form the subject of the present paper. I am not aware of the publication of any practical methods of determining the particulars relating to the Satellites of Jupiter, given in the Nautical Almanac, except in a paper by M. DELAMBRE, printed in the *Connaissance des Tems* for 1808. This ingenious paper is for the most part confined to the Configurations of the Satellites; and the method has been subsequently adopted, up to the present time, by the computers of the *Connaissance des Tems.* The roughness of the plan pursued and the variety of operations which it involves, have however induced me to extend my inquiry to the Configurations as well as the Contacts, all of which I have here endeavoured to reduce, in practice, to the most simple considerations, with a view of saving labour in the future computation of these interesting and multiplied phenomena.

GEOCENTRIC SUPERIOR CONJUNCTION.

We shall proceed first to determine the time of the geocentric conjunction of a satellite, which takes place near to any proposed heliocentric conjunction, or middle of the eclipse in the shadow of Jupiter. Let, on the day of heliocentric conjunction,

$$\left.\begin{aligned} (G) &= ♃\text{'s geocentric longitude,}\\ (H) &= ♃\text{'s heliocentric longitude,}\\ (G)-(H) = P &= ♃\text{'s annual parallax,}\end{aligned}\right\}\text{ at mean noon, expressed in degrees.}$$

(G'), (H'), P', = the same quantities at the *following* mean noon.

$$\left.\begin{aligned} (G') - (G) &= \delta(G)\\ (H') - (H) &= \delta(H)\\ \delta(G) - \delta(H) &= \delta P\end{aligned}\right\}\text{ daily differences.}$$

τ = mean time of heliocentric ☌, expressed in hours.

t = mean time of geocentric ☌, expressed in hours.

R = mean synodic revolution of the satellite, expressed in hours.

Then the parallax at the time of geocentric conjunction $= \mathrm{P} + \frac{t}{24} \delta \mathrm{P}$; and the time of moving over this, with the satellite's heliocentric synodic motion, which may be assumed the same as that of the mean, will be

$$\frac{\mathrm{R}}{360}\left(\mathrm{P} + \frac{t}{24}\delta\mathrm{P}\right).$$

This must correspond with the interval from heliocentric to geocentric conjunction; therefore, by equating it with $t - \tau$ and solving for t, we get

$$t = \frac{\tau + \frac{\mathrm{R}}{360}\cdot\mathrm{P}}{1 - \frac{\mathrm{R}}{360}\cdot\frac{\delta\mathrm{P}}{24}}$$

$$= \left(\tau + \frac{\mathrm{R}}{360}\cdot\mathrm{P}\right)\left\{1 + \frac{\frac{\mathrm{R}}{360}\cdot\frac{\delta\mathrm{P}}{24}}{1 - \frac{\mathrm{R}}{360}\cdot\frac{\delta\mathrm{P}}{24}}\right\}$$

$$= \left(\tau + \frac{\mathrm{R}}{360}\cdot\mathrm{P}\right)\left\{1 + \frac{\delta\mathrm{P}}{\frac{8460}{\mathrm{R}} - \delta\mathrm{P}}\right\}$$

or, omitting δ P in the denominator, which is very small compared with $\frac{8640}{\mathrm{R}}$,

$$t = \left(\tau + \frac{\mathrm{R}}{360}\,.\,\mathrm{P}\right)\left(1 + \frac{\mathrm{R}}{8640}\,.\,\delta\mathrm{P}\right)$$

According to Delambre, the values of R for the four satellites are

$$\begin{aligned} \mathrm{R}_1 &= 42^{\mathrm{h}}{\cdot}476651 \\ \mathrm{R}_2 &= 85{\cdot}298258 \\ \mathrm{R}_3 &= 171{\cdot}993285 \\ \mathrm{R}_4 &= 402{\cdot}085284 \end{aligned}$$

which substituted in the above, the respective formulæ for the several satellites come out as follows:

$$\left.\begin{aligned} &\text{Geo. } \circ\!\!\!\,\prime\ ☾_1 = \{\tau + 0{\cdot}1179907\ \mathrm{P}\}\ \{1 + {\cdot}0049\ \delta\mathrm{P}\} \\ &\text{Geo. } \circ\!\!\!\,\prime\ ☾_2 = \{\tau + 0{\cdot}2369396\ \mathrm{P}\}\ \{1 + {\cdot}0099\ \delta\mathrm{P}\} \\ &\text{Geo. } \circ\!\!\!\,\prime\ ☾_3 = \{\tau + 0{\cdot}4777592\ \mathrm{P}\}\ \{1 + {\cdot}0199\ \delta\mathrm{P}\} \\ &\text{Geo. } \circ\!\!\!\,\prime\ ☾_4 = \{\tau + 1{\cdot}1169036\ \mathrm{P}\}\ \{1 + {\cdot}0465\ \delta\mathrm{P}\} \end{aligned}\right\}\ \text{in hours.}$$

The first factor of each of these expressions may be regarded as an approximate time for the geocentric conjunction; we shall, for the sake of brevity, call it T. It will add to the convenience of the calculation if we use δ P in minutes, so as to bring out the corrections involved by it in minutes; and, since the values of these corrections are so small, they may be assumed equal to

$\mathrm{T}.\delta\mathrm{P} \times {\cdot}005$ or $\mathrm{T}.\delta\mathrm{P} \div 200$, in minutes, for ☾$_1$
$\mathrm{T}.\delta\mathrm{P} \times {\cdot}01$ or $\mathrm{T}.\delta\mathrm{P} \div 100$ - - - - - - ☾$_2$
$\mathrm{T}.\delta\mathrm{P} \times {\cdot}02$ or $\mathrm{T}.\delta\mathrm{P} \div 50$ - - - - - - ☾$_3$
$\mathrm{T}.\delta\mathrm{P} \times {\cdot}047$ - - - - - - - - - - - ☾$_4$

Table I, for each satellite respectively, contains the values of 0·1179907 P,

0·2369396 P, 0·4777592 P, and 1·1169035 P, or the time, on the hypothesis of the *mean* synodic motion of the satellite, in which it would describe the argument P, or the annual parallax of Jupiter. They are to be used by first taking out the part answering to the degrees, and then that answering to the minutes and tenths of the parallax. To avoid reference, the preceding factors, f, are inserted at the foot of these Tables, to which they severally belong.

We have supposed the satellite to proceed from heliocentric to geocentric conjunction with its *mean* synodic motion. Let the true motion of the satellite be assumed equal to its mean motion multiplied by $1 - X$, and X will be a small number, the same as in the *Mécanique Céleste*, but with a *different sign*. The value of P in time should therefore be multiplied by $1 + X$, or, for a correction of this time, by X alone. This correction is small except for the second and fourth satellites, for which only it is here retained.

For the *second* satellite we have

$$X = -0{\cdot}01872 \cos 2C - 0{\cdot}00058 \cos D^*$$

and for the *fourth* satellite

$$X = -0{\cdot}01455 \cos C$$

(See *Mècanique Cèleste*, Vol. IV, pages 163, 149, where the former however is improperly printed *sin.* instead of *cos.*)

The values of X will be found in Table I[a] for the second and fourth satellites. In the former, the term depending on D, being small, has been omitted.

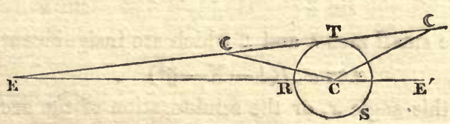
According to the tables, we compute

$$T = \tau + (\text{P in time}) + X\,.\,(\text{P in time})$$
$$\text{Geo. } ☌ = T + f.\,(T ☌ P)$$

The time of geocentric conjunction thus found is not the conjunction in longitude with reference to the orbit of Jupiter; it determines the conjunction with the centre of Jupiter in the plane of the orbit of the satellite, because the heliocentric conjunctions from which it is deduced are conjunctions on that plane. This time may therefore be assumed as the middle of the occultation, and it will then remain to find the time of the semiduration.

SEMIDURATION.

Let E be the place of the Earth; TRS an equatorial section of Jupiter, touched by the tangent ET; and let ☾ be a point in this tangent at the mean distance of the satellite from C.



Assume

$a = C☾$, the mean distance of the satellite, in equatorial semidiameters.

$s = \angle TEC$, or the apparent semidiameter of Jupiter.

$1 : 1 + \rho =$ the ratio of the polar to the equatorial axis of Jupiter.

* Delambre, in his Tables for the Eclipses, (Paris, 1817,) gives a wrong argument for the effect of this term on the semiduration. In his table, entitled '*Seconde correction des demi-durés*,' the argument D should be used instead of E, as the equation depends on the arguments D and his number N.

The elliptic section of the enveloping cone through the point ☾, as seen from Jupiter, will evidently, since his axis is nearly perpendicular to the plane of his orbit, be similar to a polar section of the planet, and have its major semi-axis seen under the angle ☾CE. Let this angle be denoted by α, and we shall have $\alpha = \mathrm{C} ☾ \mathrm{T} \pm \mathrm{TEC}$. Or,

$$\alpha = \sin^{-1}\frac{1}{a} \pm s$$

We shall distinguish the quantities, as belonging to the particular satellites, by marking the characters with the figures 1, 2, 3, and 4. Assuming thus, $a_1 = 5{\cdot}81$, $a_2 = 9{\cdot}25$, $a_3 = 14{\cdot}75$, $a_4 = 25{\cdot}95$, and $s = 0'{\cdot}3$, we find

$$\alpha_1 = \begin{cases} 9^\circ\ 55'{\cdot}0 \\ 9\ \ 54\ {\cdot}4 \end{cases} \quad \alpha_2 = \begin{cases} 6^\circ\ 12'{\cdot}7 \\ 6\ \ 12\ {\cdot}1 \end{cases} \quad \alpha_3 = \begin{cases} 3^\circ\ 53'{\cdot}6 \\ 3\ \ 53\ {\cdot}0 \end{cases} \quad \alpha_4 = \begin{cases} 2^\circ\ 12'{\cdot}8 \\ 2\ \ 12\ {\cdot}2 \end{cases}$$

The times of describing these angles by the synodic motions of the satellites will evidently give their greatest possible semiduration behind or on the disc of Jupiter. Let τ denote this time; and, following the mean synodic revolutions of DELAMBRE, viz.:

$$\begin{array}{lrrr} & ^{d} & ^{h} & ^{m} \\ R_1 = & 1 & 18 & 28{\cdot}6 \\ R_2 = & 3 & 13 & 17{\cdot}9 \\ R_3 = & 7 & 3 & 59{\cdot}6 \\ R_4 = & 16 & 18 & 5{\cdot}1 \end{array}$$

we compute, from the mean values of α,

$$\begin{array}{lrr} & ^{h} & ^{m} \\ \tau_1 = & 1 & 10{\cdot}2 \\ \tau_2 = & 1 & 28{\cdot}2 \\ \tau_3 = & 1 & 51{\cdot}5 \\ \tau_4 = & 2 & 28{\cdot}0 \end{array}$$

Let λ be the latitude of the satellite above the plane of the orbit of Jupiter, r its radius vector, α' the value of α at this distance r, β the jovicentric latitude of the Earth, and x, y, the spherical co-ordinates of the satellite, related to Jupiter as a centre, when in contact with the disc, the origin being determined by the line passing through the centres of the Earth and planet, and x by that plane through this line, whose intersection on the orbit is perpendicular both to the real and projected distance of the satellite at conjunction. Then we shall have $y = \lambda + \beta$ and, by the equation of the elliptic section,

$$\sin^2 x + (1+\rho)^2 \sin^2 y = \sin^2\alpha' = \frac{1}{r^2} = \frac{a^2}{r^2}\sin^2\alpha = (1-\mathrm{X})\sin^2\alpha$$

$$\therefore \sin^2 x = (1-\mathrm{X})\sin^2\alpha - (1+\rho)^2\sin^2 y = (1-\mathrm{X})\sin^2\alpha - (1+\rho)^2\sin^2(\lambda+\beta)$$

assume $\zeta = (1+\rho).\dfrac{\sin(\lambda+\beta)}{\sin\alpha} = (1+\rho)\dfrac{\sin\lambda + \sin\beta}{\sin\alpha}$, neglecting the second powers of the small arcs λ and β which are insignificant, and

$$x = \alpha\sqrt{(1-\mathrm{X}-\zeta^2)}$$

The time of describing this angle x, or the semiduration of the occultation or transit, is therefore

$$\tau(1+\mathrm{X})\sqrt{(1-\mathrm{X}-\zeta^2)}$$

Here the small number X need only be retained for the fourth satellite, and for this only near the limits, viz. when $\mathrm{N} < 1200$ or > 2800. The semiduration should then be computed strictly from this expression, using $\zeta = \dfrac{\mathrm{N}}{1000} - 2$. But when N falls without these limits, X may be neglected, and the table used with argument N.

By taking

$$\begin{aligned}
\mathrm{H} &= 46^{\circ}\!\cdot\!241 - 0^{\circ}\!\cdot\!01384\ (t-1750)\\
\mathrm{I} &= 74\cdot969 + 0\cdot67752\ (t-1750)\\
\mathrm{K} &= 187\cdot493 + 2\cdot53988\ (t-1750)\\
\mathrm{L} &= 256\cdot68 + 12\cdot03424\ (t-1750)
\end{aligned}$$

the theory of LAPLACE gives the latitudes of the satellites above the orbit of Jupiter, according to the following expressions, in which $☾_1$, $☾_2$, $☾_3$, $☾_4$, denote the longitudes of the satellites, which at conjunction will correspond with that of the planet.

$$\begin{aligned}
\lambda_1 &= +\,3^{\circ}\!\cdot\!0894\ \sin(☾_1+\mathrm{H})\\
&\quad -\,0\cdot0095\ \sin(☾_1+\mathrm{L})\\
&\quad -\,0\cdot0023\ \sin(☾_1+\mathrm{K})\\
\lambda_2 &= +\,3\cdot0726\ \sin(☾_2+\mathrm{H})\\
&\quad -\,0\cdot4636\ \sin(☾_2+\mathrm{L})\\
&\quad -\,0\cdot0337\ \sin(☾_2+\mathrm{K})\\
&\quad -\,0\cdot0058\ \sin(☾_2+\mathrm{I})\\
\lambda_3 &= +\,3\cdot0061\ \sin(☾_3+\mathrm{H})\\
&\quad -\,0\cdot2054\ \sin(☾_3+\mathrm{K})\\
&\quad -\,0\cdot0312\ \sin(☾_3+\mathrm{I})\\
&\quad +\,0\cdot0159\ \sin(☾_3+\mathrm{L})\\
\lambda_4 &= +\,2\cdot6786\ \sin(☾_4+\mathrm{H})\\
&\quad -\,0\cdot2491\ \sin(☾_4+\mathrm{I})\\
&\quad +\,0\cdot0404\ \sin(☾_4+\mathrm{K})\\
&\quad +\,0\cdot0004\ \sin(☾_4+\mathrm{L})
\end{aligned}$$

Let now ☉ denote the Sun's longitude, ☊ that of the ascending node of Jupiter, i the inclination of his orbit with the ecliptic, D his mean distance, and β the jovicentric latitude of the Earth; and we shall have, for the latitude β', on the supposition that Jupiter is at this mean distance,

$$\sin\beta' = \frac{\sin i}{\mathrm{D}}\sin(☉-☊)$$

If Δ be the actual distance of Jupiter from the Earth, this value must be multiplied by $\frac{\mathrm{D}}{\Delta} = 1 + \frac{\mathrm{D}-\Delta}{\Delta}$ to get the true jovicentric latitude of the Earth. Let ♃ denote the geocentric longitude of Jupiter; and, by neglecting the inclination and the ellipticities of the orbits, we shall have

$$\begin{aligned}
\Delta &= \cos(♃-☉) + \sqrt{\{\mathrm{D}^2 - \sin^2(♃-☉)\}}\\
&= \cos(♃-☉) + \sqrt{\{(\mathrm{D}^2-\tfrac{1}{2}) + \tfrac{1}{2}\cos 2\,(♃-☉)\}}
\end{aligned}$$

Take therefore a number $\mathrm{F} = \frac{\Delta}{\mathrm{D}-\Delta}$, which thus depends upon ♃ − ☉, and we get, for the true value of β,

$$\sin\beta = \sin\beta'\ (1+\frac{1}{\mathrm{F}}) = \{\frac{\sin i}{\mathrm{D}}\sin(☉-☊)\}\{1+\frac{1}{\mathrm{F}}\}$$

The recent observations of Professor STRUVE make the polar and equatorial semidiameters of Jupiter as $35''\cdot54$ to $38''\cdot33$, and hence $1+\rho = \frac{38\cdot33}{35\cdot54} = 1\cdot0785$. Assume $(1+\rho)\frac{\sin\beta}{\sin\alpha_1} = \beta_1$, $(1+\rho)\frac{\sin\beta}{\sin\alpha_2} = \beta_2$, $(1+\rho)\frac{\sin\beta}{\sin\alpha_3} = \beta_3$, $(1+\rho)\frac{\sin\beta}{\sin\alpha_4} = \beta_4$; and we deduce, from the preceding values of λ_1, λ_2, λ_3, λ_4, the following values of ζ for the four satellites,

$$\zeta_1 = \cdot 3377 \sin(☾_1 + H) - \cdot 0010 \sin(☾_1 + L) - \cdot 0003 \sin(☾_1 + K) + \beta_1$$

$$\zeta_2 = \cdot 5347 \sin(☾_2 + H) - \cdot 0807 \sin(☾_2 + L) - \cdot 0059 \sin(☾_2 + K) - \cdot 0010 \sin(☾_2 + I) + \beta_2$$

$$\zeta_3 = \cdot 8342 \sin(☾_3 + H) - \cdot 0570 \sin(☾_3 + K) - \cdot 0087 \sin(☾_3 + I) + \cdot 0044 \sin(☾_3 + L) + \beta_3$$

$$\zeta_4 = 1 \cdot 3079 \sin(☾_4 + H) - \cdot 1217 \sin(☾_4 + I) + \cdot 0197 \sin(☾_4 + K) + \cdot 0002 \sin(☾_4 + L) + \beta_4$$

The several terms of these expressions, which are called the numbers H, I, K, L, β, are contained in Tables III, III[a], and IV, for each satellite respectively: they have been computed to three places, which is as far as we can be warranted in extending them, in consequence of the present doubtful values of the mean distances of the satellites from the planet; and the numbers K, L of ζ_1, the number I of ζ_2, and the number L of ζ_4, being of little importance, have been omitted.

In Table III of the first satellite ·380 has been added to the number H.

In Tables III and III[a] of the second satellite there has been added

$$\text{to the number} \begin{cases} H \;.\;.\;.\; 0 \cdot 613 \\ K \;.\;.\;.\; 0 \cdot 006 \\ L \;.\;.\;.\; 0 \cdot 081 \end{cases} \qquad \overline{0 \cdot 700}$$

In Tables III and III[a] of the third satellite there has been added

$$\text{to the number} \begin{cases} H \;.\;.\;.\; 0 \cdot 930 \\ I \;.\;.\;.\; 0 \cdot 009 \\ K \;.\;.\;.\; 0 \cdot 057 \\ L \;.\;.\;.\; 0 \cdot 004 \end{cases} \qquad \overline{1 \cdot 000}$$

And in Tables III and III[a] of the fourth there has been added

$$\text{to the number} \begin{cases} H \;.\;.\;.\; 1 \cdot 858 \\ I \;.\;.\;.\; 0 \cdot 122 \\ K \;.\;.\;.\; 0 \cdot 020 \end{cases} \qquad \overline{2 \cdot 000}$$

The values of ζ with the constants thus added are designated the number N, so that

$$N_1 = \zeta_1 + \cdot 380, \quad N_2 = \zeta_2 + \cdot 700, \quad N_3 = \zeta_3 + 1 \cdot 000, \quad N_4 = \zeta_4 + 2 \cdot 000$$

The numbers β and F have been computed by taking $i = 1^\circ\ 18' \cdot 7$ and $D = 5 \cdot 203$.

It is evident, that a transit of the shadow of a satellite would, to an observer at the Sun, be a transit of the satellite itself, and hence the determination of the semiduration of such a transit will be of a similar nature; we have only to use the heliocentric longitude of Jupiter for ♃ instead of the geocentric, and to omit β and its correction, because the jovicentric latitude of the Sun $= 0$.

INFERIOR CONJUNCTIONS.

The determination of the time of the middle of the transit of the shadow, or the time of the inferior heliocentric conjunction, depends on a different consideration. If the satellite's motion were uniform we should merely have to take the mean of the preceding and following superior heliocentric conjunctions or middles of the eclipses in the shadow of the planet. From an inspection of the arguments we see that, in those of each satellite, C, D, E, and H, I, K, L, including *c* for the third satellite, are the

only ones which sensibly vary in the course of a revolution. The arguments H, I, K, L, vary very nearly with the motion of the satellite, and their equations help to reduce the conjunction from the plane of the orbit of the satellite to that of the planet, (see *Tables Ecliptiques des Satellites de Jupiter*. Par M. DELAMBRE. Paris, 1817, page XVII of the Introduction); therefore, from superior to inferior conjunction these arguments will acquire very nearly half a revolution, and their combinations 2 H, 2 I, 2 K, 2 L, H + I, H + K, H + L, K + I, K + L, I + L, will acquire nearly a whole revolution; and, since the equations responding to these arguments depend on the sines of these angles, they will admit of no material variation at inferior, from their values at superior, conjunction.

Also, the equations which depend on the arguments D, E, of the first satellite, are too small in themselves to affect any results in a sensible manner. The equations belonging to the other arguments have, however, irregularities which it will be necessary to take into account in deducing an inferior from the preceding and following superior conjunctions. In the course of a revolution of the satellite, the motions of these arguments will, according to DELAMBRE, be as follow:

FIRST SATELLITE.

Motion of Argument C $(= ☾_1 - ☾_2) =$ ·5020 in one revolution.

SECOND SATELLITE.

$$\text{Motion of Argument}\ \left.\begin{array}{l} C\ (= ☾_2 - ☾_3) = \cdot 5041 \\ D\ (= ☾_2 - \psi_3) = 1\cdot 001 \\ E\ (= ☾_2 - \psi_4) = 1\cdot 001 \end{array}\right\}\ \text{in one revolution.}$$

THIRD SATELLITE.

$$\text{Motion of Argument}\ \left.\begin{array}{l} c\ (= ☾_3 - ☾_4) = \cdot 572 \\ C\ (= ☾_2 - ☾_3) = 1\cdot 016 \\ D\ (= ☾_3 - \psi_3) = 1\cdot 002 \\ E\ (= ☾_3 - \psi_4) = 1\cdot 002 \end{array}\right\}\ \text{in one revolution.}$$

FOURTH SATELLITE.

$$\text{Motion of Argument}\ \left.\begin{array}{l} C\ (= ☾_4 - \psi_4) = 1\cdot 0038 \\ D\ (= ☾_4 - \psi_3) = 1\cdot 004 \\ E\ (= ☾_3 - ☾_4) = 1\cdot 338 \end{array}\right\}\ \text{in one revolution.}$$

Suppose now, with regard to the first satellite, E and E′ to represent the sum of all the equations, that of C excepted, which enter into the middle of the preceding and the following eclipse; and we shall have, in consequence of the slow and nearly equable motions of these equations, $\frac{1}{2}$ (E + E′) for the sum of the same equations at the inferior conjunction. And since the value of the argument C at this conjunction is, in accordance with its aforesaid motion in a revolution, become C + ·2510, C denoting the argument at the preceding eclipse, we get the time of the inferior conjunction

$$= \tfrac{1}{2}(E + E') + \text{Equation to Argument } (C + \cdot 2510)$$

Observing therefore that

$$E = \text{preceding hel.} \ ☌ - \text{Equ. to Arg. C}$$
$$E' = \text{following hel.} \ ☌ - \text{Equ. to Arg. } (C + \cdot 5020)$$

we deduce the time of the inferior heliocentric conjunction

$$= \tfrac{1}{2}\,(\text{prec. hel. } \sigma + \text{foll. hel. } \sigma)$$
$$+ \text{Equ. to Arg. } (C + {\cdot}2510) - \tfrac{1}{2}\{\text{Equ. to Arg. } C + \text{Equ. to Arg. } (C + {\cdot}5020)\} \quad (1)$$

In the same manner we have, for the second satellite, the time of the inferior heliocentric conjunction

$$= \tfrac{1}{2}\,(\text{prec. hel. } \sigma + \text{foll. hel. } \sigma)$$
$$+ \text{Equ. to Arg. } (C + {\cdot}2520) - \tfrac{1}{2}\{\text{Equ. to Arg. } C + \text{Equ. to Arg. } (C + {\cdot}5041)\}$$
$$+ \text{Equ. to Arg. } (D + {\cdot}500) - \text{Equ. to Arg. } D^{*}$$
$$+ \text{Equ. to Arg. } (E + {\cdot}500) - \text{Equ. to Arg. } E^{*} \quad (2)$$

For the third satellite, the time of inferior heliocentric conjunction

$$= \tfrac{1}{2}\,(\text{prec. hel. } \sigma + \text{foll. hel. } \sigma)$$
$$+ \text{Equ. to Arg. } (c + {\cdot}286) - \tfrac{1}{2}\{\text{Equ. to Arg. } c + \text{Equ. to Arg. } (c + {\cdot}572)\}$$
$$+ \text{Equ. to Arg. } (C + {\cdot}508) - \tfrac{1}{2}\{\text{Equ. to Arg. } C + \text{Equ. to Arg. } (C + 1{\cdot}016)\}$$
$$+ \text{Equ. to Arg. } (D + {\cdot}500) - \text{Equ. to Arg. } D^{*}$$
$$+ \text{Equ. to Arg. } (E + {\cdot}500) - \text{Equ. to Arg. } E^{*} \quad (3)$$

And for the fourth satellite, the time of inferior heliocentric conjunction

$$= \tfrac{1}{2}\,(\text{prec. hel. } \sigma + \text{foll. hel. } \sigma)$$
$$+ \text{Equ. to Arg. } (C + {\cdot}5019) - \tfrac{1}{2}\{\text{Equ. to Arg. } C + \text{Equ. to Arg. } (C + {\cdot}0038)\}$$
$$+ \text{Equ. to Arg. } (D + {\cdot}502) - \tfrac{1}{2}\{\text{Equ. to Arg. } D + \text{Equ. to Arg. } (D + {\cdot}004)\}$$
$$+ \text{Equ. to Arg. } (E + {\cdot}669) - \tfrac{1}{2}\{\text{Equ. to Arg. } E + \text{Equ. to Arg. } (E + {\cdot}338)\} \quad (4)$$

These several corrections, which depend on the arguments c, C, D, E, at the *preceding* eclipse, are contained in Tab. II for each satellite respectively. In the computations of those of the fourth satellite, which depend on C and D, it has been assumed that

$$\tfrac{1}{2}\{\text{Equ. to Arg. } C + \text{Equ. to Arg. } (C + {\cdot}0038)\} = \text{Equ. to Arg. } (C + {\cdot}0019)$$
$$\tfrac{1}{2}\{\text{Equ. to Arg. } D + \text{Equ. to Arg. } (D + {\cdot}004)\} = \text{Equ. to Arg. } (D + {\cdot}002)$$

Such are the simple and neat modifications in the formation of these four Tables of corrections, recently used by the late Mr. Henry Jenkins, in computing the times of the transits of the shadows over the disc of Jupiter, which he applied however on the supposition of the semiduration of the transit being the same as that of the eclipse. The computation of the time of this semiduration is afforded by the same Tables which are given for those of the occultation and transit. In taking out the component parts of the number N we have merely to form the arguments with the *heliocentric* longitude of Jupiter, and to neglect β' and its correction, as we have before observed.

We must also remark, that the time of the middle of a transit of a satellite, or the time of its inferior geocentric conjunction, is to be determined from the mean of the preceding and following geocentric conjunctions with the application of the *same* correction as that used for the transit of the shadow.

APPARENT CO-ORDINATES.

The apparent elliptic form of the orbit of the satellite is very nearly determined by its visible latitude $\lambda + \beta$, at the time when it is in its geocentric conjunction, since this

* We here assume the motions of D and E to be each 1·000 in a revolution, instead of 1·001, 1·002, which adds much to the facility of the calculation, and cannot sensibly affect the result.

latitude may be regarded as the inclination of the plane of the satellite's orbit with the visual line which joins the centre of the planet and the observer's position. If, therefore, a, b, be the semi-axes of the apparent ellipse, we shall have

$$\frac{b}{a} = \sin(\lambda + \beta) = \zeta \frac{\sin \alpha}{1+\rho}$$

$$\frac{a}{b} = \frac{1}{\zeta} \cdot \frac{1+\rho}{\sin \alpha}$$

The values of log. $\frac{1+\rho}{\sin \alpha}$ for the four respective satellites are 0·7970, 0·9990, 1·2016 and 1·4470; and since

$$\zeta_1 = N_1 - \cdot 380 \qquad \zeta_2 = N_2 - \cdot 700$$
$$\zeta_3 = N_3 - 1\cdot 000 \qquad \zeta_4 = N_4 - 2\cdot 000$$

the values of $\frac{a}{b}$ for the four satellites will be

$$\frac{a_1}{b_1} = \frac{[0\cdot 7970]}{N_1 - \cdot 380} \qquad \frac{a_2}{b_2} = \frac{[0\cdot 9990]}{N_2 - \cdot 700}$$
$$\frac{a_3}{b_3} = \frac{[1\cdot 2016]}{N_3 - 1\cdot 000} \qquad \frac{a_4}{b_4} = \frac{[1\cdot 4470]}{N_4 - 2\cdot 000}$$

As the major semi-axis (a) corresponds with (r) the distance of the satellite from the planet, the values of these ratios determine entirely its apparent orbit. When the denominator is negative, or when N is less than the number to be deducted from it, the value of $\frac{a}{b}$, as also the satellite's latitude at superior geocentric conjunction, will be negative. Thus:

$$\text{When} \left\{ \begin{array}{l} N_1 > \cdot 380 \\ N_2 > \cdot 700 \\ N_3 > 1\cdot 000 \\ N_4 > 2\cdot 000 \end{array} \right\} \text{the } \textit{Northern} \text{ surface of the orbit is visible.}$$

$$\text{When} \left\{ \begin{array}{l} N_1 < \cdot 380 \\ N_2 < \cdot 700 \\ N_3 < 1\cdot 000 \\ N_4 < 2\cdot 000 \end{array} \right\} \text{the } \textit{Southern} \text{ surface of the orbit is visible.}$$

To find the co-ordinates of the satellite at any time, let θ be the synodic angle described round the planet from its superior geocentric conjunction, which is due to the synodic motion in the interval, and xy its co-ordinates referred to the axes of the ellipse, and expressed in equatorial semidiameters of the planet. Then is

$$x = r \sin \theta$$
$$y = (r \cos \theta) \sin(\lambda + \beta)$$
$$= (r \cos \theta)\ \zeta \frac{\sin \alpha}{1+\rho}$$
$$= \zeta \frac{\cos \theta}{1+\rho}$$
$$= \zeta \left(1 - \frac{1+\rho-\cos\theta}{1+\rho}\right) = \zeta\left(1 - \frac{1}{D}\right)$$

$$\text{assuming} \quad D = \frac{1+\rho}{1+\rho-\cos\theta}$$

If x be estimated towards the West, and y towards the North, we have hence

$$x = -r \sin \theta \qquad y = \varsigma \left(1 - \frac{1}{D} \right)$$

in which θ assumes the same sign as the time from conjunction, viz. negative when *before* and positive when *after*.

For the ordinates estimated in the *same* directions, as seen through an *inverting* telescope, we must change the signs, and therefore have

$$x = r \sin \theta$$

which always takes the same sign as the time from conjunction, or the same sign as the assumed time *minus* the time of conjunction; and the ordinates y for the four satellites will be

$$y_1 = (\cdot 380 - N_1) - \frac{(\cdot 380 - N_1)}{D_1}$$

$$y_2 = (\cdot 700 - N_2) - \frac{(\cdot 700 - N_2)}{D_2}$$

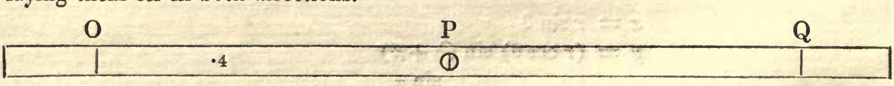
$$y_3 = (1\cdot 000 - N_3) - \frac{(1\cdot 000 - N_3)}{D_3}$$

$$y_4 = (2\cdot 000 - N_4) - \frac{(2\cdot 000 - N_4)}{D_4}$$

The Tables VI, for each satellite, (pages 35, 36,) furnish the values of x and D for successive intervals of ten minutes from the time of conjunction, and are carried to an extent which will be sufficient to determine the positions of the satellite at the times of its immersion into, and emersion from, the shadow of the planet. With the number N, which is the argument of the semiduration of the occultation, and the divisor D found from these Tables, the formula as above presents a very simple computation of the ordinate y, which will always come out with its proper sign, according to the *inverted* position of the satellite.

CONFIGURATIONS.

The Table of the elongations of the satellites, at page 39, has been computed, by supposing the equatorial semidiameter of Jupiter $= 3\cdot 7$, so as to bring the greatest for the fourth satellite a little within the limit of 100 on each side; and in the Table, 100 is applied to each of them, in order to make them positive, and avoid the necessity of laying them off in both directions.



Thus, if P be the place of Jupiter, and PO be made equal to 100, on a convenient scale, the elongations, as taken from the Table, must be measured from the line O towards the right-hand. The argument of the Table is the time from the preceding superior geocentric conjunction, expressed in decimal parts of the time of its synodic revolution, or in decimal parts of the circumference. If γ denote the argument, and r the mean distance of the satellite, its elongation to the eastward will be $= 3\cdot 7\, r \sin(\gamma . 360°)$; but by assuming the *inverted* appearance it will be to the

westward of the planet. The Table contains therefore

$$100 + 3{\cdot}7\, r \sin(\gamma . 360^\circ)$$

To find the elongation of the satellite at any time it will be necessary to know the time from the preceding geocentric conjunction, expressed in parts of the geocentric synodic revolution of the satellite at that time; and this can very readily be effected by means of Table VII, (pages 37 and 38,) which gives the decimal parts of the *mean* synodic revolution, answering to any number of hours or minutes, for each satellite.

Let G denote the time of the geocentric conjunction, which takes place near to the commencement, and G′ the time of that which takes place near to the termination, of the month; and suppose the number of revolutions which intervene to be n; and let ε denote the difference between the interval G′ — G, and n *mean* revolutions; so that if R denote the mean and R′ the actual revolution of the satellite, we shall have

$$G' - G = n\,R' = n\,R + \varepsilon$$

$$R' = R + \frac{\varepsilon}{n}$$

and consequently any time t in parts of R′

$$= \frac{t}{R'} = \frac{t}{R + \frac{\varepsilon}{n}} = \frac{t}{R}\left(\frac{1}{1 + \frac{\varepsilon}{nR}}\right)$$

$$= \frac{t}{R}\left\{1 - \frac{\frac{\varepsilon}{nR}}{1 + \frac{\varepsilon}{nR}}\right\}$$

$$\text{Or } t \text{ in parts of } R' = \frac{t}{R}\left(1 - \frac{\varepsilon}{nR'}\right)$$

$$= \left(\frac{t}{R}\right) - \frac{t}{nR'}\left(\frac{\varepsilon}{R}\right)$$

$$= \left(\frac{t}{R}\right) - \left(\frac{\varepsilon}{R}\right) \div \left(\frac{nR'}{t}\right)$$

$$= \left(\frac{t}{R}\right) - \left(\frac{\varepsilon}{R}\right) \div \left(\frac{G' - G}{t}\right)$$

In this last expression $\left(\frac{t}{R}\right)$ and $\left(\frac{\varepsilon}{R}\right)$ represent the times t and ε in parts of the mean revolution, and can be taken directly from the Tables, having previously found ε by comparing the interval G′ — G with the multiples of the mean revolutions, thus,

$$\varepsilon = (G' - G) - \text{nearest multiple of revolution.}$$

The divisor $\frac{G' - G}{t}$ expresses the number of times the time t is contained in the interval G′ — G; and if t be one day it will express the number of days in this interval. Hence, the reduction of one day into parts of the true revolution is very easily performed as follows:

Compare the interval between the geocentric conjunctions with the multiples of the mean revolutions; subtract the nearest multiple, and annex the proper algebraic sign to the small remainder. With this remainder take out the corresponding parts of the mean revolution, and divide the same by the number of days which intervene between

the conjunctions. This quotient will be a correction to be subtracted from the value of 24 hours, in parts of the *mean* revolution, to get the same in parts of the *actual* revolution.

The value thus found of a day in parts of the revolution of the satellite will serve as a daily difference, in order to deduce, by repeated addition, the arguments on successive days. It will be best to compute for the commencing one, that which is the nearest to the first conjunction, since the interval from conjunction may then, on account of its smallness, be used in parts of the mean revolution without any correction; and we must observe, that when this time is prior to the conjunction, it must be considered as negative, and we must then subtract the argument from 100000 to make it positive; also, in the successive additions of the daily difference, we must reject the units or whole revolutions as they arise. The argument computed on the day nearest to the last conjunction will serve as a check to the addition, and will indicate, by its slight deviation from the series, the degree of accuracy of the work. It is evident that the arguments can be carried back to the previous days by successively subtracting the daily difference and borrowing 100000 when necessary.

In taking out the elongations it must be observed, that when the argument is found in the left-hand column the dot is to be placed on the left-hand of the numeral; and that when the argument is found in the right-hand column the dot must be placed to the right-hand of the numeral; also, that at the beginning and end of the Table, the satellite is at its greatest elongation from the planet, and consequently stationary.

Before the elongations are taken out, however, it will be advisable to look through the times of the eclipses in Jupiter's shadow, which occur in the course of the month, and to note those which will be transpiring at the time for which the configurations are given.

When the argument approaches 000 or 500, the elongation in the Table will approach 100, and the satellite will be near to the planet. If the elongation should come within 3·7, the semidiameter of Jupiter, either in excess or defect of 100, or be between the values 96·3 and 103·7, the satellite will be

$$\left.\begin{array}{l}\textit{On the disc}\\ \textit{Behind the disc}\end{array}\right\}\text{ when the argument or dot is on the }\left\{\begin{array}{l}\textit{right hand.}\\ \textit{left hand.}\end{array}\right.$$

If the ordinate y be required it may be computed from the formula

$$y = -3 \cdot 7 \frac{\zeta}{1+\rho} \cos\theta = -(3 \cdot 43\, \zeta) \cos(\gamma . 360°)$$

in which γ is the decimal argument of the elongation; and the factor $(3 \cdot 43\, \zeta)$ may be regarded as constant for a considerable period.

This ordinate y will be the visible latitude of the satellite under the inverted appearance.

For the formation of the arguments of semidurations, the values of H, I, K, L, are here subjoined, for the commencements of the years from 1830 to 1850 inclusive. The first one H need only be used to the tenth of a degree, and the others to the nearest degree; also H, I, K, may be assumed as retaining throughout each year their values at the middle of the year, and L may be formed and used as constant for each month, the consecutive ones differing from each other one degree.

Year.	H	I	K	L
1830	45°·13	129°·2	30°·7	139°·4
1	45·12	129·8	33·2	151·5
2	45·11	130·5	35·8	163·5
3	45·09	131·2	38·3	175·5
4	45·08	131·9	40·8	187·6
5	45·06	132·6	43·4	199·6
6	45·05	133·2	45·9	211·6
7	45·04	133·9	48·5	223·7
8	45·02	134·6	51·0	235·7
9	45·01	135·3	53·5	247·7
1840	45·00	135·9	56·1	259·8
1	44·98	136·6	58·6	271·8
2	44·97	137·3	61·2	283·8
3	44·95	138·0	63·7	295·9
4	44·94	138·7	66·2	307·9
5	44·93	139·3	68·8	319·9
6	44·91	140·0	71·3	332·0
7	44·90	140·7	73·9	344·0
8	44·88	141·4	76·4	356·0
9	44·87	142·0	78·9	8·1
1850	44·86	142·7	81·5	20·1

EXAMPLES.

Let it be required to compute the superior contacts, or times of immersion and emersion, of the *third satellite*, behind the disc of Jupiter, near to the time of its eclipse, on Jan. 14, 1835; and also the co-ordinates of the satellite at the times of immersion and emersion of that eclipse, &c. &c.

To proceed with this Example, we have the following materials:

$$H = 45°·1,\ I = 133°,\ K = 43°,\ L = 200°,\ \odot = 294°\ \ \Omega = 98°$$

Middle of Eclipse Jan. $14^d\ 14^h\ 43^m·1$, *Im.* $13^h\ 29^m·4$, *Em.* $15^h\ 56^m·8$

Jupiter's Geocentric Longitude.

	° ′	Diff. ″
Jan. 14 - -	61 54·4	
15 - -	61 52·1	− 2·3
16 - -	61 49·9	2·2
17 - -	61 47·9	2·0
18 - -	61 46·2	1·7
19 - -	61 44·6	1·6
20 - -	61 43·2	1·4
21 - -	61 42·0	1·2
22 - -	61 41·1	− 0·9

Jupiter's Heliocentric Longitude.

	° ′	Diff. ″
Jan. 14 - -	70 41·1	
15 - -	70 46·4	+ 5·3
16 - -	70 51·6	5·2
17 - -	70 56·9	5·3
18 - -	71 2·1	5·2
19 - -	71 7·4	5·3
20 - -	71 12·6	5·2
21 - -	71 17·9	5·3
22 - -	71 23·1	+ 5·3

And the computations will be as here given:

OCCULTATION.

(G).... 61° 54′·4	δ(G) — 2′·3	(G) 61°·9	(G)+H 107°·0 ...	H 1728 (Numbers.)
(H)... 70 41·1	δ(H) + 5·3		(G)+I 195	I 12
P.... — 8 46·7	δP... — 7·6		(G)+K 105	K 2
P in time — 3^h 49^m·3	T.... +10·5		(G)+L 262	L 0
22·3	T.δP — 80			1742
		⊙ 294	⊙ — ☊ 196 .... β—	19
		(G—⊙) 232	F = +6 .... —	3
			Arg. semidur. N	1720

	d	h	m	
Hel. ☌ .... Jan.	14	14	43·1	T
P in time............ —		4	11·6	T
f. (T δ P)........... —			1·6	
Geo. ☌	14	10	29·9	
Semidur.		1	17·4	
Immersion	14	9	12·5	
Emersion	14	11	47·3	

Hence these phenomena take place in this order:

		d	h	m
Immersion behind....	the disc of Jupiter	Jan. 14	9	12·5
Emersion from......	the disc of Jupiter		11	47·3
Immersion in	the shadow of Jupiter		13	29·4
Emersion from......	the shadow of Jupiter		15	56·8

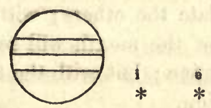
and it appears that they will be *all* visible on the Earth at such places as are free from sun-light, and have the planet sufficiently above the horizon.

When an immersion or emersion, with respect to the disc, transpires within the interval between the immersion and emersion of the eclipse, such phase will be enveloped in the shadow, and consequently invisible; similarly when either phase with respect to the shadow transpires within the duration of the occultation, it will be intercepted from the Earth by the interposition of the disc of the planet, and likewise invisible. It is evident, that in these cases two phases will be obscured, viz. one of each phenomenon; an immersion of one and an emersion of the other. Thus, if the emersion of the occultation takes place in the shadow by its falling within the duration of the eclipse, the immersion of the eclipse will also be hidden behind the disc; and if the immersion of the occultation takes place in the shadow, the emersion from the shadow will, for the same reason, take place behind the disc; and this is a very simple and accurate way of determining the visibilities of the phases of these phenomena with reference to the Earth generally. In some rare cases, near to the conjunction or opposition of the planet to the Sun, the eclipse will take place wholly behind the disc, when of course neither phase will be visible.

CO-ORDINATES AT ECLIPSE.

	h m		h m
Immersion in shadow	13 29 ·4	Emersion from shadow	15 56 ·8
Geocentric ☌......	10 29 ·9	....	10 29 ·9
Time past ☌......	+ 2 59 ·5	Time past ☌........	+ 5 26 ·9
$x = +1·61$	D = 13	$x = +2·91$	D = 11
Const...	1 ·000		
N......	1 ·720		
D...13)	− ·720	D....11)	− ·720
	− 55		− 65
	$y = -$ ·665		$y = -$ ·655
Immersion $x = +1·61$	$y = -$ ·66	Emersion $x = +2·91$	$y = -$ ·65

These serve for the delineation of the two phases, here annexed, as they will appear through an inverting telescope.



The next eclipse of the third satellite following the preceding is on Jan. 21, when we have the middle of the eclipse, Jan. 21^d 18^h 44^m·5; and, as before, we compute

	d	h	m
Geocentric ☌........Jan.	21	14	7 ·6
Semiduration		1	18 ·3
Immersion behind disc.....	21	12	49 ·3
Emersion from disc........	21	15	25 ·9

Between these two eclipses and occultations an inferior conjunction must take place, and hence a transit both of the satellite and its shadow. For the computation of these we have, in the calculation of the preceding eclipse, $c = 224$, $C = 802$, $D = 717$, $E = 523$, to find the correction for inferior conjunction, and the work will be as follows:

TRANSITS.

I. Of the Shadow.

	d	h	m
Middle preceding eclipse Jan.	14	14	43 ·1
Middle following eclipse.....	21	18	44 ·5
Sum..	36	9	27 ·6
½ Sum..	18	4	43 ·8
Corrections....................		−	5 ·5
Inferior heliocentric ☌	18	4	38 ·3

		m
c... 224	gives	− 0 ·2
C .. 802	. . .	+ 3 ·9
D .. 717	. . .	− 8 ·6
E .. 523	. . .	− 0 ·6

(these four values together = Corrections)

			Numbers.
Jan. 18 (H) = 71°·0	(H) + H = 116°·1...	H	1679
	(H) + I = 204	I	12
	(H) + K = 114	K	5
	(H) + L = 271	L	0
		N	1696

		h	m
Semidur.		1	20 ·0
Ingress of shadow...	18	3	18 ·3
Egress of shadow...	18	5	58 ·3

II. Of the Satellite.

	d	h	m
Prec. geo. ☌.......... Jan.	14	10	29 ·9
Foll. geo. ☌..............	21	14	7 ·6
Sum....	36	0	37 ·5
½ Sum....	18	0	18 ·8
Corr. ...		—	5 ·5
Inferior geo. ☌	18	0	13 ·3
Semidur..................		1	17 ·8
Ingress of satellite	17	22	55 ·5
Egress of satellite..........	18	1	31 ·1

	h	m
Semidur. prec. occult.	1	17 ·3
Semidur. foll. ———	1	18 ·3
Semidur. transit of satellite........	1	17 ·8 mean.

When a continued series is to be computed, as in the formation of page XXI of the Nautical Almanac, it will not be requisite to find the semiduration every time. With the first satellite it will be sufficient to compute the semiduration for the first in each month, and to interpolate the others; with the second, one at the beginning and another about the middle of the month will serve; and with the third, we may compute for every third conjunction; but with the fourth satellite it will be necessary to compute it at every conjunction.

All these results are in mean astronomical time, and may readily be converted into that of sidereal, in the usual manner.

CONFIGURATIONS.

In forming the times from conjunction, we may for the first, second, and third satellite, assume our epochs near to the beginning and end of the month; but for the fourth satellite it will be best to compute them near to each conjunction. As an example, take the month of January in which they are given at 9^h, mean time; and for the first satellite use the geocentric conjunctions Jan. $2^d\ 8^h\ 13^m{\cdot}7$ and Jan. $30^d\ 15^h\ 31^m{\cdot}7$. The times nearest to these conjunctions are Jan. $2^d\ 9^h$ and Jan. $30^d\ 9^h$, and we deduce the epochs of the distances from conjunction and the daily difference as hereunder.

Time.	☌	Past ☌ = Time — ☌	Epochs.
d h m	d h m	h m	
Jan. 2 9 0	2 8 13 ·7	+ 0 46 ·3 = in Parts...........	01817
30 9 0	30 15 31 ·7	— 6 31 ·7 = in Parts — 15369 or	84631

Interval....... 28 7 18 ·0
Nearest multiple 28 7 37 ·6
— 19 ·6

Interval $28^d{\cdot}3$) — 770 in Parts (....... —

Parts.	
56502	for 24 hours.
27	Corr. to deduct.
56529	daily difference.

By using, for the second satellite, the conjunctions Jan. $1^d\ 0^h\ 51^m{\cdot}7$, Jan. 29^d $10^h\ 27^m{\cdot}7$; for the third, Jan. $0^d\ 3^h\ 28^m{\cdot}8$, Jan. $28^d\ 17^h\ 50^m{\cdot}8$; and for the fourth,

Jan. $1^d\ 7^h\ 59^m{\cdot}2$, Jan. $17^d\ 23^h\ 33^m{\cdot}0$, and Jan. $34^d\ 16^h\ 9^m{\cdot}0$, we find in the same manner the epochs and daily differences to come out as follow:

SECOND SATELLITE.

Jan. 1	Epoch 09541	daily diff. 28169	
29	—— 98287		

THIRD SATELLITE.

Jan. 0	Epoch 03209	daily diff. 13987	
28	—— 94856		

FOURTH SATELLITE.

Jan. 1	Epoch 00252	daily diff. 06006
18	—— 02350	—— 05991
34	—— 98220	

We hence proceed to fill up the following Table, by first inserting the epochs marked (*), and then applying successively the daily difference.

JANUARY, AT 9 HOURS, MEAN TIME.

Date.	56529 ☾₁		28169 ☾₂		13987 ☾₃		06006 ☾₄	
	Past ☌	Elong.	Past ☌	Elong.	Past ☌	Elong.	Past ☌	Elong.
Jan. 1	45288	106·	*09541	·119	*17196	·148	*00252	·102
2	*01817	·102	37710	124·	31183	151·	06258	·137
3	58346	89·	65879	71·	45170	116·	12264	·167
4	14875	·117	94048	· 87	59157	70·	18270	·188
5	71404	79·	22217	·134	73144	46s	24276	s196
6	27933	121·	50386	99·	87131	· 60	30282	191·
7	84462	· 82	78555	· 67	01118	·104	36288	173·
8	40991	112·	06724	·114	15105	·144	42294	145·
9	97520	· 97	34893	128·	29092	153·	48300	110·
10	54049	95·	63062	75·	43079	123·	54306	74·

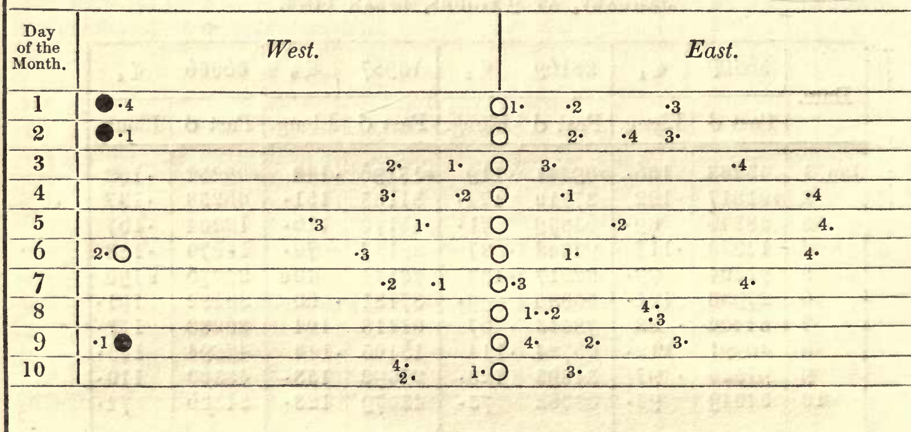
By carrying forward the successive additions of the daily differences to the distance past ☌, up to the latter epochs, which must be previously inserted, we shall find, by comparing the number which would come out by the summation with the epoch, the degree of accuracy of the work. Thus, the continued summation for the first satellite would, on the 30th day, give 84629, differing only 00002 from the epoch; the second satellite, on the 29th, would come out 98273, varying only 00014 from the epoch; the third satellite, on the 28th, would give 94845, differing only 00011; and the fourth satellite, on the 18th, would give 02354, differing only 00004. We must start again from these *epochs* for the remaining days of the month, and for the fourth satellite, we must use the new daily difference. The ten days above shown will be sufficient to illustrate the process; and it will here be only necessary to observe, that the elongations are taken out by using the nearest third figure of the argument, and that the dot is placed according to the rule given at page 12.

In mapping them, which for the Almanac is done on a scale of about 55 to the inch, we have merely to place the scale so that the division of 100 shall bisect the representation of the planet, and then to mark the places of the four satellites by means of their elongations, and lastly to place the distinguishing numeral in its proper position; keeping in mind, to look through the eclipses which occur in the course of the month, for the purpose of indicating such satellites as are immersed in the shadow at the time of the configuration. The following corresponds with the foregoing ten days, and will serve as a specimen.

JANUARY 1835.

CONFIGURATIONS OF THE SATELLITES OF JUPITER,

At 9^h, Mean Time.



Before concluding, it will be proper to observe, that the mean distances used in this paper are those which agree better than any other recorded values with the observed greatest semidurations of the eclipses, adopted in the Tables of Delambre. To correspond perfectly with these semidurations they should be 6·00, 9·49, 15·20, 26·39. They are, in fact, not yet well ascertained, and nothing seems better calculated for their accurate determination, as well as of the other elements of the orbits of the satellites, than careful observations of the contacts with the disc of the planet; and the facilities now afforded in the improved form of the Nautical Almanac will doubtless attract the attention of English observers to this as well as many other important and interesting subjects.

FIRST SATELLITE.

I. Reduction of ♃'s Annual Parallax into Synodic Time.

Degrees.	Time.
°	h m
1	0 7·1
2	0 14·2
3	0 21·2
4	0 28·3
5	0 35·4
6	0 42·5
7	0 49·6
8	0 56·6
9	1 3·7
10	1 10·8
11	1 17·9

Min.	Time.	Min.	Time.
′	m	′	m
0	0·0	30	3·5
1	0·1	31	3·7
2	0·2	32	3·8
3	0·4	33	3·9
4	0·5	34	4·0
5	0·6	35	4·1
6	0·7	36	4·2
7	0·8	37	4·4
8	0·9	38	4·5
9	1·1	39	4·6
10	1·2	40	4·7
11	1·3	41	4·8
12	1·4	42	5·0
13	1·5	43	5·1
14	1·7	44	5·2
15	1·8	45	5·3
16	1·9	46	5·4
17	2·0	47	5·5
18	2·1	48	5·7
19	2·2	49	5·8
20	2·4	50	5·9
21	2·5	51	6·0
22	2·6	52	6·1
23	2·7	53	6·3
24	2·8	54	6·4
25	3·0	55	6·5
26	3·1	56	6·6
27	3·2	57	6·7
28	3·3	58	6·8
29	3·4	59	7·0
30	3·5	60	7·1

Factor.

$$f = \cdot 005 = \frac{1}{200}$$

II. CORRECTIONS FOR INFERIOR CONJUNCTION.

Argument C, used in preceding Eclipse.

Arg. C.	Corr. C.	Diff.	Arg. C.	Corr. C.	Diff.
	m	m		m	m
0	+0·1	0·8	5000	+0·0	0·8
100	0·9	·7	5100	0·8	·8
200	1·6	·8	5200	1·6	·8
300	2·4	·7	5300	2·4	·7
400	3·1	·7	5400	3·1	·7
500	3·8	·6	5500	3·8	·6
600	4·4	·6	5600	4·4	·6
700	5·0	·4	5700	5·0	·4
800	5·4	·4	5800	5·4	·4
900	5·8	·3	5900	5·8	·3
1000	6·1	·2	6000	6·1	·2
1100	6·3	·1	6100	6·3	·1
1200	6·4	·0	6200	6·4	·0
1300	6·4	·1	6300	6·4	·1
1400	6·3	·2	6400	6·3	·2
1500	6·1	·3	6500	6·1	·3
1600	5·8	·4	6600	5·8	·4
1700	5·4	·5	6700	5·4	·5
1800	4·9	·5	6800	4·9	·6
1900	4·4	·6	6900	4·3	·6
2000	3·8	·7	7000	3·7	·7
2100	3·1	·8	7100	3·0	·7
2200	2·3	·8	7200	2·3	·8
2300	1·5	·8	7300	1·5	·8
2400	+0·7	·8	7400	+0·7	·8
2500	−0·1	·8	7500	−0·1	·7
2600	0·9	·8	7600	0·8	·8
2700	1·7	·7	7700	1·6	·8
2800	2·4	·8	7800	2·4	·7
2900	3·2	·7	7900	3·1	·7
3000	3·9	·6	8000	3·8	·6
3100	4·5	·5	8100	4·4	·5
3200	5·0	·5	8200	4·9	·5
3300	5·5	·4	8300	5·4	·4
3400	5·9	·3	8400	5·8	·3
3500	6·2	·1	8500	6·1	·2
3600	6·3	·1	8600	6·3	·1
3700	6·4	·0	8700	6·4	·0
3800	6·4	·1	8800	6·4	·1
3900	6·3	·2	8900	6·3	·2
4000	6·1	·3	9000	6·1	·3
4100	5·8	·4	9100	5·8	·4
4200	5·4	·5	9200	5·4	·5
4300	4·9	·5	9300	4·9	·6
4400	4·4	·6	9400	4·3	·6
4500	3·8	·7	9500	3·7	·7
4600	3·1	·7	9600	3·0	·7
4700	2·4	·8	9700	2·3	·8
4800	1·6	·8	9800	1·5	·8
4900	−0·8	0·8	9900	−0·7	0·8
5000	+0·0		10000	+0·1	

FIRST SATELLITE.

III. Number H. Argument (H + Long. ♃)

H + ♃		Number H.	H + ♃		Number H.	H + ♃		Number H.	H + ♃		Number H.
270°		042	315°	225°	141	0°	180°	380	45°	135°	619
271	269	042	316	224	145	1	179	386	46	134	623
272	268	042	317	223	150	2	178	392	47	133	627
273	267	042	318	222	154	3	177	398	48	132	631
274	266	043	319	221	158	4	176	403	49	131	635
275	265	043	320	220	163	5	175	409	50	130	639
276	264	044	321	219	167	6	174	415	51	129	643
277	263	044	322	218	172	7	173	421	52	128	646
278	262	045	323	217	177	8	172	427	53	127	650
279	261	046	324	216	181	9	171	433	54	126	653
280	260	047	325	215	186	10	170	439	55	125	657
281	259	048	326	214	191	11	169	444	56	124	660
282	258	049	327	213	196	12	168	450	57	123	664
283	257	051	328	212	201	13	167	456	58	122	667
284	256	052	329	211	206	14	166	462	59	121	670
285	255	053	330	210	211	15	165	467	60	120	673
286	254	055	331	209	216	16	164	473	61	119	676
287	253	057	332	208	221	17	163	479	62	118	678
288	252	058	333	207	227	18	162	484	63	117	681
289	251	060	334	206	232	19	161	490	64	116	684
290	250	062	335	205	237	20	160	496	65	115	686
291	249	064	336	204	243	21	159	501	66	114	689
292	248	067	337	203	248	22	158	507	67	113	691
293	247	069	338	202	253	23	157	512	68	112	693
294	246	071	339	201	259	24	156	517	69	111	695
295	245	074	340	200	264	25	155	523	70	110	698
296	244	076	341	199	270	26	154	528	71	109	700
297	243	079	342	198	276	27	153	533	72	108	702
298	242	082	343	197	281	28	152	539	73	107	703
299	241	084	344	196	287	29	151	544	74	106	705
300	240	087	345	195	293	30	150	549	75	105	707
301	239	090	346	194	298	31	149	554	76	104	708
302	238	093	347	193	304	32	148	559	77	103	709
303	237	096	348	192	310	33	147	564	78	102	711
304	236	100	349	191	316	34	146	569	79	101	712
305	235	103	350	190	321	35	145	574	80	100	713
306	234	106	351	189	327	36	144	579	81	99	714
307	233	110	352	188	333	37	143	583	82	98	715
308	232	114	353	187	339	38	142	588	83	97	716
309	231	117	354	186	345	39	141	593	84	96	716
310	230	121	355	185	351	40	140	597	85	95	717
311	229	125	356	184	357	41	139	602	86	94	717
312	228	129	357	183	362	42	138	606	87	93	718
313	227	133	358	182	368	43	137	610	88	92	718
314	226	137	359	181	374	44	136	615	89	91	718
315	225	141	360	180	380	45	135	619	90		718

FIRST SATELLITE.

IV. Number β

Arg. ⊙ — ☊ of Jupiter.			Divisor for Corr. β'		
⊙ — ☊		β'	♃ — ⊙		F
+	—				
°	°		°	°	
0	360	0	0	360	— 6
10	350	5	10	350	6
20	340	9	20	340	7
30	330	14	30	330	7
40	320	18	40	320	8
50	310	21	50	310	10
60	300	24	60	300	13
70	290	26	70	290	21
80	280	27	80	280	—67
90	270	28	90	270	+56
100	260	27	100	260	19
110	250	26	110	250	11
120	240	24	120	240	8
130	230	21	130	230	6
140	220	18	140	220	5
150	210	14	150	210	5
160	200	9	160	200	4
170	190	5	170	190	4
180	180	0	180	180	+ 4

V. SEMIDURATION. Argument N.

N		Semidur.	N		Semidur.	N		Semidur.
		h m			h m			h m
000	760	1 4 ·9	130	630	1 7 ·9	260	500	1 9 ·7
010	750	1 5 ·2	140	620	1 8 ·1	270	490	1 9 ·7
020	740	1 5 ·5	150	610	1 8 ·3	280	480	1 9 ·8
030	730	1 5 ·7	160	600	1 8 ·5	290	470	1 9 ·9
040	720	1 6 ·0	170	590	1 8 ·6	300	460	1 9 ·9
050	710	1 6 ·2	180	580	1 8 ·7	310	450	1 10 ·0
060	700	1 6 ·5	190	570	1 8 ·9	320	440	1 10 ·0
070	690	1 6 ·7	200	560	1 9 ·0	330	430	1 10 ·1
080	680	1 6 ·9	210	550	1 9 ·1	340	420	1 10 ·1
090	670	1 7 ·2	220	540	1 9 ·3	350	410	1 10 ·1
100	660	1 7 ·4	230	530	1 9 ·4	360	400	1 10 ·2
110	650	1 7 ·5	240	520	1 9 ·5	370	390	1 10 ·2
120	640	1 7 ·7	250	510	1 9 ·6	380	380	1 10 ·2
130	630	1 7 ·9	260	500	1 9 ·7			

For *Satellite*, use *Geo.* Long. ♃ and Arg. N = Numb. $\left(H + \beta' + \frac{\beta'}{F}\right)$

For *Shadow*, use *Hel.* Long. ♃ and Arg. N = Numb. H.

SECOND SATELLITE.

I. Reduction of ♃'s Annual Parallax into Synodic Time.

Degrees.	Time.	
°	h	m
1	0	14·2
2	0	28·4
3	0	42·7
4	0	56·9
5	1	11·1
6	1	25·3
7	1	39·5
8	1	53·7
9	2	8·0
10	2	22·2
11	2	36·4

Min.	Time.	Min.	Time.
′	m	′	m
0	0·0	30	7·1
1	0·2	31	7·3
2	0·5	32	7·6
3	0·7	33	7·8
4	0·9	34	8·1
5	1·2	35	8·3
6	1·4	36	8·5
7	1·7	37	8·8
8	1·9	38	9·0
9	2·1	39	9·2
10	2·4	40	9·5
11	2·6	41	9·7
12	2·8	42	10·0
13	3·1	43	10·2
14	3·3	44	10·4
15	3·6	45	10·7
16	3·8	46	10·9
17	4·0	47	11·1
18	4·3	48	11·4
19	4·5	49	11·6
20	4·7	50	11·9
21	5·0	51	12·1
22	5·2	52	12·3
23	5·5	53	12·6
24	5·7	54	12·8
25	5·9	55	13·0
26	6·2	56	13·3
27	6·4	57	13·5
28	6·6	58	13·7
29	6·9	59	14·0
30	7·1	60	14·2

Factor.

$$f = \cdot 01 = \frac{1}{100}$$

II. CORRECTIONS FOR INFERIOR CONJUNCTION.

Arguments C, D, E, used in preceding Eclipse.

Arg. C.	Cor. C.	Diff.	Arg. C.	Cor. C.	Diff.	Arg.	Cor. D.	Cor. E.	Arg.	Cor. D.	Cor. E.
	m	m		m	m		m	m		m	m
0	+ 1·0	3·8	5000	+ 0·5	3·8	0	+ ·0	+ ·0	500	− ·0	− ·0
100	4·8	3·8	5100	4·3	3·7	10	·1	·0	510	·1	·0
200	8·6	3·6	5200	8·0	3·7	20	·1	·1	520	·1	·1
300	12·2	3·4	5300	11·7	3·4	30	·2	·1	530	·2	·1
400	15·6	3·2	5400	15·1	3·2	40	·2	·1	540	·2	·1
500	18·8	2·8	5500	18·3	2·9	50	·3	·1	550	·3	·1
600	21·6	2·6	5600	21·2	2·6	60	·3	·2	560	·3	·2
700	24·2	2·1	5700	23·8	2·2	70	·4	·2	570	·4	·2
800	26·3	1·7	5800	26·0	1·8	80	·5	·2	580	·5	·2
900	28·0	1·3	5900	27·8	1·3	90	·5	·2	590	·5	·2
1000	29·3	0·8	6000	29·1	0·9	100	·5	·2	600	·5	·2
1100	30·1	0·4	6100	30·0	0·4	110	·6	·3	610	·6	·3
1200	30·5	0·2	6200	30·4	0·0	120	·6	·3	620	·6	·3
1300	30·3	0·6	6300	30·4	0·5	130	·7	·3	630	·7	·3
1400	29·7	1·1	6400	29·9	1·0	140	·7	·3	640	·7	·3
1500	28·6	1·5	6500	28·9	1·5	150	·8	·3	650	·8	·3
1600	27·1	1·9	6600	27·4	1·9	160	·8	·3	660	·8	·3
1700	25·2	2·4	6700	25·5	2·3	170	·9	·4	670	·9	·4
1800	22·8	2·7	6800	23·2	2·7	180	·9	·4	680	·9	·4
1900	20·1	3·1	6900	20·5	2·9	190	·9	·4	690	·9	·4
2000	17·0	3·3	7000	17·6	3·3	200	·9	·4	700	·9	·4
2100	13·7	3·5	7100	14·3	3·5	210	·9	·4	710	·9	·4
2200	10·2	3·6	7200	10·8	3·7	220	·9	·4	720	·9	·4
2300	6·6	3·8	7300	7·1	3·8	230	·9	·4	730	·9	·4
2400	+ 2·8	3·8	7400	+ 3·3	3·8	240	·9	·4	740	·9	·4
2500	− 1·0	3·9	7500	− 0·5	3·8	250	·9	·4	750	·9	·4
2600	4·9	3·7	7600	4·3	3·8	260	·9	·4	760	·9	·4
2700	8·6	3·6	7700	8·1	3·6	270	·9	·4	770	·9	·4
2800	12·2	3·4	7800	11·7	3·4	280	·9	·4	780	·9	·4
2900	15·6	3·2	7900	15·1	3·2	290	·9	·4	790	·9	·4
3000	18·8	2·8	8000	18·3	2·9	300	·9	·4	800	·9	·4
3100	21·6	2·5	8100	21·2	2·5	310	·9	·4	810	·9	·4
3200	24·1	2·2	8200	23·7	2·2	320	·9	·4	820	·9	·4
3300	26·3	1·8	8300	25·9	1·8	330	·8	·4	830	·8	·4
3400	28·1	1·3	8400	27·7	1·3	340	·8	·3	840	·8	·3
3500	29·4	0·8	8500	29·0	0·9	350	·8	·3	850	·8	·3
3600	30·2	0·4	8600	29·9	0·4	360	·7	·3	860	·7	·3
3700	30·6	0·1	8700	30·3	0·2	370	·7	·3	870	·7	·3
3800	30·5	0·5	8800	30·1	0·5	380	·6	·3	880	·6	·3
3900	30·0	1·1	8900	29·6	1·1	390	·6	·3	890	·6	·3
4000	28·9	1·5	9000	28·5	1·5	400	·6	·2	900	·6	·2
4100	27·4	1·9	9100	27·0	1·9	410	·5	·2	910	·5	·2
4200	25·5	2·3	9200	25·1	2·3	420	·5	·2	920	·5	·2
4300	23·2	2·7	9300	22·8	2·7	430	·4	·2	930	·4	·2
4400	20·5	3·0	9400	20·1	3·0	440	·3	·1	940	·3	·1
4500	17·5	3·2	9500	17·1	3·3	450	·3	·1	950	·3	·1
4600	14·3	3·5	9600	13·8	3·5	460	·2	·1	960	·2	·1
4700	10·8	3·7	9700	10·3	3·7	470	·2	·1	970	·2	·1
4800	7·1	3·8	9800	6·6	3·8	480	·1	·1	980	·1	·1
4900	− 3·3	3·8	9900	− 2·8	3·8	490	·1	·0	990	·1	·0
5000	+ 0·5		10000	+ 1·0		500	+ ·0	+ ·0	1000	− ·0	− ·0

SECOND SATELLITE.

I^{a}. Number X.
For correction of mean Synodic Time.
Arg. C used in corresponding Eclipse.

C, or C — 5000.				X
—	+	+	—	
0	2500	2500	5000	·019
50	2450	2550	4950	·019
100	2400	2600	4900	·019
150	2350	2650	4850	·018
200	2300	2700	4800	·018
250	2250	2750	4750	·018
300	2200	2800	4700	·017
350	2150	2850	4650	·017
400	2100	2900	4600	·016
450	2050	2950	4550	·016
500	2000	3000	4500	·015
550	1950	3050	4450	·014
600	1900	3100	4400	·014
650	1850	3150	4350	·013
700	1800	3200	4300	·012
750	1750	3250	4250	·011
800	1700	3300	4200	·010
850	1650	3350	4150	·009
900	1600	3400	4100	·008
950	1550	3450	4050	·007
1000	1500	3500	4000	·006
1050	1450	3550	3950	·005
1100	1400	3600	3900	·004
1150	1350	3650	3850	·002
1200	1300	3700	3800	·001
1250	1250	3750	3750	·000

SECOND SATELLITE.

III. Number H. Argument (H + Long. ♃)

H + ♃		Number H.	H + ♃		Number H.	H + ♃		Number H.	H + ♃		Number H.
°			°	°		°	°		°	°	
270		078	315	225	235	0	180	613	45	135	991
271	269	078	316	224	242	1	179	622	46	134	998
272	268	078	317	223	248	2	178	632	47	133	1004
273	267	079	318	222	255	3	177	641	48	132	1010
274	266	079	319	221	262	4	176	650	49	131	1017
275	265	080	320	220	269	5	175	660	50	130	1023
276	264	081	321	219	276	6	174	669	51	129	1029
277	263	082	322	218	284	7	173	678	52	128	1034
278	262	083	323	217	291	8	172	687	53	127	1040
279	261	085	324	216	299	9	171	696	54	126	1046
280	260	086	325	215	306	10	170	706	55	125	1051
281	259	088	326	214	314	11	169	715	56	124	1056
282	258	090	327	213	322	12	168	724	57	123	1061
283	257	092	328	212	330	13	167	733	58	122	1066
284	256	094	329	211	338	14	166	742	59	121	1071
285	255	096	330	210	346	15	165	751	60	120	1076
286	254	099	331	209	354	16	164	760	61	119	1081
287	253	101	332	208	362	17	163	769	62	118	1085
288	252	104	333	207	370	18	162	778	63	117	1090
289	251	107	334	206	379	19	161	787	64	116	1094
290	250	110	335	205	387	20	160	796	65	115	1098
291	249	114	336	204	396	21	159	805	66	114	1101
292	248	117	337	203	404	22	158	813	67	113	1105
293	247	121	338	202	413	23	157	822	68	112	1109
294	246	125	339	201	421	24	156	830	69	111	1112
295	245	128	340	200	430	25	155	839	70	110	1116
296	244	132	341	199	439	26	154	847	71	109	1119
297	243	136	342	198	448	27	153	856	72	108	1122
298	242	141	343	197	457	28	152	864	73	107	1125
299	241	145	344	196	466	29	151	872	74	106	1127
300	240	150	345	195	475	30	150	880	75	105	1130
301	239	155	346	194	484	31	149	888	76	104	1132
302	238	160	347	193	493	32	148	896	77	103	1134
303	237	165	348	192	502	33	147	904	78	102	1136
304	236	170	349	191	511	34	146	912	79	101	1138
305	235	175	350	190	520	35	145	920	80	100	1140
306	234	180	351	189	529	36	144	927	81	99	1141
307	233	186	352	188	539	37	143	935	82	98	1143
308	232	192	353	187	548	38	142	942	83	97	1144
309	231	197	354	186	557	39	141	950	84	96	1145
310	230	203	355	185	566	40	140	957	85	95	1146
311	229	209	356	184	576	41	139	964	86	94	1147
312	228	216	357	183	585	42	138	971	87	93	1147
313	227	222	358	182	594	43	137	978	88	92	1148
314	226	228	359	181	604	44	136	984	89	91	1148
315	225	235	360	180	613	45	135	991	90		1148

SECOND SATELLITE.

IIIª. Numbers K and L.
Arguments K + ♃ and L + ♃.

Com. Arg.		K	L
270°		12	162
280	260	12	160
290	250	11	157
300	240	11	151
310	230	10	143
320	220	10	133
330	210	9	121
340	200	8	109
350	190	7	95
0	180	6	81
10	170	5	67
20	160	4	53
30	150	3	41
40	140	2	29
50	130	2	19
60	120	1	11
70	110	1	5
80	100	0	2
90		0	0

IV. Number β

Arg. ⊙ − ☊ of Jupiter.			Divisor for Corr. β′		
⊙ − ☊		β′	♃ − ⊙		F
+	−				
0°	360°	0	0°	360°	− 6
10	350	8	10	350	6
20	340	15	20	340	7
30	330	22	30	330	7
40	320	28	40	320	8
50	310	34	50	310	10
60	300	38	60	300	13
70	290	41	70	290	21
80	280	43	80	280	−67
90	270	44	90	270	+56
100	260	43	100	260	19
110	250	41	110	250	11
120	240	38	120	240	8
130	230	34	130	230	6
140	220	28	140	220	5
150	210	22	150	210	5
160	200	15	160	200	4
170	190	8	170	190	4
180	180	0	180	180	+ 4

V. SEMIDURATION. Argument N.

N		Semidur.	N		Semidur.	N		Semidur.	N		Semidur.
		h m			h m			h m			h m
020	1380	1 4 ·7	190	1210	1 15 ·9	360	1040	1 23 ·0	530	870	1 27 ·0
030	1370	1 5 ·5	200	1200	1 16 ·4	370	1030	1 23 ·3	540	860	1 27 ·1
040	1360	1 6 ·3	210	1190	1 16 ·9	380	1020	1 23 ·6	550	850	1 27 ·2
050	1350	1 7 ·1	220	1180	1 17 ·4	390	1010	1 23 ·9	560	840	1 27 ·4
060	1340	1 7 ·8	230	1170	1 17 ·9	400	1000	1 24 ·2	570	830	1 27 ·5
070	1330	1 8 ·5	240	1160	1 18 ·3	410	990	1 24 ·4	580	820	1 27 ·6
080	1320	1 9 ·2	250	1150	1 18 ·8	420	980	1 24 ·7	590	810	1 27 ·7
090	1310	1 9 ·9	260	1140	1 19 ·2	430	970	1 25 ·0	600	800	1 27 ·8
100	1300	1 10 ·6	270	1130	1 19 ·7	440	960	1 25 ·2	610	790	1 27 ·9
110	1290	1 11 ·2	280	1120	1 20 ·1	450	950	1 25 ·4	620	780	1 28 ·0
120	1280	1 11 ·9	290	1110	1 20 ·5	460	940	1 25 ·7	630	770	1 28 ·0
130	1270	1 12 ·5	300	1100	1 20 ·9	470	930	1 25 ·9	640	760	1 28 ·1
140	1260	1 13 ·1	310	1090	1 21 ·2	480	920	1 26 ·1	650	750	1 28 ·1
150	1250	1 13 ·7	320	1080	1 21 ·6	490	910	1 26 ·3	660	740	1 28 ·2
160	1240	1 14 ·3	330	1070	1 22 ·0	500	900	1 26 ·5	670	730	1 28 ·2
170	1230	1 14 ·8	340	1060	1 22 ·3	510	890	1 26 ·6	680	720	1 28 ·2
180	1220	1 15 ·4	350	1050	1 22 ·6	520	880	1 26 ·8	690	710	1 28 ·2
190	1210	1 15 ·9	360	1040	1 23 ·0	530	870	1 27 ·0	700	700	1 28 ·2

For *Satellite*, use *Geo.* Long. ♃ and Arg. N = Numb. $(H + K + L + \beta' + \frac{\beta'}{F})$

For *Shadow*, use *Hel.* Long. ♃ and Arg. N = Numb. $(H + K + L)$

THIRD SATELLITE.

I. Reduction of ♃'s Annual Parallax into Synodic Time.

Degrees.	Time.	
°	h	m
1	0	28·7
2	0	57·3
3	1	26·0
4	1	54·7
5	2	23·3
6	2	52·0
7	3	20·7
8	3	49·3
9	4	18·0
10	4	46·7
11	5	15·3

Min.	Time.	Min.	Time.
′	m	′	m
0	0·0	30	14·3
1	0·5	31	14·8
2	1·0	32	15·3
3	1·4	33	15·8
4	1·9	34	16·2
5	2·4	35	16·7
6	2·9	36	17·2
7	3·3	37	17·7
8	3·8	38	18·2
9	4·3	39	18·6
10	4·8	40	19·1
11	5·3	41	19·6
12	5·7	42	20·1
13	6·2	43	20·5
14	6·7	44	21·0
15	7·2	45	21·5
16	7·6	46	22·0
17	8·1	47	22·5
18	8·6	48	22·9
19	9·1	49	23·4
20	9·6	50	23·9
21	10·0	51	24·4
22	10·5	52	24·8
23	11·0	53	25·3
24	11·5	54	25·8
25	11·9	55	26·3
26	12·4	56	26·8
27	12·9	57	27·2
28	13·4	58	27·7
29	13·9	59	28·2
30	14·3	60	28·7

Factor.

$$f = \cdot 02 = \frac{1}{50}$$

II. CORRECTIONS FOR INFERIOR CONJUNCTION.
Arguments c, C, D, E, used in preceding Eclipse.

Arg.	Corr. c.	Corr. C.	Corr. D.	Corr. E.	Arg.	Corr. c.	Corr. C.	Corr. D.	Corr. E.
	m	m	m	m		m	m	m	m
0	+0·4	−0·2	+0·1	+0·0	500	+0·2	+0·2	−0·1	−0·0
10	0·5	0·5	0·6	0·3	510	0·3	0·5	0·6	0·3
20	0·6	0·7	1·2	0·5	520	0·4	0·7	1·2	0·5
30	0·6	1·0	1·7	0·8	530	0·5	1·0	1·7	0·8
40	0·7	1·3	2·2	1·0	540	0·6	1·3	2·2	1·0
50	0·7	1·5	2·8	1·2	550	0·6	1·5	2·8	1·2
60	0·7	1·7	3·3	1·5	560	0·6	1·7	3·3	1·5
70	0·7	2·0	3·8	1·7	570	0·7	2·0	3·8	1·7
80	0·7	2·2	4·3	1·9	580	0·7	2·2	4·3	1·9
90	0·7	2·4	4·8	2·1	590	0·7	2·4	4·8	2·1
100	0·7	2·6	5·2	2·3	600	0·7	2·6	5·2	2·3
110	0·7	2·8	5·7	2·5	610	0·7	2·8	5·7	2·5
120	0·6	3·0	6·1	2·7	620	0·7	3·0	6·1	2·7
130	0·6	3·2	6·4	2·9	630	0·7	3·2	6·4	2·9
140	0·6	3·4	6·8	3·0	640	0·7	3·4	6·8	3·0
150	0·5	3·5	7·1	3·2	650	0·6	3·5	7·1	3·2
160	0·4	3·6	7·4	3·3	660	0·6	3·6	7·4	3·3
170	0·3	3·7	7·7	3·4	670	0·5	3·6	7·7	3·4
180	0·2	3·8	8·0	3·5	680	0·4	3·8	8·0	3·5
190	+0·1	3·9	8·2	3·6	690	0·4	3·9	8·2	3·6
200	0·0	4·0	8·4	3·7	700	0·3	4·0	8·4	3·7
210	−0·1	4·1	8·5	3·8	710	0·2	4·1	8·5	3·8
220	0·2	4·1	8·6	3·8	720	+0·1	4·1	8·6	3·8
230	0·3	4·1	8·7	3·9	730	0·0	4·1	8·7	3·9
240	0·4	4·1	8·8	3·9	740	−0·1	4·1	8·8	3·9
250	0·5	4·1	8·8	3·9	750	0·2	4·1	8·8	3·9
260	0·5	4·1	8·8	3·9	760	0·2	4·1	8·8	3·9
270	0·6	4·1	8·7	3·9	770	0·3	4·1	8·7	3·9
280	0·7	4·0	8·6	3·8	780	0·3	4·0	8·6	3·8
290	0·7	4·0	8·5	3·8	790	0·4	4·0	8·5	3·8
300	0·8	3·9	8·4	3·7	800	0·4	3·9	8·4	3·7
310	0·8	3·8	8·2	3·6	810	0·5	3·8	8·2	3·6
320	0·9	3·7	7·9	3·5	820	0·5	3·7	7·9	3·5
330	0·9	3·6	7·7	3·4	830	0·5	3·6	7·7	3·4
340	0·9	3·4	7·4	3·3	840	0·5	3·4	7·4	3·3
350	0·9	3·2	7·1	3·1	850	0·5	3·2	7·1	3·1
360	0·9	3·1	6·7	3·0	860	0·5	3·1	6·7	3·0
370	0·9	2·9	6·4	2·8	870	0·5	2·9	6·4	2·8
380	0·8	2·7	6·0	2·7	880	0·4	2·7	6·0	2·7
390	0·8	2·5	5·6	2·5	890	0·4	2·5	5·6	2·5
400	0·7	2·3	5·1	2·3	900	0·3	2·3	5·1	2·3
410	0·6	2·1	4·7	2·1	910	0·3	2·1	4·7	2·1
420	0·5	1·8	4·2	1·8	920	0·2	1·8	4·2	1·8
430	0·4	1·6	3·7	1·6	930	0·1	1·6	3·7	1·6
440	0·3	1·4	3·2	1·4	940	−0·1	1·4	3·2	1·4
450	0·2	1·1	2·7	1·2	950	0·0	1·1	2·7	1·2
460	0·2	0·8	2·1	1·0	960	+0·1	0·8	2·1	1·0
470	−0·1	0·6	1·6	0·7	970	0·2	0·6	1·6	0·7
480	+0·1	0·3	1·1	0·5	980	0·3	0·3	1·1	0·5
490	0·2	−0·1	+0·5	0·2	990	0·3	+0·1	−0·5	0·2
500	+0·2	+0·2	−0·1	+0·0	1000	+0·4	−0·2	+0·1	−0·0

THIRD SATELLITE.

III. Number H. Argument (H + Long. ♃)

H + ♃		Number H.	H + ♃		Number H.	H + ♃		Number H.	H + ♃		Number H.
270°		096	315°	225°	340	0°	180°	930	45°	135°	1520
271	269	096	316	224	350	1	179	945	46	134	1530
272	268	096	317	223	361	2	178	959	47	133	1540
273	267	097	318	222	372	3	177	974	48	132	1550
274	266	098	319	221	383	4	176	988	49	131	1560
275	265	099	320	220	394	5	175	1003	50	130	1569
276	264	100	321	219	405	6	174	1017	51	129	1578
277	263	102	322	218	416	7	173	1032	52	128	1587
278	262	104	323	217	428	8	172	1046	53	127	1596
279	261	106	324	216	440	9	171	1061	54	126	1605
280	260	109	325	215	452	10	170	1075	55	125	1613
281	259	111	326	214	464	11	169	1089	56	124	1622
282	258	114	327	213	476	12	168	1104	57	123	1630
283	257	117	328	212	488	13	167	1118	58	122	1637
284	256	120	329	211	500	14	166	1132	59	121	1645
285	255	124	330	210	513	15	165	1146	60	120	1652
286	254	128	331	209	526	16	164	1160	61	119	1660
287	253	132	332	208	538	17	163	1174	62	118	1667
288	252	137	333	207	551	18	162	1188	63	117	1673
289	251	141	334	206	564	19	161	1202	64	116	1680
290	250	146	335	205	577	20	160	1215	65	115	1686
291	249	151	336	204	591	21	159	1229	66	114	1692
292	248	157	337	203	604	22	158	1243	67	113	1698
293	247	162	338	202	617	23	157	1256	68	112	1703
294	246	168	339	201	631	24	156	1269	69	111	1709
295	245	174	340	200	645	25	155	1283	70	110	1714
296	244	180	341	199	658	26	154	1296	71	109	1719
297	243	187	342	198	672	27	153	1309	72	108	1723
298	242	193	343	197	686	28	152	1322	73	107	1728
299	241	200	344	196	700	29	151	1334	74	106	1732
300	240	208	345	195	714	30	150	1347	75	105	1736
301	239	215	346	194	728	31	149	1360	76	104	1740
302	238	223	347	193	742	32	148	1372	77	103	1743
303	237	230	348	192	756	33	147	1384	78	102	1746
304	236	238	349	191	771	34	146	1396	79	101	1749
305	235	247	350	190	785	35	145	1408	80	100	1751
306	234	255	351	189	799	36	144	1420	81	99	1754
307	233	264	352	188	814	37	143	1432	82	98	1756
308	232	273	353	187	828	38	142	1444	83	97	1758
309	231	282	354	186	843	39	141	1455	84	96	1760
310	230	291	355	185	857	40	140	1466	85	95	1761
311	229	300	356	184	872	41	139	1477	86	94	1762
312	228	310	357	183	886	42	138	1488	87	93	1763
313	227	320	358	182	901	43	137	1499	88	92	1764
314	226	330	359	181	915	44	136	1510	89	91	1764
315	225	340	360	180	930	45	135	1520	90		1764

THIRD SATELLITE.

IIIa. Numbers I, K, and L.

Arguments I + ♃, K + ♃, L + ♃.

Com. Arg.		I	K	L
270°		18	114	0
280	260	18	113	0
290	250	17	111	0
300	240	16	106	0
310	230	16	101	1
320	220	15	94	1
330	210	13	86	2
340	200	12	76	2
350	190	11	67	3
0	180	9	57	4
10	170	7	47	5
20	160	6	38	6
30	150	5	28	6
40	140	3	20	7
50	130	2	13	7
60	120	2	8	8
70	110	1	3	8
80	100	0	1	8
90		0	0	8

IV. Number β

Arg. ☉ − ☊ of Jupiter.			Divisor for Corr. β′		
☉ − ☊		β′	♃ − ☉		F
+ °	− °		°	°	
0	360	0	0	360	− 6
10	350	12	10	350	6
20	340	24	20	340	7
30	330	35	30	330	7
40	320	45	40	320	8
50	310	54	50	310	10
60	300	61	60	300	13
70	290	66	70	290	21
80	280	69	80	280	−67
90	270	70	90	270	+56
100	260	69	100	260	19
110	250	66	110	250	11
120	240	61	120	240	8
130	230	54	130	230	6
140	220	45	140	220	5
150	210	35	150	210	5
160	200	24	160	200	4
170	190	12	170	190	4
180	180	0	180	180	+ 4

THIRD SATELLITE.

V. SEMIDURATION. Argument N.

N		Semidur.	N		Semidur.	N		Semidur.
		h m			h m			h m
000	2000	0 0 ·0	340	1660	1 23 ·7	680	1320	1 45 ·6
010	1990	0 15 ·7	350	1650	1 24 ·7	690	1310	1 46 ·0
020	1980	0 22 ·2	360	1640	1 25 ·6	700	1300	1 46 ·3
030	1970	0 27 ·1	370	1630	1 26 ·6	710	1290	1 46 ·7
040	1960	0 31 ·2	380	1620	1 27 ·5	720	1280	1 47 ·0
050	1950	0 34 ·8	390	1610	1 28 ·3	730	1270	1 47 ·3
060	1940	0 38 ·0	400	1600	1 29 ·2	740	1260	1 47 ·6
070	1930	0 40 ·9	410	1590	1 30 ·0	750	1250	1 47 ·9
080	1920	0 43 ·7	420	1580	1 30 ·8	760	1240	1 48 ·2
090	1910	0 46 ·2	430	1570	1 31 ·6	770	1230	1 48 ·5
100	1900	0 48 ·6	440	1560	1 32 ·3	780	1220	1 48 ·7
110	1890	0 50 ·8	450	1550	1 33 ·1	790	1210	1 49 ·0
120	1880	0 52 ·9	460	1540	1 33 ·8	800	1200	1 49 ·2
130	1870	0 55 ·0	470	1530	1 34 ·5	810	1190	1 49 ·4
140	1860	0 56 ·9	480	1520	1 35 ·2	820	1180	1 49 ·6
150	1850	0 58 ·7	490	1510	1 35 ·9	830	1170	1 49 ·8
160	1840	1 0 ·5	500	1500	1 36 ·5	840	1160	1 50 ·0
170	1830	1 2 ·2	510	1490	1 37 ·2	850	1150	1 50 ·2
180	1820	1 3 ·8	520	1480	1 37 ·8	860	1140	1 50 ·4
190	1810	1 5 ·4	530	1470	1 38 ·4	870	1130	1 50 ·5
200	1800	1 6 ·9	540	1460	1 39 ·0	880	1120	1 50 ·7
210	1790	1 8 ·3	550	1450	1 39 ·5	890	1110	1 50 ·8
220	1780	1 9 ·7	560	1440	1 40 ·1	900	1100	1 50 ·9
230	1770	1 11 ·1	570	1430	1 40 ·6	910	1090	1 51 ·0
240	1760	1 12 ·4	580	1420	1 41 ·2	920	1080	1 51 ·1
250	1750	1 13 ·7	590	1410	1 41 ·7	930	1070	1 51 ·2
260	1740	1 15 ·0	600	1400	1 42 ·2	940	1060	1 51 ·2
270	1730	1 16 ·2	610	1390	1 42 ·6	950	1050	1 51 ·3
280	1720	1 17 ·4	620	1380	1 43 ·1	960	1040	1 51 ·4
290	1710	1 18 ·5	630	1370	1 43 ·6	970	1030	1 51 ·4
300	1700	1 19 ·6	640	1360	1 44 ·0	980	1020	1 51 ·4
310	1690	1 20 ·7	650	1350	1 44 ·4	990	1010	1 51 ·5
320	1680	1 21 ·7	660	1340	1 44 ·8	1000	1000	1 51 ·5
330	1670	1 22 ·7	670	1330	1 45 ·2			
340	1660	1 23 ·7	680	1320	1 45 ·6			

For *Satellite*, use *Geo.* Long. ♃ and Arg. N = Numb. $(H+I+K+L+\beta'+\frac{\beta'}{F})$

For *Shadow*, use *Hel.* Long. ♃ and Arg. N = Numb. $(H+I+K+L)$

FOURTH SATELLITE.

I. Reduction of ♃'s Annual Parallax into Synodic Time.

Degrees.	Time.	
°	h	m
1	1	7 ·0
2	2	14 ·0
3	3	21 ·0
4	4	28 ·1
5	5	35 ·1
6	6	42 ·1
7	7	49 ·1
8	8	56 ·1
9	10	3 ·1
10	11	10 ·1
11	12	17 ·2

Min.	Time.	Min.	Time.
′	m	′	m
0	0 ·0	30	33 ·5
1	1 ·1	31	34 ·6
2	2 ·2	32	35 ·7
3	3 ·4	33	36 ·9
4	4 ·5	34	38 ·0
5	5 ·6	35	39 ·1
6	6 ·7	36	40 ·2
7	7 ·8	37	41 ·3
8	8 ·9	38	42 ·4
9	10 ·1	39	43 ·6
10	11 ·2	40	44 ·7
11	12 ·3	41	45 ·8
12	13 ·4	42	46 ·9
13	14 ·5	43	48 ·0
14	15 ·6	44	49 ·1
15	16 ·8	45	50 ·3
16	17 ·9	46	51 ·4
17	19 ·0	47	52 ·5
18	20 ·1	48	53 ·6
19	21 ·2	49	54 ·7
20	22 ·3	50	55 ·9
21	23 ·5	51	57 ·0
22	24 ·6	52	58 ·1
23	25 ·7	53	59 ·2
24	26 ·8	54	60 ·3
25	27 ·9	55	61 ·4
26	29 ·0	56	62 ·6
27	30 ·2	57	63 ·7
28	31 ·3	58	64 ·8
29	32 ·4	59	65 ·9
30	33 ·5	60	67 ·0

Factor.
f = ·047

II. CORRECTIONS FOR INFERIOR CONJUNCTION.
Arguments C, D, E, used in preceding Eclipse.

Arg. C.		Cor. C.		Diff.	Arg.	Cor. D.	Cor. E.	Arg.	Cor. D.	Cor. E.
+	−	h	m	m		m	m		m	m
0	5000	0	1 ·3	7 ·0	0	−0 ·0	− ·1	500	+0 ·0	+ ·3
100	5100	0	8 ·3	6 ·9	10	0 ·2	·2	510	0 ·2	·3
200	5200	0	15 ·2	6 ·9	20	0 ·3	·2	520	0 ·3	·3
300	5300	0	22 ·1	6 ·8	30	0 ·5	·2	530	0 ·5	·3
400	5400	0	28 ·9	6 ·6	40	0 ·7	·2	540	0 ·7	·3
500	5500	0	35 ·5	6 ·6	50	0 ·8	·2	550	0 ·8	·3
600	5600	0	42 ·1	6 ·3	60	1 ·0	·3	560	1 ·0	·3
700	5700	0	48 ·4	6 ·2	70	1 ·1	·3	570	1 ·1	·3
800	5800	0	54 ·6	6 ·0	80	1 ·3	·3	580	1 ·3	·3
900	5900	1	0 ·6	5 ·7	90	1 ·4	·3	590	1 ·4	·3
1000	6000	1	6 ·3	5 ·4	100	1 ·5	·3	600	1 ·5	·3
1100	6100	1	11 ·7	5 ·2	110	1 ·7	·3	610	1 ·7	·2
1200	6200	1	16 ·9	4 ·9	120	1 ·8	·3	620	1 ·8	·2
1300	6300	1	21 ·8	4 ·5	130	1 ·9	·3	630	1 ·9	·2
1400	6400	1	26 ·3	4 ·2	140	2 ·0	·3	640	2 ·0	·2
1500	6500	1	30 ·5	3 ·9	150	2 ·1	·3	650	2 ·1	·2
1600	6600	1	34 ·4	3 ·4	160	2 ·2	·3	660	2 ·2	·1
1700	6700	1	37 ·8	3 ·1	170	2 ·3	·3	670	2 ·3	·1
1800	6800	1	40 ·9	2 ·7	180	2 ·4	·3	680	2 ·4	·1
1900	6900	1	43 ·6	2 ·3	190	2 ·5	·3	690	2 ·5	·1
2000	7000	1	45 ·9	1 ·9	200	2 ·5	·3	700	2 ·5	·1
2100	7100	1	47 ·8	1 ·4	210	2 ·6	·3	710	2 ·6	+ ·1
2200	7200	1	49 ·2	1 ·0	220	2 ·6	·3	720	2 ·6	·0
2300	7300	1	50 ·2	0 ·6	230	2 ·6	·3	730	2 ·6	·0
2400	7400	1	50 ·8	0 ·1	240	2 ·6	·3	740	2 ·6	·0
2500	7500	1	50 ·9	0 ·3	250	2 ·6	·3	750	2 ·6	·0
2600	7600	1	50 ·6	0 ·7	260	2 ·6	·2	760	2 ·6	·0
2700	7700	1	49 ·9	1 ·2	270	2 ·6	·2	770	2 ·6	·0
2800	7800	1	48 ·7	1 ·6	280	2 ·6	·2	780	2 ·6	·0
2900	7900	1	47 ·1	2 ·0	290	2 ·6	·1	790	2 ·6	·0
3000	8000	1	45 ·1	2 ·4	300	2 ·5	·1	800	2 ·5	·0
3100	8100	1	42 ·7	2 ·9	310	2 ·5	− ·1	810	2 ·5	·0
3200	8200	1	39 ·8	3 ·2	320	2 ·4	·0	820	2 ·4	·0
3300	8300	1	36 ·6	3 ·6	330	2 ·3	·0	830	2 ·3	·0
3400	8400	1	33 ·0	4 ·0	340	2 ·2	·0	840	2 ·2	·0
3500	8500	1	29 ·0	4 ·4	350	2 ·1	+ ·1	850	2 ·1	·0
3600	8600	1	24 ·6	4 ·6	360	2 ·0	·1	860	2 ·0	·0
3700	8700	1	20 ·0	5 ·0	370	1 ·9	·1	870	1 ·9	·0
3800	8800	1	15 ·0	5 ·3	380	1 ·8	·2	880	1 ·8	·0
3900	8900	1	9 ·7	5 ·6	390	1 ·7	·2	890	1 ·7	·0
4000	9000	1	4 ·1	5 ·8	400	1 ·6	·2	900	1 ·6	·0
4100	9100	0	58 ·3	6 ·0	410	1 ·4	·3	910	1 ·4	·0
4200	9200	0	52 ·3	6 ·3	420	1 ·3	·3	920	1 ·3	·0
4300	9300	0	46 ·0	6 ·4	430	1 ·1	·3	930	1 ·1	·0
4400	9400	0	39 ·6	6 ·6	440	1 ·0	·3	940	1 ·0	·0
4500	9500	0	33 ·0	6 ·7	450	0 ·8	·3	950	0 ·8	− ·1
4600	9600	0	26 ·3	6 ·9	460	0 ·6	·3	960	0 ·6	·1
4700	9700	0	19 ·4	6 ·8	470	0 ·5	·3	970	0 ·5	·1
4800	9800	0	12 ·6	6 ·9	480	0 ·3	·3	980	0 ·3	·1
4900	9900	0	5 ·7	7 ·0	490	0 ·2	·3	990	0 ·2	·1
5000	10000	0	1 ·3		500	−0 ·0	+ ·3	1000	+0 ·0	− ·1

FOURTH SATELLITE.

Iᵃ. Number X.

For correction of mean Synodic Time.

Arg. C used in corresponding Eclipse.

C				X
—	+	+	—	
00	5000	5000	10000	·015
100	4900	5100	9900	·015
200	4800	5200	9800	·014
300	4700	5300	9700	·014
400	4600	5400	9600	·014
500	4500	5500	9500	·014
600	4400	5600	9400	·014
700	4300	5700	9300	·013
800	4200	5800	9200	·013
900	4100	5900	9100	·012
1000	4000	6000	9000	·012
1100	3900	6100	8900	·011
1200	3800	6200	8800	·011
1300	3700	6300	8700	·010
1400	3600	6400	8600	·009
1500	3500	6500	8500	·009
1600	3400	6600	8400	·008
1700	3300	6700	8300	·007
1800	3200	6800	8200	·006
1900	3100	6900	8100	·005
2000	3000	7000	8000	·005
2100	2900	7100	7900	·004
2200	2800	7200	7800	·003
2300	2700	7300	7700	·002
2400	2600	7400	7600	·001
2500	2500	7500	7500	·000

FOURTH SATELLITE.

III. Number H. Argument (H + Long. ♃)

H + ♃		Number H.	H + ♃		Number H.	H + ♃		Number H.	H + ♃		Number H.
270°		550	315°	225°	933	0°	180°	1858	45°	135°	2783
271	269	550	316	224	949	1	179	1881	46	134	2799
272	268	551	317	223	966	2	178	1904	47	133	2815
273	267	552	318	222	983	3	177	1927	48	132	2830
274	266	553	319	221	1000	4	176	1949	49	131	2845
275	265	555	320	220	1017	5	175	1972	50	130	2860
276	264	557	321	219	1035	6	174	1995	51	129	2874
277	263	560	322	218	1053	7	173	2017	52	128	2888
278	262	563	323	217	1071	8	172	2040	53	127	2902
279	261	566	324	216	1089	9	171	2063	54	126	2916
280	260	570	325	215	1108	10	170	2085	55	125	2929
281	259	574	326	214	1127	11	169	2108	56	124	2942
282	258	579	327	213	1146	12	168	2130	57	123	2955
283	257	583	328	212	1165	13	167	2152	58	122	2967
284	256	588	329	211	1184	14	166	2174	59	121	2979
285	255	594	330	210	1204	15	165	2196	60	120	2990
286	254	601	331	209	1224	16	164	2218	61	119	3002
287	253	607	332	208	1244	17	163	2240	62	118	3013
288	252	614	333	207	1264	18	162	2262	63	117	3024
289	251	621	334	206	1285	19	161	2284	64	116	3034
290	250	629	335	205	1305	20	160	2305	65	115	3044
291	249	637	336	204	1326	21	159	2327	66	114	3053
292	248	645	337	203	1347	22	158	2348	67	113	3062
293	247	654	338	202	1368	23	157	2369	68	112	3071
294	246	663	339	201	1389	24	156	2390	69	111	3079
295	245	672	340	200	1411	25	155	2411	70	110	3087
296	244	682	341	199	1432	26	154	2431	71	109	3095
297	243	692	342	198	1454	27	153	2452	72	108	3102
298	242	703	343	197	1476	28	152	2472	73	107	3109
299	241	714	344	196	1498	29	151	2492	74	106	3115
300	240	726	345	195	1520	30	150	2512	75	105	3122
301	239	737	346	194	1542	31	149	2532	76	104	3128
302	238	749	347	193	1564	32	148	2551	77	103	3133
303	237	761	348	192	1586	33	147	2570	78	102	3137
304	236	774	349	191	1608	34	146	2589	79	101	3142
305	235	787	350	190	1631	35	145	2608	80	100	3146
306	234	800	351	189	1653	36	144	2627	81	99	3150
307	233	814	352	188	1676	37	143	2645	82	98	3153
308	232	828	353	187	1699	38	142	2663	83	97	3156
309	231	842	354	186	1721	39	141	2681	84	96	3159
310	230	856	355	185	1744	40	140	2699	85	95	3161
311	229	871	356	184	1767	41	139	2716	86	94	3163
312	228	886	357	183	1789	42	138	2733	87	93	3164
313	227	901	358	182	1812	43	137	2750	88	92	3165
314	226	917	359	181	1835	44	136	2767	89	91	3166
315	225	933	360	180	1858	45	135	2783	90		3166

FOURTH SATELLITE.

III^a. Numbers I and K. Arguments I+♃, K+♃.				IV. Number β. Arg. ⊙ − ☊ of Jupiter.			Divisor for Corr. β'.		
Com. Arg.		I	K	⊙ − ☊		β'	♃ − ⊙		F
				+	−				
270°		244	0	0°	360°	0	0°	360°	− 6
280	260	242	1	10	350	21	10	350	6
290	250	236	1	20	340	42	20	340	7
300	240	227	3	30	330	62	30	330	7
310	230	215	5	40	320	79	40	320	8
320	220	200	7	50	310	94	50	310	10
330	210	183	10	60	300	107	60	300	13
340	200	164	13	70	290	116	70	290	21
350	190	143	17	80	280	121	80	280	−67
0	180	122	20	90	270	123	90	270	+56
10	170	101	23	100	260	121	100	260	19
20	160	80	27	110	250	116	110	250	11
30	150	61	30	120	240	107	120	240	8
40	140	44	33	130	230	94	130	230	6
50	130	29	35	140	220	79	140	220	5·4
60	120	17	37	150	210	62	150	210	4·8
70	110	8	39	160	200	42	160	200	4·5
80	100	2	39	170	190	21	170	190	4·3
90		0	40	180	180	0	180	180	+ 4·2

FOURTH SATELLITE.

V. SEMIDURATION. Argument N.

N		Semidur.		N		Semidur.		N		Semidur.	
		h	m			h	m			h	m
1000	3000	0	0·0	1340	2660	1	51·2	1680	2320	2	20·2
1010	2990	0	20·9	1350	2650	1	52·5	1690	2310	2	20·7
1020	2980	0	29·5	1360	2640	1	53·7	1700	2300	2	21·2
1030	2970	0	36·0	1370	2630	1	54·9	1710	2290	2	21·6
1040	2960	0	41·4	1380	2620	1	56·1	1720	2280	2	22·1
1050	2950	0	46·2	1390	2610	1	57·3	1730	2270	2	22·5
1060	2940	0	50·5	1400	2600	1	58·4	1740	2260	2	22·9
1070	2930	0	54·4	1410	2590	1	59·5	1750	2250	2	23·3
1080	2920	0	58·0	1420	2580	2	0·6	1760	2240	2	23·7
1090	2910	1	1·4	1430	2570	2	1·6	1770	2230	2	24·0
1100	2900	1	4·5	1440	2560	2	2·6	1780	2220	2	24·4
1110	2890	1	7·5	1450	2550	2	3·6	1790	2210	2	24·7
1120	2880	1	10·3	1460	2540	2	4·6	1800	2200	2	25·0
1130	2870	1	13·0	1470	2530	2	5·5	1810	2190	2	25·3
1140	2860	1	15·5	1480	2520	2	6·4	1820	2180	2	25·6
1150	2850	1	18·0	1490	2510	2	7·3	1830	2170	2	25·8
1160	2840	1	20·3	1500	2500	2	8·2	1840	2160	2	26·1
1170	2830	1	22·5	1510	2490	2	9·0	1850	2150	2	26·3
1180	2820	1	24·7	1520	2480	2	9·8	1860	2140	2	26·5
1190	2810	1	26·8	1530	2470	2	10·6	1870	2130	2	26·7
1200	2800	1	28·8	1540	2460	2	11·4	1880	2120	2	26·9
1210	2790	1	30·7	1550	2450	2	12·2	1890	2110	2	27·1
1220	2780	1	32·6	1560	2440	2	12·9	1900	2100	2	27·3
1230	2770	1	34·4	1570	2430	2	13·6	1910	2090	2	27·4
1240	2760	1	36·2	1580	2420	2	14·3	1920	2080	2	27·5
1250	2750	1	37·9	1590	2410	2	15·0	1930	2070	2	27·6
1260	2740	1	39·5	1600	2400	2	15·6	1940	2060	2	27·7
1270	2730	1	41·1	1610	2390	2	16·3	1950	2050	2	27·8
1280	2720	1	42·7	1620	2380	2	16·9	1960	2040	2	27·9
1290	2710	1	44·2	1630	2370	2	17·5	1970	2030	2	27·9
1300	2700	1	45·7	1640	2360	2	18·1	1980	2020	2	28·0
1310	2690	1	47·1	1650	2350	2	18·6	1990	2010	2	28·0
1320	2680	1	48·5	1660	2340	2	19·2	2000	2000	2	28·0
1330	2670	1	49·9	1670	2330	2	19·7				
1340	2660	1	51·2	1680	2320	2	20·2				

For *Satellite*, use *Geo.* Long. ♃ and Arg. $N = \text{Numb.}\ (H + I + K + \beta' + \frac{\beta'}{F})$

For *Shadow*, use *Hel.* Long. ♃ and Arg. $N = \text{Numb.}\ (H + I + K)$

TABLES FOR FINDING THE CO-ORDINATES OF THE SATELLITES AT THE TIMES OF THEIR ECLIPSES,

OR AT ANY TIME NEAR TO THE GEOCENTRIC CONJUNCTION.

VI. CO-ORDINATES.

FIRST SATELLITE.

Time from Conjunction. h	m	x	Divisor for y.
0	0	0 ·00	14
	10	0 ·15	14
	20	0 ·29	14
	30	0 ·44	13
	40	0 ·58	13
	50	0 ·72	12
1	0	0 ·86	12
	10	1 ·00	12
	20	1 ·14	11
	30	1 ·28	11
	40	1 ·42	10
	50	1 ·55	9
2	0	1 ·69	9
	10	1 ·82	8
	20	1 ·96	8
	30	2 ·09	7
	40	2 ·23	7

$$y = \cdot380 - N - \frac{\cdot380 - N}{D}$$

SECOND SATELLITE.

Time from Conjunction. h	m	x	Divisor for y.	Time from Conjunction. h	m	x	Divisor for y.
0	0	0 ·00	14	2	10	1 ·47	12
	10	0 ·12	14		20	1 ·58	12
	20	0 ·23	14		30	1 ·69	11
	30	0 ·34	14		40	1 ·80	11
	40	0 ·45	14		50	1 ·92	11
	50	0 ·57	13	3	0	2 ·03	11
1	0	0 ·68	13		10	2 ·14	10
	10	0 ·80	13		20	2 ·25	10
	20	0 ·91	13		30	2 ·36	10
	30	1 ·02	13		40	2 ·47	9
	40	1 ·13	12		50	2 ·58	9
	50	1 ·25	12	4	0	2 ·69	9
2	0	1 ·36	12		10	2 ·80	9
	10	1 ·47	12		20	2 ·90	8

$$y = \cdot700 - N - \frac{\cdot700 - N}{D}$$

THIRD SATELLITE.

Time from Conjunction. h	m	x	Divisor for y.	Time from Conjunction. h	m	x	Divisor for y.	Time from Conjunction. h	m	x	Divisor for y.
0	0	0 ·00	14	2	40	1 ·44	13	5	20	2 ·85	11
	10	0 ·09	14		50	1 ·52	13		30	2 ·94	11
	20	0 ·18	14	3	0	1 ·61	13		40	3 ·03	11
	30	0 ·27	14		10	1 ·70	13		50	3 ·12	11
	40	0 ·36	14		20	1 ·79	13	6	0	3 ·21	11
	50	0 ·45	14		30	1 ·88	12		10	3 ·30	10
1	0	0 ·54	14		40	1 ·97	12		20	3 ·38	10
	10	0 ·63	14		50	2 ·06	12		30	3 ·47	10
	20	0 ·72	14	4	0	2 ·15	12		40	3 ·56	10
	30	0 ·81	13		10	2 ·24	12		50	3 ·64	10
	40	0 ·90	13		20	2 ·32	12	7	0	3 ·73	10
	50	0 ·99	13		30	2 ·41	12		10	3 ·82	10
2	0	1 ·08	13		40	2 ·50	12		20	3 ·90	9
	10	1 ·17	13		50	2 ·59	11		30	3 ·99	9
	20	1 ·26	13	5	0	2 ·68	11		40	4 ·08	9
	30	1 ·35	13		10	2 ·77	11		50	4 ·17	9
	40	1 ·44	13		20	2 ·85	11	8	0	4 ·25	9

$$y = 1\cdot000 - N - \frac{1\cdot000 - N}{D}$$

VI. CO-ORDINATES.

FOURTH SATELLITE.

Time from Conjunction.		x	Divisor for y.	Time from Conjunction.		x	Divisor for y.	Time from Conjunction.		x	Divisor for y.
h	m			h	m			h	m		
0	0	0 ·00	14	5	20	2 ·16	13	10	40	4 ·31	12
	10	0 ·07	14		30	2 ·23	13		50	4 ·37	12
	20	0 ·14	14		40	2 ·30	13	11	0	4 ·44	12
	30	0 ·21	14		50	2 ·36	13		10	4 ·51	12
	40	0 ·28	14	6	0	2 ·43	13		20	4 ·57	11
	50	0 ·35	14		10	2 ·50	13		30	4 ·64	11
1	0	0 ·41	14		20	2 ·56	13		40	4 ·71	11
	10	0 ·48	14		30	2 ·63	13		50	4 ·77	11
	20	0 ·55	14		40	2 ·70	13	12	0	4 ·84	11
	30	0 ·62	14		50	2 ·76	13		10	4 ·91	11
	40	0 ·68	14	7	0	2 ·83	13		20	4 ·97	11
	50	0 ·75	14		10	2 ·90	13		30	5 ·04	11
2	0	0 ·81	14		20	2 ·97	13		40	5 ·10	11
	10	0 ·88	14		30	3 ·03	13		50	5 ·17	11
	20	0 ·95	14		40	3 ·10	13	13	0	5 ·23	11
	30	1 ·02	14		50	3 ·17	13		10	5 ·30	11
	40	1 ·08	14	8	0	3 ·24	12		20	5 ·37	11
	50	1 ·15	14		10	3 ·31	12		30	5 ·43	11
3	0	1 ·22	14		20	3 ·37	12		40	5 ·50	11
	10	1 ·29	14		30	3 ·44	12		50	5 ·56	11
	20	1 ·36	14		40	3 ·51	12	14	0	5 ·63	10
	30	1 ·42	13		50	3 ·57	12		10	5 ·70	10
	40	1 ·49	13	9	0	3 ·64	12		20	5 ·76	10
	50	1 ·56	13		10	3 ·71	12		30	5 ·83	10
4	0	1 ·62	13		20	3 ·77	12		40	5 ·90	10
	10	1 ·69	13		30	3 ·83	12		50	5 ·96	10
	20	1 ·76	13		40	3 ·90	12	15	0	6 ·03	10
	30	1 ·83	13		50	3 ·97	12		10	6 ·10	10
	40	1 ·89	13	10	0	4 ·04	12		20	6 ·16	10
	50	1 ·96	13		10	4 ·11	12		30	6 ·23	10
5	0	2 ·03	13		20	4 ·17	12		40	6 ·29	10
	10	2 ·10	13		30	4 ·24	12		50	6 ·36	10
	20	2 ·16	13		40	4 ·31	12	16	0	6 ·42	10

$$y = 2{\cdot}000 - \mathrm{N} - \frac{2{\cdot}000 - \mathrm{N}}{\mathrm{D}}$$

VII. TABLES FOR CONFIGURATIONS.

FIRST SATELLITE.

Time in Parts of a Mean Revolution.

Hours.	Parts.	Min.	Parts.	Min.	Parts.
1	·02354	1	00039	30	01177
2	·04708	2	078	31	1216
3	·07063	3	118	32	1256
4	·09417	4	157	33	1295
5	·11771	5	196	34	1334
6	·14125	6	235	35	1373
7	·16480	7	275	36	1413
8	·18834	8	314	37	1452
9	·21188	9	353	38	1491
10	·23542	10	392	39	1530
11	·25897	11	432	40	1569
12	·28251	12	471	41	1609
13	·30605	13	510	42	1648
14	·32959	14	549	43	1687
15	·35314	15	589	44	1726
16	·37668	16	628	45	1766
17	·40022	17	667	46	1805
18	·42376	18	706	47	1844
19	·44731	19	746	48	1883
20	·47085	20	785	49	1923
21	·49439	21	824	50	1962
22	·51793	22	863	51	2001
23	·54148	23	902	52	2040
24	·56502	24	942	53	2080
		25	00981	54	2119
		26	01020	55	2158
		27	1059	56	2197
		28	1099	57	2237
		29	1138	58	2276
		30	01177	59	2315
				60	02354

Multiples of Mean Revolutions.

d	h	m	d	h	m	d	h	m
1	18	28·6	14	3	48·8	26	13	9·0
3	12	57·2	15	22	17·4	28	7	37·6
5	7	25·8	17	16	46·0	30	2	6·2
7	1	54·4	19	11	14·6	31	20	34·8
8	20	23·0	21	5	43·2	33	15	3·4
10	14	51·6	23	0	11·8	35	9	32·0
12	9	20·2	24	18	40·4	37	4	0·6

SECOND SATELLITE.

Time in Parts of a Mean Revolution.

Hours.	Parts.	Min.	Parts.	Min.	Parts.
1	·01172	1	00020	30	00586
2	·02345	2	039	31	606
3	·03517	3	059	32	625
4	·04689	4	078	33	645
5	·05862	5	098	34	664
6	·07034	6	117	35	684
7	·08206	7	137	36	703
8	·09379	8	156	37	723
9	·10551	9	176	38	742
10	·11724	10	195	39	762
11	·12896	11	215	40	782
12	·14068	12	234	41	801
13	·15241	13	254	42	821
14	·16413	14	274	43	840
15	·17585	15	293	44	860
16	·18758	16	313	45	879
17	·19930	17	332	46	899
18	·21102	18	352	47	918
19	·22275	19	371	48	938
20	·23447	20	391	49	957
21	·24619	21	410	50	977
22	·25792	22	430	51	00996
23	·26964	23	449	52	01016
24	·28137	24	469	53	1036
		25	488	54	1055
		26	508	55	1075
		27	527	56	1094
		28	547	57	1114
		29	567	58	1133
		30	00586	59	1153
				60	01172

Multiples of Mean Revolutions.

d	h	m	d	h	m	d	h	m
3	13	17·9	17	18	29·5	31	23	41·1
7	2	35·8	21	7	47·4	35	12	59·0
10	15	53·7	24	21	5·3	39	2	16·9
14	5	11·6	28	10	23·2	42	15	34·7

VII. TABLES FOR CONFIGURATIONS.

THIRD SATELLITE.

Time in Parts of a Mean Revolution.

Hours.	Parts.	Min.	Parts.	Min.	Parts.
1	·00581	1	00010	30	00291
2	·01163	2	19	31	300
3	·01744	3	29	32	310
4	·02326	4	39	33	320
5	·02907	5	48	34	329
6	·03489	6	58	35	339
7	·04070	7	68	36	349
8	·04651	8	78	37	359
9	·05233	9	87	38	368
10	·05814	10	00097	39	378
11	·06396	11	00107	40	388
12	·06977	12	116	41	397
13	·07558	13	126	42	407
14	·08140	14	136	43	417
15	·08721	15	145	44	426
16	·09303	16	155	45	436
17	·09884	17	165	46	446
18	·10466	18	174	47	455
19	·11047	19	184	48	465
20	·11628	20	194	49	475
21	·12210	21	203	50	485
22	·12791	22	213	51	494
23	·13373	23	223	52	504
24	·13954	24	233	53	514
		25	242	54	523
		26	252	55	533
		27	262	56	543
		28	271	57	552
		29	281	58	562
		30	00291	59	572
				60	00581

Multiples of Mean Revolutions.

d h m	d h m	d h m
7 3 59·6	21 11 58·8	35 19 58·0
14 7 59·2	28 15 58·4	42 23 57·6

FOURTH SATELLITE.

Time in Parts of a Mean Revolution.

Hours.	Parts.	Min.	Parts.	Min.	Parts.
1	·00249	1	00004	30	00124
2	·00497	2	08	31	128
3	·00746	3	12	32	133
4	·00995	4	17	33	137
5	·01244	5	21	34	141
6	·01492	6	25	35	145
7	·01741	7	29	36	149
8	·01990	8	33	37	153
9	·02238	9	37	38	158
10	·02487	10	41	39	162
11	·02736	11	46	40	166
12	·02984	12	50	41	170
13	·03233	13	54	42	174
14	·03482	14	58	43	178
15	·03731	15	62	44	182
16	·03979	16	66	45	187
17	·04228	17	70	46	191
18	·04477	18	75	47	195
19	·04725	19	79	48	199
20	·04974	20	83	49	203
21	·05223	21	87	50	207
22	·05471	22	91	51	211
23	·05720	23	95	52	216
24	·05969	24	00099	53	220
		25	00104	54	224
		26	108	55	229
		27	112	56	232
		28	116	57	236
		29	120	58	240
		30	00124	59	245
				60	00249

Multiples of Mean Revolutions.

d h m	d h m	d h m
16 18 5·1	33 12 10·2	50 6 15·3

VIII. ELONGATIONS.

Common Arg.		☾₁		☾₂		☾₃		☾₄		Common Arg.	
γ										γ	
750	750	78·4	0·1	65·8	0·1	45·4	0·1	3·9	0·2	750	750
760	740	78·5	0·1	65·9	0·2	45·5	0·3	4·1	0·5	760	740
770	730	78·6	0·2	66·1	0·3	45·8	0·5	4·6	1·0	770	730
780	720	78·8	0·3	66·4	0·5	46·3	0·8	5·6	1·3	780	720
790	710	79·1	0·4	66·9	0·6	47·1	0·9	6·9	1·7	790	710
800	700	79·5	0·5	67·5	0·7	48·0	1·2	8·6	2·0	800	700
810	690	80·0	0·5	68·2	0·9	49·2	1·4	10·6	2·4	810	690
820	680	80·5	0·6	69·1	1·0	50·6	1·5	13·0	2·7	820	680
830	670	81·1	0·7	70·1	1·0	52·1	1·8	15·7	3·1	830	670
840	660	81·8	0·8	71·1	1·2	53·9	1·9	18·8	3·4	840	660
850	650	82·6	0·8	72·3	1·3	55·8	2·1	22·2	3·7	850	650
860	640	83·4	0·9	73·6	1·5	57·9	2·3	25·9	4·0	860	640
870	630	84·3	0·9	75·1	1·5	60·2	2·4	29·9	4·3	870	630
880	620	85·2	1·1	76·6	1·6	62·6	2·6	34·2	4·5	880	620
890	610	86·3	1·0	78·2	1·7	65·2	2·7	38·7	4·8	890	610
900	600	87·3	1·2	79·9	1·8	67·9	2·8	43·5	5·0	900	600
910	590	88·5	1·1	81·7	1·8	70·7	3·0	48·5	5·2	910	590
920	580	89·6	1·2	83·5	1·9	73·7	3·0	53·7	5·3	920	580
930	570	90·8	1·3	85·4	2·0	76·7	3·2	59·0	5·6	930	570
940	560	92·1	1·2	87·4	2·0	79·9	3·2	64·6	5·7	940	560
950	550	93·3	1·3	89·4	2·1	83·1	3·3	70·3	5·8	950	550
960	540	94·6	1·4	91·5	2·1	86·4	3·4	76·1	5·9	960	540
970	530	96·0	1·4	93·6	2·1	89·8	3·4	82·0	5·9	970	530
980	520	97·4	1·2	95·7	2·2	93·2	3·4	87·9	6·1	980	520
990	510	98·6	1·4	97·9	2·1	96·6	3·4	94·0	6·0	990	510
000	500	100·0	1·4	100·0	2·1	100·0	3·4	100·0	6·0	000	500
010	490	101·4	1·2	102·1	2·2	103·4	3·4	106·0	6·1	010	490
020	480	102·6	1·4	104·3	2·1	106·8	3·4	112·1	5·9	020	480
030	470	104·0	1·4	106·4	2·1	110·2	3·4	118·0	5·9	030	470
040	460	105·4	1·3	108·5	2·1	113·6	3·3	123·9	5·8	040	460
050	450	106·7	1·2	110·6	2·0	116·9	3·2	129·7	5·7	050	450
060	440	107·9	1·3	112·6	2·0	120·1	3·2	135·4	5·6	060	440
070	430	109·2	1·2	114·6	1·9	123·3	3·0	141·0	5·3	070	430
080	420	110·4	1·1	116·5	1·8	126·3	3·0	146·3	5·2	080	420
090	410	111·5	1·2	118·3	1·8	129·3	2·8	151·5	5·0	090	410
100	400	112·7	1·0	120·1	1·7	132·1	2·7	156·5	4·8	100	400
110	390	113·7	1·1	121·8	1·6	134·8	2·6	161·3	4·5	110	390
120	380	114·8	0·9	123·4	1·5	137·4	2·4	165·8	4·3	120	380
130	370	115·7	0·9	124·9	1·5	139·8	2·3	170·1	4·0	130	370
140	360	116·6	0·8	126·4	1·3	142·1	2·1	174·1	3·7	140	360
150	350	117·4	0·8	127·7	1·2	144·2	1·9	177·8	3·4	150	350
160	340	118·2	0·7	128·9	1·0	146·1	1·8	181·2	3·1	160	340
170	330	118·9	0·6	129·9	1·0	147·9	1·5	184·3	2·7	170	330
180	320	119·5	0·5	130·9	0·9	149·4	1·4	187·0	2·4	180	320
190	310	120·0	0·5	131·8	0·7	150·8	1·2	189·4	2·0	190	310
200	300	120·5	0·4	132·5	0·6	152·0	0·9	191·4	1·7	200	300
210	290	120·9	0·3	133·1	0·5	152·9	0·8	193·1	1·3	210	290
220	280	121·2	0·2	133·6	0·3	153·7	0·5	194·4	1·0	220	280
230	270	121·4	0·1	133·9	0·2	154·2	0·3	195·4	0·5	230	270
240	260	121·5	0·1	134·1	0·1	154·5	0·1	195·9	0·2	240	260
250	250	121·6		134·2		154·6		196·1		250	250
		☾₁		☾₂		☾₃		☾₄			

ON THE

COMPUTATION OF AN EPHEMERIS OF A COMET FROM ITS ELEMENTS.

BY Mr. W. S. B. WOOLHOUSE.

In computing the places of a comet from its elements, every operation admits of a strict and direct method, except the formation of the anomalies; but in this, as in most other calculations, we can have recourse to facilities in the management of a series which are not attainable in working an independent case. It is here intended to give such formulæ as may be best suited to the purposes of actual computation.

Let m denote the mean daily motion of the comet, e the excentricity of its orbit, u its excentric anomaly from the perihelion, and t the number of days from the perihelion passage; and, by the elliptic theory,

$$u - e \sin u = mt \ldots\ldots (1)$$

which is the equation from which u is to be deduced from the known value of the mean anomaly mt. Differentiate with respect to t, and

$$\frac{du}{dt}(1 - e \cos u) = m$$

Let r be the radius vector which corresponds with u, and a the major semi-axis of the elliptic orbit, so that $r = a\ (1 - e \cos u)$, and we shall thus have

$$\frac{du}{dt} = \left(\frac{a}{r}\right) m$$

which expresses the motion of u when the comet is at the point in the orbit whose radius vector is r. Let n be the number of days which intervene between the successive places of the comet; and suppose we already know the present values of u, r, and also u_{-1}, r_{-1}, the preceding values, which correspond with $t - n$ days from the perihelion; and let it be required to find the next value u_1, which answers to $t + n$ days from the perihelion. Now $\frac{u - u_{-1}}{n}$ expresses the motion of u near to the middle time between $t - n$ and t, or near to the time $t - \frac{n}{2}$ days; and $\frac{u_1 - u}{n}$ similarly expresses the motion near to the mean of the times t and $t + n$. Hence

$$\log \frac{u - u_{-1}}{n} \qquad \log \frac{du}{dt} = \log\left(\frac{a}{r}m\right) \qquad \log \frac{u_1 - u}{n}$$

are logarithms of motions of u at three times, very nearly in arithmetical progression, and are themselves nearly in arithmetical progression, because the equation

$$\log \frac{du}{dt} = \log\,(am) - \log r$$

shows that the logarithms of the motions of u at different times will difference the same as log r, only with different signs, and log r is a quantity which, at equal intervals, has a small second difference compared with the first. If we therefore assume the above logarithms to be in arithmetical progression, we get

$$\log \frac{u_1 - u}{n} = 2 \log\left(\frac{a}{r}m\right) - \log \frac{u - u_{-1}}{n}$$

$$\therefore \log (u_1 - u) = 2 \log \frac{a}{r} + 2 \log (mn) - \log (u - u_{-1})$$

Or, without logarithms,

$$u_1 - u = \left(\frac{a}{r}\right)^2 \frac{(mn)^2}{u - u_{-1}} \dots\dots\dots (2)$$

To compute a series of excentric anomalies by means of this formula we must obviously know previously two values to begin with; and to effect this we may commence close to the perihelion. If $\rho = a(1-e)$, the perihelion distance, we shall have at this point

$$\frac{du}{dt} = \frac{a}{\rho} m = \frac{m}{1-e} \dots\dots\dots (3)$$

which motion may be taken as uniform very near to the perihelion.

When the value of u_1 is found as above, let the corresponding mean anomaly be computed from the expression $u - e \sin u$, and let $\delta(mt)$ be the error of the result, and the error of u_1 will be

$$\delta u_1 = \left(\frac{du_1}{dt}\right) \delta t = \left(\frac{a}{r_1} m\right) \delta t = \frac{a}{r_1} \delta\,(mt)$$

But as before we have nearly

$$\log \left(\frac{a}{r_1}\right) = 2 \log \left(\frac{a}{r}\right) - \log \left(\frac{a}{r_{-1}}\right) \quad \text{or} \quad \frac{a}{r_1} = \left(\frac{a}{r}\right)^2 \div \left(\frac{a}{r_{-1}}\right)$$

and hence

$$\delta u_1 = \frac{\left(\frac{a}{r}\right)^2}{\left(\frac{a}{r_{-1}}\right)} . \,\delta\,(mt) \dots\dots\dots (4)$$

which, along with (2), is a convenient form for the computation of the correction to be applied to u_1, and will mostly produce a result sufficiently accurate, as will be corroborated by again trying this last value of u in the equation (1).

We ought here to observe, that in finding the value of the term $e \sin u$, it must be reduced to seconds of arc by dividing by $\sin 1''$. Thus, if a constant c be formed $= \frac{e}{\sin 1''}$, we shall have $\log c + \log \sin u$ for the logarithm of the arc in seconds which is to be deducted from u; and this arc can be read off with the greatest facility from the logarithms of Callet.

For the radius vector we have

$$\begin{aligned} r &= a\,(1 - e \cos u) \\ &= a\,\{1 - e\,(\cos^2 \tfrac{1}{2} u - \sin^2 \tfrac{1}{2} u)\} \\ &= a\,\{\cos^2 \tfrac{1}{2} u + \sin^2 \tfrac{1}{2} u - e\,(\cos^2 \tfrac{1}{2} u - \sin^2 \tfrac{1}{2} u)\} \\ &= a\,\{(1-e) \cos^2 \tfrac{1}{2} u + (1+e) \sin^2 \tfrac{1}{2} u\} \end{aligned}$$

Assume

$$h^2 = a\,(1-e) \qquad k^2 = a(1+e) \dots\dots\dots (5)$$

and

$$r = (h \cos \tfrac{1}{2} u)^2 + (k \sin \tfrac{1}{2} u)^2$$

But if v denote the true anomaly of the comet, we have by the ellipse,

$$\tan \tfrac{1}{2} v = \frac{\sin \frac{1}{2} v}{\cos \frac{1}{2} v} = \frac{\sin \frac{1}{2} u}{\cos \frac{1}{2} u} \sqrt{\frac{1+e}{1-e}} = \frac{k \sin \frac{1}{2} u}{h \cos \frac{1}{2} u} \dots\dots (6)$$

$$\therefore \text{ also } r = \left(\frac{h \cos \frac{1}{2} u}{\cos \frac{1}{2} v}\right)^2 = \left(\frac{k \sin \frac{1}{2} u}{\sin \frac{1}{2} v}\right)^2 \ldots\ldots (7)$$

which are very simple formulæ and well adapted to the calculation of the radius vector and the true anomaly.

For a *parabolic* orbit, the true anomaly, and thence the radius vector, will be best computed by means of a table for the perihelion distance = *unity*, constructed for that purpose, and to be found in most works on Astronomy.

The arguments of latitude, θ, or the distances of the comet from the ascending node, estimated in direction of the order of signs, may now be determined thus,

$$\theta = v + \pi - \Omega$$

in which v must be used with the negative sign when the comet is on that side of the perihelion, which is contrary to the order of signs.

If now the planes of the equator, solstitial colure, and equinoctial colure, be respectively taken for the co-ordinate planes (xy), (yz), (zx), we shall have the intersection of the equator and the ecliptic, or the line which points out the first point of Aries, or the Zero point of right ascension, for the axis of x; the line in the equator which points out 90°, or 6^h of right ascension, for the axis of y, and the axis of the equator for that of z, the positive values of z being north. It may be shown either by the principles of spherics, or of analytical geometry, as was first determined by GAUSS, that the ordinates $x\,y\,z$ of the comet, estimated from the Sun as an origin, may be found from the following formulæ, in which ω denotes the obliquity of the ecliptic:

$$\tan A = -\frac{\cot \Omega}{\cos i} \qquad \sin a = \frac{\cos \Omega}{\sin A}$$

$$\tan \psi = \frac{\tan i}{\cos \Omega}$$

$$\tan B = \frac{\sin \Omega \sin \psi \cos \omega}{\sin i \cos (\psi + \omega)} \qquad \sin b = \frac{\sin \Omega \cos \omega}{\sin B}$$

$$\tan C = \frac{\sin \Omega \sin \psi \sin \omega}{\sin i \sin (\psi + \omega)} \qquad \sin c = \frac{\sin \Omega \sin \omega}{\sin C}$$

$$x = r \sin a \sin (A + \theta)$$
$$y = r \sin b \sin (B + \theta)$$
$$z = r \sin c \sin (C + \theta)$$

The constants A, B, C, sin a, sin b, sin c, will perhaps be more readily found by the following formulæ, as the computer will have less consideration in the determination of the particular quadrants in which the angles A, B, C, will fall.

$$\left.\begin{array}{l|l|l} \multicolumn{3}{c}{\tan \psi = \dfrac{\tan i}{\cos \Omega} \qquad s = \dfrac{\sin i}{\sin \psi}} \\ \sin a \cos A = -\sin \Omega \cos i & \sin b \cos B = s \cos (\psi + \omega) & \sin c \cos C = s \sin (\psi + \omega) \\ \sin a \sin A = \cos \Omega & \sin b \sin B = \sin \Omega \cos \omega & \sin c \sin C = \sin \Omega \sin \omega \end{array}\right\} (8)$$

The arc ψ is to be taken out in the first quadrant with its proper sign; and sin a, sin b, sin c, being always positive, by attending to the algebraic signs we shall deduce those of both the cosine and the sine of the angles A, B, C. It will not be necessary to take out the arcs a, b, c, as their log. sines only are wanted.

We may also avoid the computation of the successive values of θ by taking

$$A' = A + (\pi - \Omega) \quad B' = B + (\pi - \Omega) \quad C' = C + (\pi - \Omega) \ldots (9)$$

for then

$$\left.\begin{array}{l} x = r \sin a \sin (A' + v) \\ y = r \sin b \sin (B' + v) \\ z = r \sin c \sin (C' + v) \end{array}\right\} \ldots\ldots\ldots\ldots (10)$$

In these computations we may use the descending node ☋ instead of the ascending; and the algebraic sign of the inclination i must be taken thus:

1. If the ascending node ☊ is used

when the motion is $\left\{\begin{array}{l}\text{Direct}\\ \text{Retrograde}\end{array}\right\}$ use i with the $\left\{\begin{array}{l}\textit{positive}\\ \textit{negative}\end{array}\right\}$ sign.

2. If the descending node ☋ is used

when the motion is $\left\{\begin{array}{l}\text{Direct}\\ \text{Retrograde}\end{array}\right\}$ use i with the $\left\{\begin{array}{l}\textit{negative}\\ \textit{positive}\end{array}\right\}$ sign.

Let us now assume the centre of the Earth for the origin of co-ordinates, and denote those of the Sun by X, Y, Z; then if ☉ denote the sun's longitude, and R its radius vector or distance from the earth, we shall have

$$\left.\begin{array}{rl} X &= R \cos \odot \\ Y &= R \sin \odot \cos \omega \\ Z &= R \sin \odot \sin \omega \\ &= Y \tan \omega \end{array}\right\} \ldots\ldots\ldots\ldots (11)$$

However, when great accuracy is not required, the values of the Sun's co-ordinates may be more readily taken from Weisse's Tables of Co-ordinates of the Sun and Planets. For the year 1835 it will be necessary to correct his epoch, which should be 1·0278, instead of 0·0278, to which it will also be necessary to add 0·0065 for difference of longitude between Greenwich and Paris.

The co-ordinates of the comet estimated from the Earth will thus be $X + x$, $Y + y$, $Z + z$. And hence, α denoting the comet's geocentric right ascension, δ its declination, and ρ its distance from the earth,

$$\left.\begin{array}{rl} \tan \alpha &= \dfrac{Y + y}{X + x} \\ \tan \delta &= \dfrac{Z + z}{(X + x) \sec \alpha} \\ &= \dfrac{Z + z}{X + x} \cos \alpha \\ \rho &= \dfrac{Z + z}{\sin \delta} \end{array}\right\} \ldots\ldots\ldots\ldots (12)$$

We shall here recapitulate the whole of the formulæ, in the order and form of computation, and then for an example take the comet of Halley, which will pass its perihelion Nov. 7, 1835.

1. *Anomalies and Radii Vectores.*

$$\text{Constants}\left\{\begin{array}{ll} m = \dfrac{[3\cdot55002^*]}{a^{\frac{3}{2}}} & c = \dfrac{e}{\sin 1''} \\ h = \sqrt{a(1-e)} & k = \sqrt{a(1+e)} \end{array}\right.$$

$$u - c \sin u = mt$$

$$u_1 - u = \left(\frac{a}{r}\right)^2 \frac{(mn)^2}{u - u_{-1}} \qquad \delta u_1 = \frac{\left(\frac{a}{r}\right)^2}{\left(\frac{a}{r_{-1}}\right)} \cdot \delta\,(mt)$$

$$\tan \tfrac{1}{2} v = \frac{k \sin \frac{1}{2} u}{h \cos \frac{1}{2} u} \qquad \sqrt{r} = \frac{h \cos \frac{1}{2} u}{\cos \frac{1}{2} v}$$

2. *Right Ascensions, Declinations, &c.*

$$\text{Constants}\left\{\begin{array}{lll} p = \cos \Omega & p' = \sin \Omega \cos \omega & p'' = \sin \Omega \sin \omega \\ & \tan \psi = \dfrac{\tan i}{\cos \Omega} \quad s = \dfrac{\sin i}{\sin \psi} & \\ q = -\sin \Omega \sin i & q' = s \cos (\psi + \omega) & q'' = s \sin (\psi + \omega) \\ \tan A = \dfrac{p}{q} & \tan B = \dfrac{p'}{q'} & \tan C = \dfrac{p''}{q''} \\ \sin a = \dfrac{p}{\sin A} & \sin b = \dfrac{p'}{\sin B} & \sin c = \dfrac{p''}{\sin C} \\ A' = A + (\pi - \Omega) & B' = B + (\pi - \Omega) & C' = C + (\pi - \Omega) \end{array}\right.$$

$$x = r \sin a \sin (A' + v) \qquad y = r \sin b \sin (B' + v) \qquad z = r \sin c \sin (C' + v)$$

$$X = R \cos \odot \qquad Y = R \sin \odot \cos \omega \qquad Z = Y \tan \omega$$

$$\tan \alpha = \frac{Y + y}{X + x} \qquad \tan \delta = \frac{Z + z}{X + x} \cos \alpha \qquad \rho = \frac{Z + z}{\sin \delta}$$

EXAMPLE.

The Elements of Halley's Comet for its perihelion passage in the year 1835 are, as deduced by Pontécoulant in the *Connais. des Tems* for 1832, p. 33,

Perihelion passage, Nov. $7^{d}\cdot2$, mean Paris time.

Semi-axis major ...	a	17·98705
Excentricity	e	0·9675212
Perihelion on orbit	π	304° 31′ 43″
Ascending node ...	Ω	55 30
Inclination	i	17 44 24

Motion RETROGRADE.

* 3·55002 is the logarithm of the sun's mean daily motion.

To compute the places of this comet at intervals of 4 days, we have $n = 4$, and we proceed as follows:

log a	1·25496	log a	1·25496
$\frac{1}{2}$ log a	0·62748	log $(1-e)$ 0·032479	8·51160
	1·88244	log h^2..............	9·76656
Const.	3·55002	log h	9·88328
log m..............	1·66758	log $(1+e)$ 1·96752	0·29392
log n	0·60206	log k^2..............	1·54888
log (mn)..........	2·26964	log k	0·77444
log $(mn)^2$	4·53928	$mn =$	3′ 6″·05

log e.............. 9·9856605
log sin 1″......... 4·6855749

c................ 5·3000856

Perihelion passage Nov. 7ᵈ·2000 mean Paris time
·0065 Diff. long.

7·1935 mean Greenwich time.

Thus, as we compute for mean noon at Greenwich, we have

On Nov. 7, time preceding perihelion $= + 0^d$·1935
log 0·1935 .... 9·28668
log m .... 1·66758

∴ Nov. 7, $mt = + 0'$ 9″·00.... 0·95426

The comet's motion being retrograde, we thence deduce, as under, the mean anomalies for the following days, by the successive *subtraction* of the value of (mn), which is the mean motion of the comet in four days

	(mt) ′ ″
1835, Nov. 7....	+ 0 9·00
11....	− 2 57·05
15....	− 6 3·10
&c.	&c.

With these we now begin the calculation of the Anomalies and Radii Vectores.

Nov. 7, $mt = +$ 0′ 9″·00			0·95426
		$(1-e)$	8·51160
$u = 0°$ 4′ 37″			2·44266
		c	5·3000856
		sin u	7·1280545
c sin u .... 0 4 28·00			2·4281401
	0 9·00	$\frac{1}{2}u = 0°$ 2′ 18″·5	
mt	0 9·00		
	*		
sin $\frac{1}{2}u$	6·82702	cos $\frac{1}{2}u$..........	0·00000
k sin $\frac{1}{2}u$............	7·60146	h cos $\frac{1}{2}u$..........	9·88328
h cos $\frac{1}{2}u$............	9·88328		
tan................	7·71818	cos $\frac{1}{2}v$..........	9·99999
$\frac{1}{2}v$	0° 18′·0	$\frac{1}{2}$ log r............	9·88329

$v = \ldots\ldots + 0° \ 36'{\cdot}0$ $\log r = \ldots\ldots$ 9·76658
Nov. 7. $a \ldots\ldots$ 1·25496

$\frac{a}{r} \ldots\ldots$ 1·48838

Nov. 11, $mt = -\ 2'\ 57''{\cdot}05 \ldots\ldots$ 2·24810
$(1-e) \ldots\ldots$ 8·51160

$u = 1°\ 30'\ 51'' \ldots\ldots$ 3·73650

$c \ldots\ldots$ 5·3000856
$\sin u \ldots\ldots$ 8·4220005

$c \sin u \ldots\ldots$ 1 27 53·34 $\ldots\ldots$ 3·7220861

2 57·66
$mt \ldots\ldots$ 2 57·05 $(1-e) \ldots\ldots$ 8·5116
$\delta\,(mt) \ldots\ldots$ 0·61 $\ldots\ldots$ 9·7853

Corr. $u \ldots\ldots$ 19 $\ldots\ldots$ 1·2737

$u \ldots\ldots = 1°\ 30'\ 32''$ $\ldots$ sin $\ldots\ldots$ 8·4204844
$c \sin u \ldots\ldots$ 1 27 34·97 $\ldots\ldots$ 3·7205700

2 57·03 $\frac{1}{2} u = 0°\ 45'\ 16''$
$mt \ldots\ldots$ 2 57·05

*

$\sin \frac{1}{2} u \ldots\ldots$ 8·11949 $\cos \frac{1}{2} u \ldots\ldots$ 9·99996
$k \sin \frac{1}{2} u \ldots\ldots$ 8·89393 $h \cos \frac{1}{2} u \ldots\ldots$ 9·88324
$h \cos \frac{1}{2} u \ldots\ldots$ 9·88324

$\left\{\begin{array}{l}\tan \ldots\ldots\\ \frac{1}{2} v \ldots\ldots\end{array}\right.$ 9·01069 / 5° 51′·1 $\cos \frac{1}{2} v \ldots\ldots$ 9·99773
$\frac{1}{2} \log r \ldots\ldots$ 9·88551
$v = \ldots\ldots -11\ \ 42{\cdot}2$ $\log r = \ldots\ldots$ 9·77102
Nov. 11. $a \ldots\ldots$ 1·25496

$\frac{a}{r} \ldots\ldots$ 1·48394

$\left(\frac{a}{r}\right)^2 \ldots\ldots$ 2·96788

	°	′	″		
$u = \ldots\ldots$	-1	30	32	$(mn)^2 \ldots\ldots$	4·53928
$u_{-1} = \ldots\ldots$	$+0$	4	37		7·50716
$u - u_{-1} \ldots\ldots$	-1	35	9	log $\ldots\ldots$	3·75656
$u_1 - u \ldots\ldots$	-1	33	51	$\ldots\ldots$	3·75060
$u_1 \ldots\ldots$	-3	4	23	$c \ldots\ldots$	5·3000856
				$\sin u_1 \ldots\ldots$	8·7292395
$c \sin u_1 \ldots\ldots$	2	58	18·55	$\ldots\ldots$	4·0293251
	6	4·45			$1{\cdot}488 = \frac{a}{r_{-1}}$
$mt \ldots\ldots$	6	3·10		$\left(\frac{a}{r}\right)^2 \div \frac{a}{r_{-1}} \ldots$	1·480
	1·35				

		$\left(\frac{a}{r}\right)^2 \div \frac{a}{r_{-1}}$	1·480
$\delta\,(mt)$	1·35	....	0·130
Corr. u_1	41	....	1·610
$u_1 =$	3 3 42	.... sin	8·7276286
Corr. u_1	2 57 38·94	....	4·0277142
	6 3·06	$\frac{1}{2}u =$ −	1° 31′ 51″
mt	6 3·10		
	*		

$\sin\frac{1}{2}u$	8·42675	$\cos\frac{1}{2}u$	9·99984
$k\sin\frac{1}{2}u$	9·20119	$h\cos\frac{1}{2}u$	9·88312
$h\cos\frac{1}{2}u$	9·88312		
tan	9·31807	$\cos\frac{1}{2}v$	9·99080
$\frac{1}{2}v$	11° 45′·0	$\frac{1}{2}\log r$	9·89232
$v =$	− 23 30·0	$\log r =$	9·78464
		a	1·25496
	&c.		
Nov. 15.		$\frac{a}{r}$	1·47032
			&c.

The remaining anomalies to the end of the intended Ephemeris may now be determined in exactly the same manner as this for Nov. 15.

The computation of the co-ordinate constants comes next, and will be as follows:

$p = \cos\Omega$ +	9·7531280	$\sin\Omega$ ... +	9·9159937	.... +	9·9159937
q { $\sin\Omega$ +	9·9159937	$\cos\omega$ .... +	9·9625258	$\sin\omega$ .... +	9·6000211
q { $\cos i$ +	9·9788417	p' ...... +	9·8785195	p'' ...... +	9·5160148
$\tan i$ .... −	9·5050280	$\sin i$ ... −	9·4838697	Here i is *negative*, see p. 43	
tan ..... −	9·7519000	$\sin\psi$ ... −	9·6917764		
ψ ...... −	29° 27′ 29″	s ...... +	9·7920933	.... +	9·7920933
ω	23 27 40				
$\psi + \omega$ .. −	5 59 49	cos ..... +	9·9976168	sin ..... −	9·0190141
q .... −	9·8948354	q' ...... +	9·7897101	q'' ...... −	8·8111074
tan ..... −	9·8582926	tan ..... +	0·0888094	tan ..... −	0·7049074
A	144° 11′ 10″	B	50° 49′ 4″	C	101° 9′ 37″
sin ..... +	9·7672704	sin ..... +	9·8893805	sin ..... +	9·9917087
$\sin a$	9·9858576	$\sin b$	9·9891390	$\sin c$	9·5243061
$\pi - \Omega$ ..	249 1 43	....	249 1 43	....	249 1 43
A′	33 12 53	B′	299 50 47	C′	350 11 20

Hence we have for the heliocentric co-ordinates of the comet,

$$\log x = \log \sin\,(33^\circ\ 12'\cdot9 + v) + 9\cdot98586 + \log r$$
$$\log y = \log \sin\,(299\ \ 50\ \cdot8 + v) + 9\cdot98914 + \log r$$
$$\log z = \log \sin\,(350\ \ 11\ \cdot3 + v) + 9\cdot52431 + \log r$$

We shall take Nov. 15 for the completion of an example in which X, Y, Z, are taken from Weisse's tables.

November 15.

A′........	33° 12′·9	B′........	299° 50′·8	C′........	350° 11′·3
v ....—	23 30 ·0	v ....—	23 30 ·0	v ....—	23 30 ·0
A′+v.....	9 42 ·9	B′+v.....	276 20 ·8	C′+v.....	326 41 ·3
sin	9 ·22724	sin	9 ·99733	sin	9 ·73972
	9 ·98586		9 ·98914		9 ·52431
r..........	9 ·78464	r..........	9 ·78464	r	9 ·78464
log	8 ·99774	log	9 ·77111	log	9 ·04867
x ....+	0 ·09948	y ....—	0 ·59035	z ....—	0 ·11186
X........—	0 ·60254	Y........—	0 ·71888	Z........—	0 ·31194
X+x.....—	0 ·50306	Y+y.....—	1 ·30923	Z+z.....—	0 ·42380
		log (Y+y) —	0 ·11701	log (Z+z)	9 ·62716
		log (X+x) —	9 ·70162	....	9 ·70162
α ......=	248° 58′·9	tan α .....+	0 ·41539		9 ·92554
		sin α	9 ·97010	cos α	9 ·55471
=	16h 35m·9	δ	—16° 48′·8	tan δ	9 ·48025
		same sign as Z+z		cos δ	9 ·98103
				sin δ	9 ·46128
				Log. dist. from ⊕ ..	0 ·16588

When α comes out near 90° or 270° its sine is taken out, as in this example, for the purpose of getting the cosine more accurately, by subtracting the tangent from the sine; but when α is nearer to 0° or 180° it will be better not to regard the sine, as the cosine itself can then be taken more accurately from the tables.

COMPARISON OF RESULTS

DEDUCED FROM

M. BURCKHARDT'S AND M. DAMOISEAU'S LUNAR TABLES.

Professor Schumacher has given, in his Ephemeris of the Distances of the four planets, Venus, Mars, Jupiter, and Saturn, for the year 1834, (Copenhagen, 1832.) an Ephemeris of the Moon for every third hour of mean time at Greenwich, founded upon Damoiseau's Lunar Tables; and the following Table contains the differences between his results and those in the Nautical Almanac for 1834, which have been derived from Burckhardt's Tables.

The Differences of Right Ascensions and Declinations are for the Mean Noon of each day, and those of the Horizontal Parallaxes for Mean Midnight.

Denoting Professor Schumacher's results by D, and those in the Nautical Almanac by B; the numbers in the Table represent, in all cases, D — B, or the quantity to be added to the Nautical Almanac results (South Declinations being considered —) to produce those of Professor Schumacher.

According to Burckhardt,

$$\frac{☾\text{'s Sem.}}{☾\text{'s Hor. Par.}} = 0{\cdot}2725$$ (Burckhardt's *Tables de la Lune*, p. 73. Paris, 1812.)

According to Damoiseau,

$$\frac{☾\text{'s Sem.}}{☾\text{'s Hor. Par.}} = 0{\cdot}27263$$ (Schumacher's *Ephemeris* for 1834, p. 134.)

TABLE,

Showing the Difference between the results, for the year 1834, derived from BURCKHARDT'S and DAMOISEAU'S LUNAR TABLES.

Days of the Month.	JANUARY.			FEBRUARY.			MARCH.			APRIL.		
	R. A.	Dec.	H. P.	R. A.	Dec.	H. P.	R. A.	Dec.	H. P.	R. A.	Dec.	H. P.
	″	″	″	″	″	″	″	″	″	″	″	″
1	−13·6	+ 9·7	+ 2·0	−13·7	+9·3	+2·1	− 6·9	+4·6	+3·8	− 3·0	−0·8	+1·9
2	14·5	10·1	−11·8	13·0	8·1	1·0	7·3	4·0	3·1	− 0·2	1·4	1·4
3	15·3	10·8	+ 2·0	10·1	6·5	0·9	6·3	3·0	2·3	+ 3·4	1·2	+0·4
4	14·8	10·3	1·8	− 3·2	4·8	+0·7	− 3·0	2·3	1·3	7·0	1·3	−0·1
5	11·5	8·8	1·6	+ 4·2	3·6	−0·1	+ 2·9	1·5	0·8	10·3	0·8	0·6
6	6·3	5·8	1·5	10·0	2·9	0·2	9·0	1·9	+0·1	13·3	−0·3	0·9
7	− 0·5	3·3	1·3	12·5	3·0	0·5	13·6	2·5	−0·4	14·5	+0·6	0·7
8	+ 4·0	2·1	0·8	12·0	1·9	1·3	15·5	2·5	1·2	13·4	+0·8	−0·3
9	5·8	+ 0·9	+ 0·3	9·0	+0·2	1·3	15·4	1·5	0·9	9·8	0·0	0·0
10	4·2	− 0·6	− 0·1	6·5	−1·7	1·4	13·5	+0·8	1·4	+ 3·7	−1·7	+0·5
11	2·0	2·3	0·5	4·1	3·1	1·6	10·7	−0·3	1·1	− 1·9	2·8	0·8
12	+ 0·2	4·7	1·2	2·7	4·0	1·3	6·5	2·1	0·9	6·0	2·8	0·7
13	− 0·1	6·5	1·0	1·8	4·3	1·4	+ 2·4	3·1	0·4	6·5	1·6	+0·4
14	+ 0·3	7·6	1·1	+ 1·1	3·5	2·6	− 1·4	3·5	0·5	3·8	−0·3	−0·3
15	1·8	7·2	1·1	− 1·2	3·7	2·2	3·0	2·9	0·9	2·2	+0·2	1·2
16	4·2	6·8	1·5	4·7	3·9	2·4	2·9	1·7	1·9	3·2	0·5	1·4
17	3·5	6·1	1·8	10·2	4·2	2·3	3·8	0·7	2·4	7·0	1·4	1·9
18	+ 1·2	6·2	2·3	15·7	2·8	2·5	8·1	−0·6	2·8	10·3	2·9	2·3
19	− 2·7	6·3	2·2	18·9	−0·6	1·3	14·1	+0·4	2·7	9·8	3·6	1·9
20	6·3	5·8	2·0	18·5	+1·8	−0·4	19·6	2·0	2·3	6·5	3·6	1·4
21	8·3	4·0	− 0·8	14·5	4·1	+1·2	19·6	5·1	1·4	3·9	2·9	−0·6
22	9·3	− 1·8	+ 0·3	10·1	5·8	2·6	12·7	6·0	−0·1	3·2	2·8	+0·3
23	10·1	+ 0·3	1·0	7·0	6·6	3·7	5·0	4·7	+1·3	6·4	3·4	1·4
24	11·6	2·8	2·3	8·2	7·4	4·6	2·1	4·3	2·3	10·0	4·2	2·0
25	14·0	5·3	2·9	11·0	9·4	5·2	3·8	5·2	3·4	14·4	4·2	2·3
26	16·0	8·2	3·2	12·3	9·2	5·4	7·8	5·9	4·1	15·9	2·5	2·4
27	15·9	10·3	3·8	10·8	8·0	5·3	11·6	6·4	4·4	14·2	+0·1	1·6
28	15·4	11·2	3·9	− 8·3	+6·1	+5·0	12·2	5·3	4·4	9·9	−2·0	1·7
29	13·8	10·3	3·8	- -	- -	- -	10·5	3·3	4·3	6·8	3·3	1·3
30	12·1	9·3	3·8	- -	- -	- -	8·2	1·7	3·1	− 4·6	−4·2	+1·0
31	−12·9	+ 9·4	+ 2·9	- -	- -	- -	− 4·9	+0·6	+2·7	- -	- -	- -

TABLE,

Showing the Difference between the results, for the year 1834, derived from BURCKHARDT'S and DAMOISEAU'S LUNAR TABLES.

Days of the Month.	MAY.			JUNE.			JULY.			AUGUST.		
	R. A.	Dec.	H. P.	R. A.	Dec.	H. P.	R. A.	Dec.	H. P.	R. A.	Dec.	H. P.
	″	″	″	″	″	″	″	″	″	″	″	″
1	− 3·1	−4·7	+0·4	+0·6	−6·1	−1·1	+ 3·0	−4·1	−0·5	+ 1·7	−2·1	+1·5
2	− 1·0	5·3	−0·3	4·3	3·7	1·3	2·8	3·1	−0·2	+ 1·7	1·9	1·6
3	+ 2·6	4·6	0·6	7·2	−1·4	−1·0	3·1	−1·5	+0·3	− 1·4	−1·2	1·4
4	+ 6·3	2·8	0·7	7·7	+0·5	0·0	3·4	+0·5	0·6	8·7	+0·8	+1·0
5	−10·4	−0·2	1·0	6·4	1·8	+0·6	+ 1·8	1·1	1·1	14·4	3·7	−0·3
6	12·5	+1·3	−0·8	+3·5	2·8	0·9	− 1·5	2·6	1·2	17·3	6·4	0·5
7	11·3	1·5	0·0	0·0	3·7	1·7	6·7	3·9	0·5	18·2	7·3	0·9
8	6·6	1·3	+0·6	−3·3	4·8	1·6	9·2	5·8	0·7	16·6	6·7	0·9
9	+ 0·6	0·7	1·3	6·1	6·0	1·6	10·4	7·5	1·0	12·8	4·6	0·5
10	− 5·0	0·7	1·7	7·7	7·4	2·1	9·7	8·2	1·2	8·3	2·4	0·2
11	7·7	1·3	1·7	7·1	8·3	1·6	7·8	7·7	1·7	− 3·5	+0·1	0·5
12	8·8	2·9	1·5	5·0	8·2	2·3	5·1	6·4	1·7	+ 0·3	−0·6	0·3
13	7·5	3·6	1·6	−2·2	7·1	1·8	− 1·9	4·5	1·8	3·0	0·9	−0·4
14	4·5	3·8	0·8	+0·7	5·4	1·7	+ 1·5	3·5	1·1	5·5	0·4	+0·2
15	2·4	3·4	+0·3	2·7	4·4	+0·7	3·2	3·1	0·6	5·9	−0·1	0·4
16	0·8	2·9	−0·2	+2·0	4·5	−0·1	2·7	3·5	0·4	6·2	+0·2	0·8
17	1·2	2·8	1·1	−0·2	5·7	0·9	+ 0·6	4·4	0·1	4·0	−0·5	1·0
18	1·6	3·0	1·3	2·6	6·2	0·7	− 0·3	4·6	0·4	+ 0·8	2·5	1·1
19	2·5	4·0	1·7	4·3	6·0	0·4	1·6	3·7	0·9	− 2·9	4·5	0·3
20	3·6	4·2	1·4	4·4	4·9	−0·1	3·1	+1·6	+0·8	4·7	6·3	+0·1
21	5·4	4·0	−0·8	4·1	3·3	+0·3	3·4	−0·9	−0·1	4·2	7·0	−0·7
22	7·4	3·8	0·0	4·4	+1·4	−0·1	3·2	2·4	0·5	− 2·0	6·2	0·8
23	9·0	2·9	+0·4	3·7	−1·5	0·8	2·7	4·4	1·3	+ 0·7	4·2	−0·3
24	10·3	+1·0	0·6	1·9	2·9	1·3	1·7	5·3	2·1	2·2	3·6	0·0
25	10·3	−0·9	+0·4	1·9	4·9	1·7	− 0·1	5·1	1·7	2·3	2·9	+0·5
26	8·0	2·7	−0·1	2·7	6·2	1·8	+ 2·7	4·4	1·4	+ 0·4	2·9	1·5
27	5·5	4·3	0·5	2·4	7·1	1·6	3·7	3·9	−0·8	− 1·7	2·7	2·1
28	4·9	5·9	0·4	−0·8	7·4	1·5	3·4	4·0	0·0	2·9	2·4	2·9
29	5·1	7·5	0·4	+0·8	6·4	1·1	1·5	4·4	+0·8	2·2	1 9	3·1
30	5·3	8·1	1·1	+1·8	−5·4	−0·5	0·2	4·2	0·7	1·6	1·9	3·5
31	− 3·1	−8·1	−0·8	- -	- -	- -	+ 0·5	−3·4	+1·1	− 2·3	−1·4	+3·3

TABLE,

Showing the Difference between the results, for the year 1834, derived from Burckhardt's and Damoiseau's Lunar Tables.

Days of the Month.	SEPTEMBER.			OCTOBER.			NOVEMBER.			DECEMBER.		
	R. A.	Dec.	H. P.	R. A.	Dec.	H. P.	R. A.	Dec.	H. P.	R. A.	Dec.	H. P.
	″	″	″	″	″	″	″	″	″	″	″	″
1	− 6·0	+0·1	+2·4	−10·2	+ 5·5	+2·8	−10·9	+6·6	+1·2	−2·9	+1·4	0·0
2	12·3	2·9	1·2	16·3	9·4	+1·2	12·0	5·6	−0·3	7·0	+0·3	−1·0
3	19·3	7·2	+0·1	21·2	11·1	0·0	9·9	3·2	1·0	7·8	−1·8	1·7
4	23·3	9·5	−0·9	19·5	9·0	−1·0	− 4·3	+0·7	2·3	−3·9	3·6	2·2
5	22·0	8·9	1·5	12·3	4·4	1·8	+ 0·3	−0·5	3·0	+0·6	4·3	2·6
6	17·5	5·7	2·4	− 3·7	+ 0·7	3·1	4·2	1·4	3·9	4·2	4·3	2·4
7	9·9	+1·4	2·4	+ 2·2	− 1·0	3·4	7·2	1·8	3·5	5·1	4·8	2·2
8	− 3·4	−1·6	2·7	5·0	1·4	4·0	8·0	2·4	3·3	5·4	5·1	1·6
9	+ 0·1	2·8	2·4	4·6	1·5	4·0	7·7	2·6	3·0	5·9	4·6	1·5
10	1·5	2·9	2·5	5·5	2·0	3·7	8·5	2·7	2·2	6·6	5·0	0·9
11	4·2	2·7	2·3	8·1	2·0	2·8	8·6	2·4	1·5	6·5	3·7	0·7
12	6·9	2·0	1·2	9·6	1·7	2·0	8·9	1·8	1·1	4·6	3·1	−0·3
13	9·8	2·1	−0·4	10·0	1·8	1·0	6·9	1·8	−0·6	2·6	2·3	+0·2
14	10·1	1·9	0·0	8·8	2·3	0·6	+ 3·8	1·8	0·0	+1·4	−1·0	1·1
15	7·9	3·4	+0·5	5·9	3·2	−0·5	0·0	1·6	+0·1	−0·3	+0·2	1·6
16	+ 3·7	4·7	0·4	+ 2·3	4·3	+0·1	− 2·6	−0·7	0·4	2·2	1·7	2·0
17	− 0·5	6·7	0·4	− 0·7	4·7	−0·1	4·0	+1·1	0·5	3·0	3·6	1·9
18	2·7	7·9	0·5	3·1	3·8	0·0	2·5	3·1	0·6	2·3	4·7	1·2
19	3·1	7·1	0·3	3·9	2·7	+0·4	− 1·0	4·7	0·8	−1·2	5·1	1·4
20	2·7	5·5	0·2	4·1	− 1·2	0·6	+ 1·2	4·8	0·6	+0·1	4·9	0·5
21	1·5	3·9	0·3	3·3	+ 0·2	0·8	+ 0·6	4·8	0·8	−1·0	5·2	0·5
22	1·4	3·0	0·8	1·9	1·9	1·2	− 1·5	5·3	1·6	3·9	5·9	0·9
23	1·3	1·6	1·6	1·7	2·4	1·9	4·6	5·8	2·7	6·6	6·7	1·6
24	2·2	0·8	2·3	4·4	3·0	3·0	6·6	6·7	3·2	7·4	6·7	1·5
25	4·0	−0·5	3·0	8·7	3·9	3·8	5·7	6·3	4·0	5·8	5·4	0·7
26	6·1	+0·1	3·8	10·5	4·8	5·0	− 1·8	3·8	3·7	−1·9	3·2	+0·7
27	7·7	0·5	4·7	8·6	5·3	5·3	+ 2·1	1·8	3·0	+2·5	1·9	−0·1
28	7·9	1·4	4·8	4·9	4·6	5·0	4·8	0·8	2·3	6·0	1·1	0·6
29	6·5	2·5	4·8	1·8	3·1	4·2	4·6	0·4	1·7	5·5	1·5	0·6
30	− 6·3	+3·4	+3·8	1·8	3·4	3·4	+ 0·8	+1·1	+1·1	3·7	+0·9	1·1
31	- -	- -	- -	− 6·1	+ 5·6	+1·9	- -	- -	- -	+0·1	−0·4	−1·4

ON ECLIPSES.

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Eclipses, in all the varieties of aspect which they present to different places on the Earth, form an entertaining subject for discussion; and, without considering the public interest generally excited by their prediction and appearance, the use of them, as a test of the degree of perfection of the lunar and solar tables, and in the determination and corroboration of geographical positions, &c., renders their accurate calculation an object of some importance. The popularity of the phenomena naturally called the attention of astronomers, at an early period, into the field of investigation, and several methods of calculation have been adopted by different authors at various periods.

For the general circumstances which take place on the Earth, the plan of orthographic projection, though it can only be recognised as affording good approximations, seems to have predominated, and to have been almost exclusively adopted in actual calculations. This method is explained in the Astronomical treatises of De La Lande and Delambre, and more recently by Hallaschka, in his *Elementa Eclipsium*, (Pragæ, 1816), where an example is to be found at length. Various particulars are laid down in a more accurate manner in *Mémoires sur l'Astronomie Pratique*. Par M. J. Monteiro Da Rocha, traduits du Portugais (Paris, 1808).

The circumstances of an Eclipse for a particular place are usually calculated by the "Method of the Nonagesimal," which refers the bodies to the Ecliptic, and an example of which may be seen in the work of Hallaschka above mentioned. This part of the subject has also been discussed analytically by Lagrange, in the *Astron. Jahrbuch* for 1782; and Professor Bessel has since made some important additions to the theory, in a paper inserted in the *Astronomische Nachrichten*, vol. vii., No. 151, which is to be found translated in the *Philosophical Magazine*, vol. viii.

As the numerous calculations which may be required for an eclipse, such as of the Maps, &c. given in the Nautical Almanac, could not be performed without many perplexing references to different authors, it has been presumed that a complete and systematic set of formulæ would be generally acceptable; and such a conviction has led to the drawing up of the following paper, which contains an extensive classification of useful remarks and formulæ, developed and arranged with a careful view to their practical application, and with the endeavour to establish a direct and uniform mode of conducting each species of calculation.

LIMITS WHICH DETERMINE THE OCCURRENCES OF ECLIPSES.

ELEMENTS.

The following elements, used in the calculation of the limits, have been derived from the Tables of Damoiseau, Burckhardt, and Carlini, viz.:—

		°	′	″
Moon's Horizontal Parallax	greatest		61	32
	least		52	50
Sun's Horizontal Parallax	greatest		0	9
	least		0	8
Moon's Semidiameter	greatest		16	46
	least		14	24
Sun's Semidiameter	greatest		16	18
	least		15	45
Moon's Hourly motion in Longitude	greatest		38	35
	least		27	47
Sun's Hourly motion in Longitude	greatest		2	33
	least		2	23
Moon's Hourly motion in Latitude	greatest		3	4
	least		0	0
Inclination of Moon's Orbit with Ecliptic	greatest	5°	20	6
	least	4	57	22

LIMITS.

For the occurrence of an eclipse of the Moon:—

1. The greatest possible distance of the centres of the Moon and Earth's Shadow at the time of contact, is 63′ 29″.

2. At the time of true ecliptic conjunction of the Moon and Earth's Shadow, or at the time of opposition or Full Moon, the greatest possible latitude of the Moon is 63′ 45″.

3. At the time of opposition, or Full Moon, the greatest possible distance of the centre of the Moon or of the Earth's Shadow from the ascending or descending node of the Moon's orbit is 12° 24′.

For the occurrence of an eclipse of the Sun:—

1. The greatest possible distance of the centres of the Sun and Moon, at the time of contact, is 1° 34′ 28″.

2. At the time of true conjunction of the Sun and Moon, the greatest possible latitude of the Moon is 1° 34′ 52″.

3. At the time of true conjunction of the Sun and Moon, or the time of New Moon, the greatest possible distance of the centre of the Sun or Moon from one of the nodes of the Moon's orbit is 18° 36′.

The third of these limits applies to the true place of the node, which may differ considerably from the mean place.

The most convenient and certain limits, however, will be those of the Moon's latitude (β), and will be as follows:—

1. At the time of Full Moon an eclipse of the Moon will be

$$\left.\begin{array}{l}\text{certain}\\ \text{impossible}\end{array}\right\}\text{when } \beta \left\{\begin{array}{l}< 51'\ 57''\\ > 63\ \ 45\end{array}\right.$$

and doubtful between these limits.

For the doubtful cases, an eclipse will result when

$$\beta < \frac{61}{60}(P + \pi - \sigma) + s + 16''$$

in which P, s, denote the equatorial horizontal parallax and semidiameter of the Moon, and π, σ, those of the Sun.

2. At the time of New Moon an eclipse of the Sun will be

$$\left.\begin{array}{l}\text{certain}\\ \text{impossible}\end{array}\right\}\text{when } \beta \left\{\begin{array}{l}< 1^\circ\ 23'\ 15''\\ > 1\ \ 34\ \ 52\end{array}\right.$$

and doubtful between these limits.

For the doubtful cases, an eclipse will happen when

$$\beta < (P - \pi) + \sigma + s + 25''$$

PARALLAX.

If a straight line be drawn from the centre of the Earth to any assumed place, it will be the radius of the Earth for that place, and this radius we shall designate by the letter ρ. This radius ρ, produced upwards towards the heavens, will determine what we shall call the *central zenith*, being that point which spherically determines our true position in relation to the centre of the Earth. The apparent zenith, however, is naturally determined by a line which is vertical to the observer, and therefore a normal to the spheroidal surface of the Earth. The small angular deviation of this normal from the radius of the Earth, or the angular distance between the central and apparent zeniths, is what astronomers call "the angle of the vertical;" and, the Earth being an oblate spheroid, it is evident that the central zenith will be nearer to the equator than the apparent, and also that the horizontal parallax will always be less than that at the equator, in consequence of the diminution of the Earth's radius in proceeding towards the poles. The effect of parallax on the position of a body above the horizon is to augment its zenith distance, and for this we have the well-known relation—

"sin. par. in zen. dist. = sin. Hor. Par. × sin. app. zen. dist."

This relation will hold strictly for the spheroidal figure of the Earth, provided we adopt the central zenith, and that horizontal parallax which appertains to the radius ρ of the place of observation.

Consider the equatorial semidiameter of the Earth as unity, and let γ denote the polar semidiameter, which, adopting the mean between La Lande and Delambre, will be $\frac{304}{305}$. Let also l be the latitude of the central zenith, or what is usually called the "geocentric latitude," and l' that of the apparent zenith, which may be termed the spheroidal or geographical latitude. Then the co-ordinates of this place, referred, in the plane of its meridian, to the polar axis, will be

$$x = \rho \sin l \qquad\qquad y = \rho \cos l$$

By the generating ellipse

$$\frac{x^2}{\gamma^2} + y^2 = 1$$

and therefore, for the angle l' which the normal makes with y or the tangent with x, we have

$$\tan l' = -\frac{dy}{dx} = \frac{1}{\gamma^2} \cdot \frac{x}{y} = \frac{\tan l}{\gamma^2}$$

$$\therefore \tan l = \gamma^2 \tan l' \quad \text{- - - - - - - - - - - - - - - (1)}$$

Again, the values of x and y, substituted in the above equation of the ellipse, give

$$\rho^2 \left(\frac{\sin^2 l}{\gamma^2} + \cos^2 l \right) = 1$$

and hence

$$\rho = \frac{1}{\sqrt{\left(\frac{\sin^2 l}{\gamma^2} + \cos^2 l \right)}} = \frac{1}{\sqrt{(1 + \frac{1-\gamma^2}{\gamma^2} \sin^2 l)}} \quad \text{- - - - - - - - - - (2)}$$

To these may be added the following, which are sometimes useful, and directly deducible from the equations (1), (2).

$$x = \rho \sin l = \frac{\gamma^2 \tan l'}{\sqrt{(1 + \gamma^2 \tan^2 l')}} = \frac{(1 - e^2) \sin l'}{\sqrt{(1 - e^2 \sin^2 l')}} \quad \text{- - - - - - - - - (3)}$$

$$y = \rho \cos l = \frac{1}{\sqrt{(1 + \gamma^2 \tan^2 l')}} = \frac{\cos l'}{\sqrt{(1 - e^2 \sin^2 l')}} \quad \text{- - - - - - - - - (4)}$$

where $e = \sqrt{(1 - \gamma^2)}$ is the excentricity of the meridian.

Also

$$\rho = \sqrt{(x^2 + y^2)} = \sqrt{\frac{1 + \gamma^4 \tan^2 l'}{1 + \gamma^2 \tan^2 l'}} = \sqrt{\frac{\cos l'}{\cos l \cos (l' - l)}} \quad \text{- - - - - - - - - (5)}$$

The equations (1), (2) are convenient, and the latter may be simply resolved by logarithms, thus:

$$\left. \begin{aligned} \tan \psi &= \sin l \sqrt{\frac{1-\gamma^2}{\gamma^2}} \\ \rho &= \cos \psi \end{aligned} \right\} \quad \text{- - - - - - - - - (6)}$$

From (1) may also be deduced

$$\left. \begin{aligned} \tan \chi = \gamma \tan l' &= \frac{\tan l}{\gamma} = \sqrt{(\tan l \tan l')} \\ \tan (l' - l) &= \frac{1-\gamma^2}{2\gamma} \sin 2\chi \end{aligned} \right\} \quad \text{- - - - - - - (7)}$$

Here we may remark, that in reducing the geographical latitude to the geocentric with the argument l', the auxiliary arc χ, being between the values of l and l', will be a very small quantity in defect of the argument; and that, on the contrary, in reducing the geocentric to the geographical latitude, the arc χ will exceed the argument by nearly the same quantity. Therefore, if we assume χ as an argument for the difference $l' - l$, a table formed from the equation

$$\tan (l' - l) = \left(\frac{1-\gamma^2}{2\gamma} \right) \sin 2\chi$$

$$\text{or } l' - l = \left(\frac{1-\gamma^2}{2\gamma \tan 1''} \right) \sin 2\chi, \text{ in seconds,}$$

will be equally adapted to both reductions, giving nearly the mean between them; and a table so constructed, with the argument χ, signifying either latitude, will answer every necessary degree of accuracy, since the reduction itself is so small. In numbers we have

$$\frac{1-\gamma^2}{2\gamma} = \frac{609}{2 \times 304 \times 305}, \text{ and its logarithm} = 7\cdot51641 \therefore \log\left(\frac{1-\gamma^2}{2\gamma \tan 1''}\right) = 2\cdot83084,$$

and hence

$$l'-l = [2\cdot83084] \sin 2\chi$$

Thus the following table has been derived:

Difference between the Geographical and Geocentric Latitudes.

Argument: χ, either Latitude.

χ		$l'-l$		χ		$l'-l$		χ		$l'-l$	
°	°	′	″	°	°	′	″	°	°	′	″
0	90	0	0	15	75	5	39	30	60	9	47
1	89	0	24	16	74	5	59	31	59	9	58
2	88	0	47	17	73	6	19	32	58	10	9
3	87	1	11	18	72	6	38	33	57	10	19
4	86	1	34	19	71	6	57	34	56	10	28
5	85	1	58	20	70	7	15	35	55	10	37
6	84	2	21	21	69	7	33	36	54	10	44
7	83	2	44	22	68	7	51	37	53	10	51
8	82	3	7	23	67	8	7	38	52	10	57
9	81	3	29	24	66	8	23	39	51	11	3
10	80	3	52	25	65	8	39	40	50	11	7
11	79	4	14	26	64	8	54	41	49	11	11
12	78	4	36	27	63	9	8	42	48	11	14
13	77	4	57	28	62	9	22	43	47	11	16
14	76	5	18	29	61	9	34	44	46	11	17
15	75	5	39	30	60	9	47	45	45	11	17

The difference is to be subtracted from the geographical, or added to the geocentric latitude, whether it be North or South.

It is evident, from what has been said, page 55, that if Z denote the true distance of the Moon from the central zenith as it would appear at the centre of the Earth, and Z′ the apparent distance from the same zenith, as seen from the place on the surface, where the radius of the Earth is ρ; and furthermore, P the equatorial horizontal parallax, and $z = Z' - Z$, the parallax in altitude, we shall have

$$\sin z = \rho \sin P \sin Z' \quad \text{- - - - - - - - - - - -} \quad (8)$$

Substituting $Z + z$ in the place of Z′, and dividing by $\cos z$, we find

$$\tan z = \frac{\rho \sin P \sin Z}{1 - \rho \sin P \cos Z} \quad \text{- - - - - - - - - - -} \quad (9)$$

which are the usual formulæ for the parallax in altitude.

For the radius ρ of the Earth we have $\log \sqrt{\frac{1-\gamma^2}{\gamma^2}} = 8{\cdot}909435$, and $\therefore$ by (6)

$$\tan \psi = [8{\cdot}909435] \sin l$$

$$\rho = \cos \psi$$

The values of ρ so computed are given in the annexed table.

Log, Radius of the Earth.

Argument: Geocentric Latitude.

l	$\log \rho$	l	$\log \rho$	l	$\log \rho$
°		°		°	
0	0·00000	30	9·99964	60	9·99893
1	0·00000	31	9·99962	61	9·99891
2	0·00000	32	9·99960	62	9·99889
3	0·00000	33	9·99958	63	9·99887
4	9·99999	34	9·99955	64	9·99885
5	9·99999	35	9·99953	65	9·99883
6	9·99998	36	9·99951	66	9·99881
7	9·99998	37	9·99948	67	9·99879
8	9·99997	38	9·99946	68	9·99877
9	9·99997	39	9·99943	69	9·99876
10	9·99996	40	9·99941	70	9·99874
11	9·99995	41	9·99938	71	9·99872
12	9·99994	42	9·99936	72	9·99871
13	9·99993	43	9·99934	73	9·99870
14	9·99992	44	9·99931	74	9·99868
15	9·99990	45	9·99929	75	9·99867
16	9·99989	46	9·99926	76	9·99866
17	9·99988	47	9·99924	77	9·99865
18	9·99986	48	9·99921	78	9·99864
19	9·99985	49	9·99919	79	9·99863
20	9·99983	50	9·99916	80	9·99862
21	9·99982	51	9·99914	81	9·99861
22	9·99980	52	9·99911	82	9·99860
23	9·99978	53	9·99909	83	9·99859
24	9·99976	54	9·99907	84	9·99859
25	9·99974	55	9·99904	85	9·99858
26	9·99973	56	9·99902	86	9·99858
27	9·99971	57	9·99900	87	9·99858
28	9·99968	58	9·99897	88	9·99858
29	9·99966	59	9·99895	89	9·99857
30	9·99964	60	9·99893	90	9·99857

PHENOMENA WHICH TAKE PLACE ON THE EARTH GENERALLY.

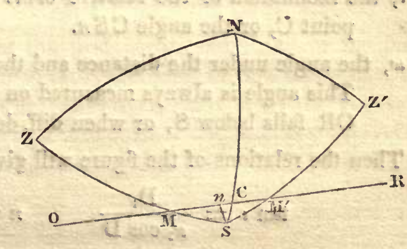
The place on the surface of the Earth where the limbs of the Sun and Moon first appear in contact will be where the penumbra first touches the Earth, and, consequently, at this place the apparent contact will be in the horizon, the disc of the Moon being wholly above the horizon, and that of the Sun below it. The point of contact will be in the same vertical with the two centres; and, therefore, the real as well as the apparent places will be in the same vertical circle; and the lower limb of the Moon, being in the horizon, will be depressed by the whole amount of the horizontal parallax which belongs at that time to the latitude of the place. Similarly, the place which first has a central eclipse will be where the straight line through the centres of the Sun and Moon comes first in contact with the Earth, and at this place the centres of both objects will be in the horizon, that of the Moon experiencing the whole effect of the horizontal parallax.

The same circumstances will have place where the phenomena finally quit the Earth.

Since the apparent places of the Sun and Moon are so contiguous, and the parallax of the Sun so small, it is evident that the relative positions will be the same if we give to the Moon the effect of the difference of the parallaxes $P - \pi$, and retain the Sun in his true position. This difference $P - \pi$ is therefore the relative parallax, or that which influences the relative position of the bodies. If ρ be the radius of the Earth for the place on its surface, the parallax which ought to be used is $\rho\,(P - \pi)$. But in the following investigations, where a place is generally the object of determination, we cannot previously so reduce this relative parallax $P - \pi$. In order, therefore, to secure the chance of least deviation from the truth in this respect, we shall in these cases reduce the parallax in the first instance to a mean latitude of 45°, so that it will be $[9{\cdot}99929]\,(P - \pi)$. We shall consequently, to simplify the analytical expressions, hereafter denote this quantity by the letter P' only; except in one or two instances, where the latitude of the place is known, and where it is always distinctly specified to represent the parallax properly reduced to that latitude, or $\rho\,(P - \pi)$.

I. Places where the different Phases are first and last seen on the Earth.

Let the whole be referred to the surface of a sphere concentric with the Earth; and let O R be the relative orbit of the Moon, which is generated by the differences of the motions in right ascension and declination, or by the relative motion of the Moon; N, the North Pole; S, the Sun; S n, perpendicular to the relative orbit, the nearest approach which we denote by n; C, the point where the Moon comes in conjunction in right ascension, and C S the difference of declination at that time, which we shall denote by contraction, diff. dec. Let also M, M′, be the positions of the Moon, when a distance of the centres equal to Δ' first appears on, and finally quits, the Earth; $MS = M'S = \Delta$, the corresponding true distance as seen from the centre of the Earth; Z, Z′, the zeniths of these places on the Earth, which must be respectively in the continuations of S M, S M′, in order that the full effect of parallax may be communicated in causing the bodies to approach.



As the apparent zenith distance of the points which experience the greatest effect must be $90°$, we may evidently assume $ZS = 90°$: for contact of either limb of the Moon with the contiguous limb of the Sun, we have accurately $ZS = (90° - \pi) + \sigma$; for contact of either limb of the Moon with the remote limb of the Sun, $ZS = (90° - \pi) - \sigma$; and for contact of the centres $ZS = 90° - \pi$. By making $ZS = 90°$, the phase will begin with sunrise and end with sunset; and it is evident that no sensible augmentation can affect the semidiameter of the Moon so near the horizon. The true distance SM of the centres being Δ, and P' the relative horizontal parallax, the apparent distance Δ' will be $P' - \Delta$; and, by estimating positive distances from S towards M, in order to have the first occurrence of the phase, it will be $\Delta - P'$

$$\therefore \Delta = P' + \Delta'$$

Here we may notice three limiting aspects:—

(1) When simple or exterior contact of limbs first takes place, $\Delta' = s + \sigma$ and

$$\Delta = P' + s + \sigma.$$

(2) When interior contact of limbs first takes place, $\Delta' = s - \sigma$: when $s > \sigma$, a total contact first commences with $\Delta' = s - \sigma$; when $s < \sigma$, an annular contact first commences with $\Delta' = \sigma - s$. Therefore,

If $s > \sigma$, a total eclipse first begins on the Earth when

$$\Delta = P' + s - \sigma$$

If $s < \sigma$, an annular eclipse first begins on the Earth when

$$\Delta = P' - s + \sigma$$

(3) When contact of centres first takes place on the Earth, $\Delta' = 0$, and

$$\Delta = P'$$

For the time of true conjunction in right ascension, assume

D, the true declination of the Moon.

α, the true difference of right ascension, or ☽'s right ascension *minus* ☉'s right ascension, *in space.*

D_1, the relative motion in declination, or the motion of the Moon in declination *minus* that of the Sun, at that time.

α_1, the relative motion in right ascension at the same time.

ι, the inclination of the relative orbit OR with a parallel of declination through the point C, or the angle CSn.

ω, the angle under the distance and the line of nearest approach, or the angle MSn. This angle is always measured on the northern side of the distance, so that when OR falls below S, or when diff. dec. CS is negative, it will exceed $90°$.

Then the relations of the figure will give these equations,

$$\tan \iota = \frac{D_1}{\alpha_1 \cos D} \qquad n = (\text{diff. dec.}) \cos \iota \text{ - - - - - } (1)$$

$$\text{Hourly motion in the orbit} = \frac{D_1}{\sin \iota}$$

$$\text{arc } nC = n \tan \iota$$

For the time of describing the arc nC, or the interval between the middle of the general eclipse and the time of conjunction, it must be divided by the hourly motion in the orbit. Therefore, t denoting this interval,

$$t = \left(\frac{n \sin \iota}{D_1}\right) \tan \iota$$

Assume

$$\left.\begin{array}{l} c = 3600'' \times \dfrac{n \sin \iota}{D_1} = [3{\cdot}55630]\ \dfrac{n \sin \iota}{D_1} \\ \text{and} \\ t \textit{ in seconds} = c \tan \iota \end{array}\right\} \text{- - - - - - (2)}$$

The sign of t will be determined by combining the signs of diff. dec. and D_1; and then,

$$\text{Time of middle} = \text{Time of } ☌ - t \quad \text{- - - - - - - - - (3)}$$

Also,

$$\cos \omega = \frac{n}{\Delta} \quad \text{- - - - - - - - - - (4)}$$

$$M n = n \tan \omega$$

Let τ denote the semiduration of the phase, or the time of describing $M n$, and

$$\left.\begin{array}{l} \tau \textit{ in seconds} = c \tan \omega \\ \text{Time of} \left\{\begin{array}{l}\text{beginning} \\ \text{ending}\end{array}\right\} = \text{Time of middle} \left\{\begin{array}{c} - \\ + \end{array}\right\} \tau \end{array}\right\} \text{- (5)}$$

Again let, at the beginning, the $\angle$ NSZ $= a$, and for the ending, the $\angle$ NSZ$'$ $= b$; and, these angles being estimated from NS towards the East, we shall have

$$a = (-\iota) - \omega \qquad b = (-\iota) + \omega \quad \text{- - - - - - - - (6)}$$

and, the Sun being supposed in the horizon, ZS $= 90°$, Z$'$S $= 90°$,

$$\cos NZ = \cos NSZ \sin NS \qquad \tan ZNS = -\frac{\tan NSZ}{\cos NS}$$

$$\cos NZ' = \cos NSZ' \sin NS \qquad \tan Z'NS = -\frac{\tan NSZ'}{\cos NS}$$

or,

$$\left.\begin{array}{ll} \sin l = \cos a \cos \delta & \tan h = -\dfrac{\tan a}{\sin \delta} \\ \sin l' = \cos b \cos \delta & \tan h' = -\dfrac{\tan b}{\sin \delta} \end{array}\right\} \text{- - - - - - (7)}$$

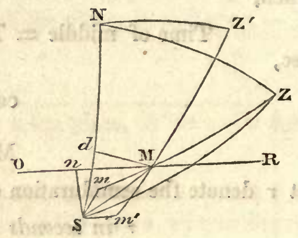
the latitude and hour angle l, h, relating to the first place, and l', h', to the last. These hour angles are measured from the Sun towards the East, so that the longitudes of the places will be determined by subtracting respectively from them the apparent Greenwich times of beginning and ending reduced into degrees and minutes, observing that positive differences will indicate East longitudes and negative differences West longitudes.

In the preceding formulæ we must use,

$$\text{For beginning and ending of a} \left\{\begin{array}{l}\text{Partial} \\ \text{Total} \\ \text{Annular} \\ \text{Central}\end{array}\right\} \text{Eclipse,} \ \Delta = \left\{\begin{array}{l} P' + s + \sigma \\ P' + s - \sigma \\ P' - s + \sigma \\ P' \end{array}\right.$$

II. Rising and Setting Limits.

The places Z, Z′, thus found, are the two extreme points of a series of places where, at the intermediate times, the same phase will appear in the horizon; and, for the phase of external contact of limbs, the curves which these places assume form one of the principal geographical limits of the general eclipse. In the annexed diagram let M be the place of the Moon at a time between the beginning and ending of the partial eclipse. Make $Sm = \Delta'$, $Mm = P'$, and $mZ = 90°$; then at the place Z the Moon will appear at m, and have simple external contact with the Sun in the horizon. The two triangles SmM, $Sm'M$ will give two such places at each instant, which, on considering the passage of the penumbra over the terrestrial disc, evidently ought to be the case. Since $Mm = P'$ and $Sm = \Delta'$, the possibility of forming the triangles SmM, $Sm'M$ will depend on two conditions for the value of SM, viz., $SM < Mm + Sm$, $SM > Mm - Sm$, or $\Delta < P' + \Delta'$ and $> P' - \Delta'$, that is, Δ must be between the values $P' - \Delta'$ and $P' + \Delta'$: this leads to two species of curves.



1. *When the nearest approach is greater than* $P' - \Delta'$.

Here the formation of the triangles SmM, $Sm'M$, will always be possible during the appearance of the phase on the Earth. At the first appearance and final departure of the phase, $SM = Mm + Sm$, the triangle SmM will be simply the line SM, and only one place Z will result. By taking positions of M on both sides of the middle point n, it will also appear that the relative positions of the places Z, Z′, become inverted, and that the curves described by them must intersect each other at some intermediate place. Hence it appears that the curve of risings and settings commences with a single point, which immediately after divides itself into two points moving in opposite directions on the Earth, and which describe two curves intersecting each other, and finally meeting again in a single point, the whole forming one continued curve, returning into itself, and assuming the figure of an 8 much distorted. At the place where they intersect, the phase will begin at sunrise and end at sunset, or it will begin at sunset and end at sunrise.

2. *When the nearest approach is less than* $P' - \Delta'$.

In this case the triangles SmM, $Sm'M$ will resolve into the line SM when $\Delta = P' + \Delta'$ and also when $\Delta = P' - \Delta'$, each of which positions will give only one place Z. Thus it appears that the points Z will form two distinct, oval, and isolated curves, the former curve being generated between the decreasing values $\Delta = P' + \Delta'$ and $\Delta = P' - \Delta'$, and the latter between the increasing values $\Delta = P' - \Delta'$ and $\Delta = P' + \Delta'$. The leading point of the first oval and the terminating point of the second oval are the places where the phase begins and ends on the Earth. The terminating point of the first oval and the leading point of the second oval are simply determined by using $\Delta = P' - \Delta'$, and computing the same as for the beginning and ending of a phase on the Earth.

Let us now turn our attention to the determination of the two places Z, Z′, at any time, or for any position of M. Join Z S and draw M d perpendicular to N S.

We shall, throughout our investigation, usually denote Sd by (x), d M by (y), and the $\angle d$SM by S, this angle being estimated from SN towards the East.

To determine these quantities, let the declination of the point $d =$ (D), which will a little exceed that of M, and which is distinguished from it by being placed within a parenthesis; then, supposing N M to be joined, the right-angled spherical triangle NdM will give $\tan\,(\mathrm{D}) = \dfrac{\tan \mathrm{D}}{\cos \alpha}$. As α is always small, the difference of the declinations $(\mathrm{D}) - \mathrm{D} = \tan^{-1} \dfrac{\tan \mathrm{D}}{\cos \alpha} - \mathrm{D}$ may be arranged in a small table as annexed.

Difference between (D) and D, or α corr.
Arguments: D and α.

D	α 10′	20′	30′	40′	50′	60′	70′	80′	90′	100′
°	″	″	″	″	″	″	″	″	″	″
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	1	1	1	2
2	0	0	0	0	1	1	1	2	2	3
3	0	0	0	1	1	2	2	3	4	5
4	0	0	1	1	2	2	3	4	5	6
5	0	0	1	1	2	3	4	5	6	8
6	0	0	1	1	2	3	4	6	7	9
7	0	0	1	2	3	4	5	7	9	11
8	0	0	1	2	3	4	6	8	10	12
9	0	1	1	2	3	5	7	9	11	13
10	0	1	1	2	4	5	7	10	12	15
11	0	1	1	3	4	6	8	10	13	16
12	0	1	2	3	4	6	9	11	14	18
13	0	1	2	3	5	7	9	12	15	19
14	0	1	2	3	5	7	10	13	17	20
15	0	1	2	3	5	8	11	14	18	22
16	0	1	2	4	6	8	11	15	19	23
17	0	1	2	4	6	9	12	16	20	24
18	0	1	2	4	6	9	13	16	21	26
19	0	1	2	4	7	10	13	17	22	27
20	0	1	3	4	7	10	14	18	23	28
21	0	1	3	5	7	11	14	19	24	29
22	0	1	3	5	8	11	15	19	25	30
23	0	1	3	5	8	11	15	20	25	31
24	0	1	3	5	8	12	16	21	26	32
25	0	1	3	5	8	12	16	21	27	33
26	0	1	3	6	9	12	17	22	28	34
27	0	1	3	6	9	13	17	23	29	35
28	0	1	3	6	9	13	18	23	29	36
29	0	1	3	6	9	13	18	24	30	37

The number of seconds given by this table, which we have denoted by the term α corr. is to be applied so as to increase D, whether it be North or South.

The value of (D) being found by so correcting D with this table, we shall evidently have

$$\left.\begin{array}{c} (x) = (\mathrm{D}) - \delta \qquad\qquad (y) = \alpha \cos(\mathrm{D}) \\ \tan \mathrm{S} = \dfrac{(y)}{(x)} \\ \Delta = \dfrac{(y)}{\sin \mathrm{S}} = \dfrac{(x)}{\cos \mathrm{S}} \end{array}\right\} \text{- - - - (A)}$$

the quadrant in which S is to be taken being determined by (x) and (y) as co-ordinates.

We shall afterwards have frequent occasion to use these quantities.

If t denote the time from the middle of the general eclipse, they may be determined more easily, though less accurately, by means of the following formulæ, which may readily be inferred from what has preceded.

$$\left.\begin{array}{c} \tan \omega = \dfrac{t}{c} \qquad\qquad \Delta = \dfrac{n}{\cos \omega} \\ \mathrm{S} = (-\iota) \mp \omega \\ (x) = \Delta \cos \mathrm{S} \qquad\qquad (y) = \Delta \sin \mathrm{S} \end{array}\right\} \text{- - - - (B)}$$

the upper sign being for the time t before the middle, and the under sign for the same time after the middle.

Denote the $\angle m\mathrm{MS}$ by m. In the triangle $m\mathrm{MS}$, which may, on account of its smallness, be considered as a plane one, we also have $\mathrm{M}m = \mathrm{P}'$, $\mathrm{S}m = \Delta'$, and $\mathrm{SM} = \Delta$. Assume

$$\left.\begin{array}{l} p = \dfrac{\mathrm{P}' - \Delta'}{2} \qquad\qquad q = \dfrac{\mathrm{P}' + \Delta'}{2} \\ \text{and then} \\ \sin \dfrac{m}{2} = \sqrt{\dfrac{\left(\frac{\Delta}{2} - p\right)\left(q - \frac{\Delta}{2}\right)}{\mathrm{P}' . \Delta}} \end{array}\right\} \text{- - - - (1)}$$

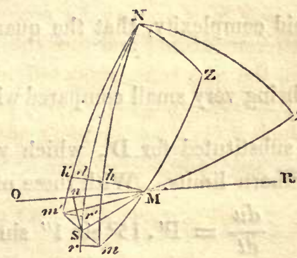
As ZS, $\mathrm{Z}m$ may be considered as quadrantal arcs, they will be parallel at the extremities S, m; and thus the $\angle \mathrm{ZSM} = \angle m\mathrm{MS} = m$. Therefore the $\angle \mathrm{NSZ} = \mathrm{S} \pm m$; and the Sun being supposed in the horizon, the spherical triangle NSZ will have $\mathrm{ZS} = 90°$, and hence the places Z, Z′, will depend on the following formulæ, in which Z is called the place advancing, and Z′ the place following.

$$\left.\begin{array}{ll} \textit{Place following,} & \\ \sin l = \cos(\mathrm{S} - m) \cos \delta & \tan h = -\dfrac{\tan(\mathrm{S} - m)}{\sin \delta} \\ \textit{Place advancing,} & \\ \sin l = \cos(\mathrm{S} + m) \cos \delta & \tan h = -\dfrac{\tan(\mathrm{S} + m)}{\sin \delta} \end{array}\right\} \text{- (2)}$$

In these expressions the symbol δ represents the declination of the Sun at the time for which we calculate; but for common purposes the value of δ at the time of conjunction may be used in all cases.

III. Northern and Southern Limits for any Phase.

The determination of the extreme latitudinal limits of a phase, or of the terrestrial lines whereon that phase will appear as the middle of the local eclipse, is the most complex and unmanageable of all operations which relate to a general eclipse. For any given phase, at different places on the Earth, the Moon must be so reduced by parallax as to touch a given concentric circle on the solar disc; and if we consider this circle, by way of illustration, to represent, instead of the Sun, the disc of the luminous body, the places on the Earth which severally see the given phase must be situated in the surface of the penumbral or umbral cone, according as the interfering limb of the Moon only approaches or projects over the centre of the Sun; that is, the places must all be found in the intersection of this cone with the surface of the Earth. This intersection will assume a complete or partial oval form, according as the cone falls wholly or partially on the Earth's illuminated disc. When it falls only partially on the Earth, the extreme points will evidently see the Sun in the horizon, and be therefore two points belonging to the horizon limits; but in the other case the phase cannot at that instant be seen in the horizon. It is evident then, that these two cases have been already characterized in the discussion of the rising and setting limits. Let us now suppose the bodies to assume consecutive positions, answering to very small intervals of time, the Earth also turning round its axis, and we shall have a series of these ovals. It is obvious that the extreme geographical limits of the phase will be represented by curves which envelope all these ovals;—that at each instant the place of limit, by reason of the compound of the motions, will be proceeding relatively in the direction of the tangent to the oval;—that there will be two of these limits when the oval becomes entire during the eclipse, but only one when it is always partial. This is the most popular and natural idea that can be formed of the nature of these limits; and we may here remark, as an inference from what has been said, that if the rising and setting limits of any phase do not extend throughout the general partial eclipse, there will be both a Northern and Southern limit to that phase; but that, on the contrary, when the rising and setting limits continue throughout the eclipse, there will be only one of these limits to the phase, viz.: a Southern limit when the difference of declination at conjunction is positive, and a Northern one when that difference is negative.



As before, let the system be referred to a sphere concentric with the Earth, and let M be the place of the Moon; Z, Z′, the zeniths of the places which are respectively in the Northern and Southern limits; and m, m', the corresponding apparent places of the Moon. Draw the meridians Nm', N S, Nm, N Z, N Z′; also $m\,r$, $m'\,r'$, and M$h\,d\,h'$ perpendicular to N S; and assume $\mathrm{S}d = (x)$, $d\mathrm{M} = (y)$, $mh = x$, $h\mathrm{M} = y$, $\mathrm{S}r = u$, $mr = v$, $\mathrm{S}m = \Delta'$, $\mathrm{Z}m = \mathrm{Z}$, $\angle \mathrm{N}m\mathrm{Z} = \mathrm{M}$, $\angle m\mathrm{NS} = \alpha'$, declination of $m = \mathrm{D}'$, and the latitude of $\mathrm{Z} = l$. Then the $\angle m\mathrm{NZ} = h - \alpha'$, $m\mathrm{M} = \mathrm{P}' \sin \mathrm{Z}$, $x = m\mathrm{M} \cos \mathrm{M} = \mathrm{P}' \sin \mathrm{Z} \cos \mathrm{M}$ and $y = m\mathrm{M} \sin \mathrm{M} = \mathrm{P}' \sin \mathrm{Z} \sin \mathrm{M}$; these by spherics resolve thus:—

$$x = \mathrm{P}' \sin \mathrm{Z} \cos \mathrm{M}$$
$$= \mathrm{P}' \{\sin l \cos \mathrm{D}' - \cos l \sin \mathrm{D}' \cos (h - \alpha')\}$$
$$y = \mathrm{P}' \sin \mathrm{Z} \sin \mathrm{M}$$
$$= \mathrm{P}' \cos l \sin (h - \alpha')$$

From these we deduce

$$\left.\begin{aligned} u &= x - (x) \\ &= \mathrm{P}' \sin \mathrm{Z} \cos \mathrm{M} - (x) \\ &= \mathrm{P}' \{\sin l \cos \mathrm{D}' - \cos l \sin \mathrm{D}' \cos (h - \alpha')\} - (x) \\ v &= (y) - y \\ &= (y) - \mathrm{P}' \sin \mathrm{Z} \sin \mathrm{M} \\ &= (y) - \mathrm{P}' \cos l \sin (h - \alpha') \end{aligned}\right\} \text{ - - (1)}$$

Let us now keep our attention to the same place Z on the Earth, and suppose the system to be in motion as in nature. The hour angle h will increase at the rate of 15° per hour, and the latitude l will by hypothesis remain unchanged; so that the following equations will ensue:

$$\frac{du}{dt} = - \mathrm{P}' \sin 1'' \frac{d\mathrm{D}'}{dt} \{\sin l \sin \mathrm{D}' + \cos l \cos \mathrm{D}' \cos (h - \alpha')\}$$

$$+ \mathrm{P}' \sin 1'' \left(15° - \frac{d\alpha'}{dt}\right) \cos l \sin \mathrm{D}' \sin (h - \alpha') - \frac{d\,(x)}{dt}$$

$$= - \mathrm{P}' \sin 1'' \frac{d\mathrm{D}'}{dt} \cos \mathrm{Z} + \mathrm{P}' \sin 1'' \left(15° - \frac{d\alpha'}{dt}\right) \sin \mathrm{D}' \sin \mathrm{Z} \sin \mathrm{M} - \frac{d\,(x)}{dt}$$

$$\frac{dv}{dt} = \frac{d\,(y)}{dt} - \mathrm{P}' \sin 1'' \left(15° - \frac{d\alpha'}{dt}\right) \cos l \cos (h - \alpha')$$

$$= \frac{d\,(y)}{dt} - \mathrm{P}' \sin 1'' \left(15° - \frac{d\alpha'}{dt}\right) (\cos \mathrm{Z} \cos \mathrm{D}' - \sin \mathrm{Z} \sin \mathrm{D}' \cos \mathrm{M})$$

Now, in order that m may be the apparent place of the Moon at the middle of the eclipse, and consequently her nearest apparent contiguity with the Sun, we must have $\frac{d\,\Delta'}{dt} = 0$; or since $u^2 + v^2 = \Delta'^2$, $u \frac{du}{dt} + v \frac{dv}{dt} = 0$, which is the condition of limit.

Before we substitute the preceding values of $\frac{du}{dt}, \frac{dv}{dt}$, it may be observed, to avoid complexity, that the quantities $\mathrm{P}' \sin 1'' \frac{d\mathrm{D}'}{dt}$, $\mathrm{P}' \sin 1'' \frac{d\alpha'}{dt}$ may be neglected as being very small compared with $\mathrm{P}' . 15° \sin 1''$, $\frac{d\,(x)}{dt}$ and $\frac{d\,(y)}{dt}$; also that δ may be substituted for D′, which will equally serve the purpose of both Northern and Southern limits. With these modifications we have

$$\left.\begin{aligned} \frac{du}{dt} &= \mathrm{P}' . 15° \sin 1'' \sin \delta \sin \mathrm{Z} \sin \mathrm{M} - \frac{d\,(x)}{dt} \\ \frac{dv}{dt} &= \frac{d\,(y)}{dt} - \mathrm{P}' . 15° \sin 1'' (\cos \mathrm{Z} \cos \delta - \sin \mathrm{Z} \sin \delta \cos \mathrm{M}) \end{aligned}\right\} \text{ (2)}$$

and, for the condition of limit,

$$u \left\{\mathrm{P}' . 15° \sin 1'' \sin \delta \sin \mathrm{Z} \sin \mathrm{M} - \frac{d\,(x)}{dt}\right\}$$

$$+ v \left\{\frac{d\,(y)}{dt} - \mathrm{P}' . 15° \sin 1'' (\cos \mathrm{Z} \cos \delta - \sin \mathrm{Z} \sin \delta \cos \mathrm{M})\right\} = 0$$

Instead of $P' \sin Z \cos M$ put $(x) + u$, and for $P' \sin Z \sin M$ put $(y) - v$, and it becomes

$$u\left\{15^\circ \sin 1'' (y) \sin \delta - \frac{d(x)}{dt}\right\} + v\left\{15^\circ \sin 1'' (x) \sin \delta + \frac{d(y)}{dt}\right\}$$
$$- P'v\, 15^\circ \sin 1'' \cos Z \cos \delta = 0$$

$$\therefore \cos Z = \frac{\frac{u}{P'}\left\{(y) \sin \delta - \frac{\frac{d(x)}{dt}}{15^\circ \sin 1''}\right\} + \frac{v}{P'}\left\{(x) \sin \delta + \frac{\frac{d(y)}{dt}}{15^\circ \sin 1''}\right\}}{v \cos \delta}$$

But, if α_1 denote the true relative motion in right ascension, and D_1 the true relative motion in declination, and D the declination of the Moon, at the time of true conjunction,

$$\frac{d(x)}{dt} = D_1 \qquad \frac{d(y)}{dt} = \alpha_1 \cos D$$

$$\therefore \cos Z = \frac{\frac{u}{P'}\left\{(y) \sin \delta - \frac{D_1}{15^\circ \sin 1''}\right\} + \frac{v}{P'}\left\{(x) \sin \delta + \frac{\alpha_1 \cos D}{15^\circ \sin 1''}\right\}}{v \cos \delta}$$

Make now the following assumptions:

$$\left.\begin{aligned} (A) &= \frac{\alpha_1 \cos D}{15^\circ \sin 1''} = [0 \cdot 58204]\, \alpha_1 \cos D \\ (B) &= \frac{D_1}{15^\circ \sin 1''} = [0 \cdot 58204]\, D \end{aligned}\right\} \text{------} \quad (C)$$

$$\left.\begin{aligned} \lambda \sin \nu &= \frac{(B) - (y) \sin \delta}{P' \cos \delta} \\ \lambda \cos \nu &= \frac{(A) + (x) \sin \delta}{P' \cos \delta} \end{aligned}\right\} \text{--------} \quad (3)$$

in which (A), (B) may be used as constant quantities throughout the eclipse, and we get

$$\cos Z = \frac{\lambda}{v}(-u \sin \nu + v \cos \nu)$$

The angle rSm is equal to the inclination of the apparent relative orbit with the parallel of declination; denote it by ι', and then $u = \Delta' \cos \iota'$, $v = \Delta' \sin \iota'$, and

$$\therefore \cos Z = \lambda \frac{\sin(\iota' - \nu)}{\sin \iota'} \text{-----------} \quad (4)$$

which is a concise form of the condition to be fulfilled by Z and ι', in order that the place Z may be situated in the limit of a phase.

Since the $\angle\, MSd = S$, and the $\angle\, MSm = 180^\circ - (S + \iota')$, $\angle\, MSm' = S + \iota'$, we have for the triangle MSm

$$Mm^2 = \Delta^2 + \Delta'^2 \pm 2\Delta\Delta' \cos(S + \iota')$$

Divide this by P'^2 and we get

$$\sin^2 Z = \frac{\Delta^2 + \Delta'^2}{P'^2} \pm \frac{2\Delta\Delta'}{P'^2} \cos(S + \iota') \text{--------} \quad (5)$$

for the geometrical relation between S and ι', the upper sign applying to the Northern, and the under sign to the Southern limit. Add this to the square of the preceding equation (4), and there results

$$\lambda^2 \frac{\sin^2(\iota' - \nu)}{\sin^2 \iota'} \pm 2\, \frac{\Delta\,\Delta'}{P'^2} \cos(S + \iota') + \frac{\Delta^2 + \Delta'^2}{P'^2} = 1 \quad - - - (6)$$

for the determination of the angle ι'.

The solution of this equation is by no means very practicable; but as a small error in the value of Z will not sensibly affect the angle ι', we may have recourse to the following indirect process, in which we first consider the angle ι' to be equal to ι, which in most instances is very nearly so. The letter M designates the angle Mmh.

$$\left.\begin{array}{ll} u = \Delta' \cos\iota & D' = \delta \mp u \\ v = \Delta' \sin\iota & \alpha' = \pm \dfrac{v}{\cos D'} \\ \multicolumn{2}{c}{(D) = D + (\alpha - \alpha')\ \text{corr.}} \\ y = (\alpha - \alpha')\cos(D) & x = (D) - D' \\ \tan M = \dfrac{y}{x} & \sin Z = \dfrac{x}{P' \cos M} = \dfrac{y}{P' \sin M} \end{array}\right\} - - - - - - (7)$$

the upper signs being for the Northern, and the under signs for the Southern limit.

Or, if t be the time from the middle of the general eclipse, and ω' the angle under Mm and the line of nearest approach, we shall have

$$Mm \sin\omega' = n \tan\omega = n\,\frac{t}{c}, \text{ and } Mm \cos\omega' = n \pm \Delta',$$

which, observing that $Mm = P' \sin Z$, give the following equations, wherein E and F are constant for all the computations.

$$\left.\begin{array}{l} \left\{\begin{array}{ll} E = \dfrac{n}{c\,(n \pm \Delta')} & F = \dfrac{n \pm \Delta'}{P'} \\ \multicolumn{2}{l}{\text{use} \left\{\begin{array}{l}\text{upper}\\ \text{under}\end{array}\right\} \text{sign for} \left\{\begin{array}{l}\text{northern}\\ \text{southern}\end{array}\right\} \text{limit.}} \end{array}\right. \\ \left\{\begin{array}{lll} \tan\omega' = t \,.\, E & \sin Z = \dfrac{F}{\cos\omega'} & M = (-\iota) \mp \omega' \\ \multicolumn{3}{l}{\text{use} \left\{\begin{array}{l}\text{upper}\\ \text{under}\end{array}\right\} \text{sign for the interval } t \left\{\begin{array}{l}\text{before}\\ \text{after}\end{array}\right\} \text{the middle.}} \end{array}\right. \end{array}\right\} - (8)$$

The sign of the constants E, F, are the same as that of $n \pm \Delta'$; and when this is negative, the angle ω' will be in the second quadrant.

The value of Z determined in this manner will be sufficiently approximate for the purposes of a general map; and where greater minuteness is wanted, it will serve very well to get the angle ι' from the equation (4). For this we have

$$\cot\iota' = \cot\nu - \frac{\cos Z}{\lambda \sin\nu}$$

which may be resolved thus:

$$\sin\phi = \sqrt{\frac{\cos Z}{2\lambda\cos\nu}} \qquad \tan\iota' = \frac{\tan\nu}{\cos 2\phi} \quad - - - - - (9)$$

After ι' is so found, which is only wanted roughly, the accuracy of the calculation may be tested by the equation (4); and then we may proceed to a correct computation of M, Z, by the equations (7), only using ι' instead of ι. We shall thus have in the spherical triangle ZmN, $Zm = Z$, $Nm = 90° - D'$, and the angle $ZmN = M$; and by spherics the following formulæ:

$$\left.\begin{array}{l} \tan\theta = \tan Z \cos M \\ \tan(h-\alpha') = \dfrac{\sin\theta}{\cos(\theta+D')}\tan M \qquad \tan l = \tan(\theta + D')\cos(h-\alpha') \\ \text{check - - - } \dfrac{\sin\theta}{\cos(\theta+D')} = \dfrac{\sin Z \cos M}{\cos(h-\alpha')\cos l} \end{array}\right\} (10)$$

For a map the equations (8) and (10) will alone be amply sufficient. In fact, where a very accurate calculation is wanted, the most satisfactory method will consist in first computing the places roughly; then to reduce the horizontal parallax to the latitude by means of the radius ρ, from the table at page 58, and with the use of the value of Z, to find the augmented semidiameter of the Moon by means of the table at page 83, and thence the proper value of Δ', and then to follow the equations (3), (9), (4), (7), (10).

The first and last points of these limits will have $Z = 90°$. For these places we have therefore by (5)

$$P'^2 = \Delta^2 + \Delta'^2 \pm 2\Delta\Delta'\cos(S+\iota')$$

If we assume $\iota' = \iota$, we shall obviously have $S + \iota' = S + \iota = \omega$, and $\Delta\cos(S+\iota') = n$, ω being the angle under the distance Δ and the nearest approach n, as before used.

$$\therefore P'^2 = \Delta^2 + \Delta'^2 \pm 2\Delta' n$$
$$= \Delta^2 - n^2 + (\Delta' \pm n)^2$$

Consequently

$$\Delta^2 \sin^2\omega = \Delta^2 - n^2 = P'^2 - (n \pm \Delta')^2$$

which divided by $\Delta^2\cos^2\omega = n^2$, gives

$$\tan\omega = \frac{1}{n}\sqrt{\{P'^2 - (n\pm\Delta')^2\}}$$

Therefore by taking the constant c used in the computation of the beginning and ending of a phase on the Earth, we shall have

$$\text{semiduration} = c\tan\omega = \frac{c}{n}\sqrt{\{P'^2 - (n\pm\Delta')^2\}}$$

which may be arranged for calculation as follows:

$$\left.\begin{array}{l} \cos\omega' = \dfrac{n\pm\Delta'}{P'} \qquad \text{semiduration} = c\,\dfrac{P'}{n}\sin\omega' \\ \text{Time of}\left\{\begin{array}{l}\text{entrance}\\ \text{departure}\end{array}\right\} = \text{time of middle}\left\{\begin{array}{c}-\\+\end{array}\right\}\text{semiduration} \end{array}\right\} \text{- - - } (11)$$

The places of entrance and departure of the limits, by continuing the assumption $\iota' = \iota$, may be hence calculated as for the beginning and ending of a phase, only using $\delta \mp u$ instead of δ, thus:

$$\left.\begin{array}{l} \qquad\qquad \delta \mp u = D' \\ a = (-\iota) - \omega \qquad\qquad b = (-\iota) + \omega \\ \textit{For place of entrance,} \\ \sin l = \cos a \cos D' \qquad \tan h = -\dfrac{\tan a}{\sin D'} \\ \textit{For place of departure,} \\ \sin l = \cos b \cos D' \qquad \tan h = -\dfrac{\tan b}{\sin D'} \end{array}\right\} \text{- - - } (12)$$

Having assumed $\iota' = \iota$, the times and places so computed will only be approximate, though sufficiently near for general purposes. For an accurate calculation we must first determine the true value of ι'. Since $Z = 90°$, the equations (9) give $\iota' = \nu$, which is also shown by (4). We may therefore, with the quantities taken out for the respective times of entrance and departure, proceed with the equations (C), (3), use ν instead of ι in (7), and then the final results will be determined by (10). It ought however to be observed, that it will be advisable to take the time of entrance in excess to the next higher integral minute, and to reject fractions of a minute in the time of departure; since by fixing on a time a trifle without the actual limits, the value of $\sin Z$ would come out greater than unity, and the calculation rendered useless in consequence. The places so computed will be accurately situated in the limiting lines, and though not strictly the first and last points of these lines, they will be very nearly so.

IV. Determination of the Place where a given Phase will appear both at Sunrise and Sunset.

We have seen (page 62) that when the rising and setting lines of a phase extend throughout the eclipse, they will compose the figure of an 8 much distorted. The point of intersection or nodus is a place where the phase will be seen to begin and end in the horizon; that is, it will either commence at sunrise and end at sunset, or commence at sunset and end at sunrise. At the time of the middle of the eclipse, the sun will therefore be very nearly on the meridian: if diff. dec. and δ are of the same sign, it will be midnight, because the pole of the earth will have the zenith and sun on opposite sides of it; but when those values are of different signs, it will be noon at the place, for then the zenith and sun will be both on the same side of the pole. If τ denote the semiduration of the eclipse, which begins and ends with the given phase, $\tau \frac{dh}{dt}$ will express the semidiurnal arc of the Sun; and $\therefore -\tan l \tan \delta$ $= \cos\left(\tau \frac{dh}{dt}\right) = \cos(\tau \,.\, 15°)$, which being nearly unity, we must have $l - \delta$ or Z nearly $= 90°$. Consequently for the values of $u, v, \frac{du}{dt}, \frac{dv}{dt}$, at the time of the middle of the eclipse, which will be either noon or midnight, we may assume $\sin Z =$ *unity*, and $M = 0°$ or $180°$. So we get, from the equations (1) and (2), page 66,

$$u = -(x) \pm P' \qquad v = (y)$$

$$\frac{du}{dt} = -\frac{d(x)}{dt} \qquad \frac{dv}{dt} = \frac{d(y)}{dt} \pm P' \,.\, 15° \sin 1'' \sin \delta$$

Let μ denote the hourly motion on the apparent relative orbit, and ι' the inclination with a parallel of declination; then $\mu \cos \iota' = \frac{dv}{dt}$, $\mu \sin \iota' = -\frac{du}{dt}$. Or,

$$\left.\begin{aligned} \mu \sin \iota' &= D_1 \\ \mu \cos \iota' &= \alpha_1 \cos D \pm [9{\cdot}41796]\, P' \sin \delta \end{aligned}\right\} \text{-----} (1)$$

The condition for the greatest phase is $u\frac{du}{dt} + v\frac{dv}{dt} = 0$, or $u \sin \iota' - v \cos \iota' = 0$;

that is,

$$\{-(x) \pm P'\} \sin \iota' - (y) \cos \iota' = 0$$

If t denote the interval past the time of the true conjunction, we shall have $(x) =$ diff. dec. $+ t D_1$ and $(y) = t \alpha_1 \cos D$

$$\therefore \{-\text{diff. dec.} \pm P'\} \sin \iota' - t \{D_1 \sin \iota' + \alpha_1 \cos D \cos \iota'\} = 0$$

Or, since $D_1 = \left(\frac{D_1}{\sin \iota}\right) \sin \iota$, $\alpha_1 \cos D = \left(\frac{D_1}{\sin \iota}\right) \cos \iota$,

$$\{-\text{diff. dec.} \pm P'\} \sin \iota' - t \frac{D_1}{\sin \iota} \cos (\iota' - \iota) = 0$$

Assume

$$k = \frac{-\text{diff. dec.} \pm P'}{\cos (\iota' - \iota)} \quad \text{- - - - - - - - -} \quad (2)$$

and then $t = \dfrac{k \sin \iota' \sin \iota}{D_1}$ or, since $D_1 = \mu \sin \iota'$,

$$t = \frac{k \sin \iota}{\mu} \text{ or } t \textit{ in seconds} = [3{\cdot}55630] \frac{k \sin \iota}{\mu} \quad \text{- - - - - -} \quad (3)$$

When diff. dec. is negative, $M = 180°$ and the lower sign of P' must be used; or, as a general rule, P' must be used with the same sign as that of diff. dec. and, since l nearly $= 90° - \delta$, we can previously correct the horizontal parallax for the place by reducing it to a latitude equal to the complement of δ. The value of t being found, we shall have at the place

When diff. dec. and δ have $\left\{\begin{matrix}\text{the same}\\ \text{different}\end{matrix}\right\}$ signs, app. time of true ☌ $= \left\{\begin{matrix}12^h\\ 0^h\end{matrix}\right\} - t$ - - (4)

which compared with the Greenwich apparent time of the true conjunction will show the longitude of the place.

For the values of u and v we have

$$u = (-\text{diff. dec.} \pm P') - t D_1 = k \cos (\iota' - \iota) - k \sin \iota' \sin \iota = k \cos \iota' \cos \iota$$
$$v = t \alpha_1 \cos D = t D_1 \cot \iota = k \sin \iota' \cos \iota$$

Let n' be the nearest apparent approach of the centres; and the semiduration τ will be determined by the equations

$$n' = \frac{v}{\sin \iota'} \qquad \cos \omega = \frac{n'}{\Delta'} \qquad \tau = \frac{\Delta' \sin \omega \sin \iota'}{D_1}$$

and thence the latitude by the equation, $\tan l = \pm \dfrac{\cos (\tau . 15°)}{\tan \delta}$

Or, using the above value of v,

$$\cos \omega = \frac{k \cos \iota}{\Delta'} \qquad \tau = \frac{\Delta' \sin \omega}{\mu} \qquad \tan l = \pm \frac{\cos (\tau . 15°)}{\tan \delta} \quad \text{- -} \quad (5)$$

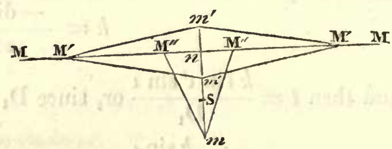
the latitude being of the same name as diff. dec.

The middle of the eclipse will not have the Sun in the horizon, except $k \cos \iota = \Delta'$, $\tau = 0$, $l = 90° - \delta$, and therefore, unless these particular values should happen, the place will not range exactly in the line whereon the middle of the eclipse is seen at sunrise or sunset; this line, which we are about to notice, will pass the intersection at a higher latitude, and will form a very small triangle with the Rising and Setting limits.

V. PLACES WHICH WILL HAVE THE MIDDLE OF THE ECLIPSE WITH THE SUN IN THE HORIZON.

In the first place, we shall suppose the inclination of the apparent orbit to be the same as that of the true. The condition for the middle of the eclipse will then be simply to have the apparent place of the Moon somewhere on the line of nearest approach.

On both sides of S take $Sm = Sm', = s + \sigma$, and m, m' will be the limits between which the apparent place must be, in order that an eclipse may result. On the orbit make $M'm' = P'$. Then if m' falls between S and n this will be the first position in which the eclipse can take place. But, if m' falls beyond the point n, the first position of the Moon will be at M where $Mn = P'$; and in this case, for each position between M and M′ there will evidently be a position of m' on both sides of the orbit, and consequently two corresponding places on the Earth; when the Moon arrives at M′ the remote point m' will be receding from S, and will at that time get beyond the limit of an eclipse, so that the other point m' only will produce an eclipse under the assigned conditions.



Again, when mn is greater than P', it is evident that these limits will continue throughout the whole duration of M′M′ or MM. When mn is less than P', by making $mM'' = P'$ the limits for an eclipse will end at the point M″, and it will be impossible throughout the duration of M″M″. These two cases are the same as those distinguished in the rising and setting limits, page 62, $s + \sigma$ being the value of Δ'.

To determine the times between which these phases are possible, or the semidurations answering to the positions M, M′, M″, we shall in each instance denote the angle Mmn by the character ω, and the following equations will be readily deduced.

(1) WHEN $n < P' - (s + \sigma)$

$$\left.\begin{array}{ll} \omega_1 = 90^\circ & \tau_1 = \dfrac{cP'}{n} \\ \cos\omega_2 = \dfrac{n + (s+\sigma)}{P'} & \tau_2 = \left(\dfrac{cP'}{n}\right)\sin\omega_2 \end{array}\right\} \quad \text{------} \quad (1)$$

$\omega_2 > 90^\circ$ when diff. dec. is negative

These semidurations will give two times of beginning and ending; the one answering to the point M and the other to the point M″. The middle of an eclipse in the horizon will take place from the first beginning to the second beginning, and from the second ending to the first ending.

The places will be determined by producing mM to a distance of 90° from m. If a great circle be drawn through S, so as to be at this point parallel to mM, it will evidently intersect the former at a distance of 90° and determine the same place. We shall therefore, in supposing the places to be determined in this manner, have the following formulæ:

First place of beginning, $\omega_1 = 90°$

$$\left.\begin{array}{ll} \sin l = -\sin \iota \cos \delta & \tan h = -\dfrac{\cot \iota}{\sin \delta} \\ \multicolumn{2}{l}{h \text{ must be taken in the 2nd semicircle, or between } 0° \text{ and } -180°} \end{array}\right\} \text{ - - - (2)}$$

First place of ending,

Change the name of the latitude of the place of beginning, and to the hour angle h apply $\pm 180°$. The results will determine the place of ending.

$$\left.\begin{array}{ll} \text{Second place of beginning,} & \\ a = -\iota - \omega_2 & b = -\iota + \omega_2 \\ \sin l = \cos a \cos \delta & \tan h = -\dfrac{\tan a}{\sin \delta} \\ \text{Second place of ending,} & \\ \sin l = \cos b \cos \delta & \tan h = -\dfrac{\tan b}{\sin \delta} \end{array}\right\} \text{ - - - - - - - - - (3)}$$

The second places of beginning and ending will be two of the extreme points of the lines traced on the Earth. The other two extremes may be determined by computing $\cos \omega = \dfrac{(s+\sigma)-n}{P'}$ and proceeding as before, observing that n must be considered positive, and $\omega > 90°$ when diff. dec. is *positive*. These four extreme points are the same as those of the Northern and Southern limits, the phase being simply external contact.

(2) When $n > P' - (s+\sigma)$ and $< s+\sigma$
The places will be determinable throughout the whole of the first duration found as above. - (4)

(3) When $n > s+\sigma$

$$\cos \omega = \frac{n-(s+\sigma)}{P'} \qquad \tau = \left(\frac{c\,P'}{n}\right) \sin \omega$$

n must here be considered a positive quantity, and ω will be $>90°$ when diff. dec. is negative.
The phase will continue throughout the whole duration, and the extreme places may be computed from this value of ω according to the equations (3). - (5)

Having found the limits between which the phase is possible, the places for any intermediate times may be determined thus,

t denoting the time from the middle,

$$\sin \omega = \left(\frac{n}{c\,P'}\right) t$$

$\omega > 90°$ when diff. dec. is *negative*,
and the places by the equations (3).

If $n < s+\sigma$, suppose n to be positive, and compute

$$\cos \omega = \frac{n-(s+\sigma)}{P'} \qquad \tau = \left(\frac{c\,P'}{n}\right) \sin \omega$$

Then for times, without the limits of this duration, we may determine four places; two with $\omega < 90^\circ$ and two with $\omega > 90^\circ$, which will all fulfil the necessary conditions.

The preceding results have been derived on the assumption of $\iota' = \iota$. They will be sufficiently approximate for a general drawing of the lines on a map, and more particularly as these phenomena cannot be subjcct to minute observation. When, however, from local circumstances or otherwise, greater accuracy is wanted, we must use the proper value of ι' and the relative horizontal parallax reduced to the latitude thus determined. Since $Z = 90^\circ$, the condition for the middle of the eclipse, according to the equation (4) page 67, is $\iota' - \nu = 0$ or $\iota' = \nu$. Let the figure at page 65 represent the positions which answer to the particulars of the present case. Then as $Mm = Mm' = P'$, the $\angle Mmm' = \angle Mm'm$. Denote this angle by θ; the angles NmM, $Nm'M$ by M, M'; and we shall have

$$\begin{array}{ll} \angle NmS = \nu & \angle Nm'S = 180^\circ - \nu \\ M = \theta - \nu & M' = 180^\circ - \nu - \theta \\ \angle MSm = 180^\circ - S - \nu & \angle MSm' = S + \nu \\ \angle SMm = S + \nu - \theta & \angle SMm' = 180^\circ - (S + \nu + \theta) \end{array}$$

With the triangles MSm, MSm', we hence find

$$\sin\theta = \frac{\Delta}{P'}\sin(S+\nu)$$

$$Sm = P'\,\frac{\sin(S+\nu-\theta)}{\sin(S+\nu)} \qquad Sm' = P'\,\frac{\sin(S+\nu+\theta)}{\sin(S+\nu)}$$

which, for computation, may be thus arranged:

$$\left.\begin{array}{ll} g = \dfrac{\sin(S+\nu)}{P'} & \sin\theta = g\,.\,\Delta \\ \multicolumn{2}{c}{\theta \text{ to be } + \text{ or } - \text{ but less than } 90^\circ} \\ Sm = \dfrac{\sin(S+\nu-\theta)}{g} & M = \theta - \nu \\ Sm' = \dfrac{\sin(S+\nu+\theta)}{g} & M' = (180^\circ - \theta) - \nu \end{array}\right\} \text{- - (6)}$$

The points m, m', may in some cases be both on the same side of S, and the value of Sm is only necessary to indicate whether any portion of the Sun is eclipsed or not. To have an eclipse, Sm, taken as a positive quantity, must be less than $s + \sigma$, and we must only determine a place from the angle M when the corresponding value of Sm is within this limit. If Sm, Sm', taken as positive quantities, are both greater than $s + \sigma$, the middle of an eclipse cannot be seen on the Earth under the assumed conditions; on the contrary, if Sm, Sm' so taken are both less than $s + \sigma$, the angles M, M' may both be used, and consequently two places will be determined. In each case, similarly to (3), we adopt the formulæ

$$\sin l = \cos M \cos\delta \qquad \tan h = -\frac{\tan M}{\sin\delta} \Bigg\} \text{- - - - (7)}$$

VI. Central Line.

The places which in succession see a central eclipse are evidently determined by producing S M to a distance Z from S, so that

$$\sin Z = \frac{\Delta}{P'} \quad \text{- - - - - - - - - - - - - - - -} \quad (1)$$

for then the relative parallax P′ will bring the centres to a coincidence. To determine the position of the place on the Earth for any given time we have in the triangle NSZ, thus formed, NS $= 90° - \delta$, $\angle$ NSZ $=$ S, SZ $=$ Z, and hence the following formulæ

$$\left.\begin{array}{l} \tan\theta = \tan Z \cos S \\ \theta \text{ to be } + \text{ or } - \text{ and less than } 90° \\ \tan h = \dfrac{\sin\theta}{\cos(\theta+\delta)} \tan S \qquad \tan l = \tan(\theta+\delta)\cos h \\ h \text{ to be in the same semicircle with S} \\ \text{check - - } \dfrac{\sin\theta}{\cos(\theta+\delta)} = \dfrac{\sin Z \cos S}{\cos h \cos l} \end{array}\right\} \text{ - - } (2)$$

In the course of the general central eclipse, one of the places on the Earth will have the central eclipse at noon. At this instant the bodies will obviously have true as well as apparent conjunction in right ascension, and ∴ $\Delta =$ diff. dec. and S $= 0$. This place is hence determined thus:

$$\left.\begin{array}{l} \sin Z = \dfrac{\text{diff. dec.}}{P'} \qquad l = \delta + Z \\ \text{Z to have the same sign as diff. dec.} \\ \text{App. time of true ☌} = \text{West Long. of place} \end{array}\right\} \text{- - - - - - - - -} (3)$$

These equations (1), (2), (3), involve the horizontal parallax P′, answering to a mean latitude of 45°, which will be sufficiently near for ordinary purposes. Where an accurate result is wanted the calculation must be repeated with the use of the equatorial relative parallax properly reduced to the latitude thus determined.

The first and last places on the Earth, which see a central eclipse, are to be found by the formulæ at pages 60 and 61.

The preceding discussions comprise all that is necessary for the calculation of the lines which are shown in the maps now inserted in the Nautical Almanac, and which are quite sufficient to indicate the general character of the eclipse that may be expected for any particular place. We might now proceed to show the application of these equations in the resolution of innumerable other curious and interesting problems; but such a field of speculation would not conform with the object of this paper, and may the more willingly be abandoned on the consideration that the means of solution may, in most cases, be readily elicited from the equations already established. The following classification of these equations will be found to exhibit, in a comprehensive form, all that will be requisite to direct and facilitate the operations of the calculator, and relieve the mind from any unnecessary reference or consideration.

NOTATION.

D = the ☽'s true declination.
δ = the ☉'s true declination.
α = the true difference of right ascension *in arc*,
 or ☽'s right ascension — ☉'s right ascension.
D_1 = the ☽'s relative motion in declination,
 or ☽'s motion in declination — ☉'s motion in declination.
α_1 = the ☽'s relative motion in right ascension,
 or the motion of the ☽ — that of the ☉.
Diff. dec. = the true difference of declination at ☌ in right ascension,
 viz. ☽'s declination — ☉'s declination, at that time.
P = the ☽'s equatorial horizontal parallax.
π = the ☉'s equatorial horizontal parallax.
$P' = [9{\cdot}99929]\,(P - \pi)$.
s = the ☽'s true semidiameter.
σ = the ☉'s true semidiameter.
Δ = the true distance of the centres.
D', α', s', Δ', the apparent values of D, α, s, Δ.
ω = the angle under Δ and n: in all cases this angle is to be taken positively and between 0° and 180°.

I.—BEGINNING AND ENDING OF A PHASE ON THE EARTH.
1. (D, D_1 and α_1 at ☌) $\quad \tan\iota = \dfrac{D_1}{\alpha_1 \cos D} \qquad n = \text{diff. dec.} \times \cos\iota$ ι of the same sign as D_1 n of the same sign as diff. dec.
2. $c = \dfrac{n \sin\iota\,[3{\cdot}55630]}{D_1} \qquad t = c \tan\iota$ $\sin\iota$ to be found by combining the preceding values of $\cos\iota$ and $\tan\iota$ sign of t to be determined by diff. dec. × D_1
3. Time of middle = Time of ☌ — t For $\left\{\begin{array}{l}\text{Partial}\\ \text{Central}\\ \text{Total}\\ \text{Annular}\end{array}\right\}$ Eclipse, $\Delta = \left\{\begin{array}{l}P' + s + \sigma\\ P'\\ P' + s - \sigma\\ P' - s + \sigma\end{array}\right.$ $\cos\omega = \dfrac{n}{\Delta} \qquad \tau = c\tan\omega$ Time of $\left\{\begin{array}{c}\text{beginning}\\ \text{ending}\end{array}\right\}$ = Time of middle $\left\{\begin{array}{c}-\\ +\end{array}\right\}\tau$ $a = (-\iota) - \omega \qquad b = (-\iota) + \omega$

4. Place of beginning, (δ at σ)

$$\sin l = \cos a \cos \delta \qquad \tan h = -\frac{\tan a}{\sin \delta}$$

H = Apparent Greenwich time of beginning

Longitude E. $= h -$ H

h to be in the same semicircle with a

---

5. Place of ending, (δ at σ)

$$\sin l = \cos b \cos \delta \qquad \tan h = -\frac{\tan b}{\sin \delta}$$

H = Apparent Greenwich time of ending

Longitude E. $= h -$ H

h to be in the same semicircle with b

---

6. For more accurate calculations, reduce the true relative horizontal parallax, by means of the table at page 58, to the latitudes so determined, and recompute.

II. RISING AND SETTING LINES.

For partial eclipse, $\Delta' = s + \sigma$

---

7. When $n > P' - \Delta'$

These limits will extend throughout the entire duration of the general eclipse, and form the distorted figure of an 8, the first and last points being the places of beginning and ending on the Earth.

---

8. When $n < P' - \Delta'$

With $P' - \Delta'$, instead of Δ', compute as for the times of beginning and ending on the Earth; and let these times be t_1, t_2. Then

$$\text{The Risings} \begin{Bmatrix}\text{begin}\\ \text{end}\end{Bmatrix} \text{ at } \begin{Bmatrix}\text{partial beginning}\\ t_1\end{Bmatrix}$$

in which interval the first oval will be completed.

$$\text{The Settings} \begin{Bmatrix}\text{begin}\\ \text{end}\end{Bmatrix} \text{ at } \begin{Bmatrix}t_2\\ \text{partial ending}\end{Bmatrix}$$

in which interval the second oval will be completed.

The limiting places at the times t_1, t_2, are to be found in the same manner as the places of beginning and ending of a phase on the Earth.

9. Places for any times within the limits.

Prepare the constants, $p = \frac{P' - \Delta'}{2}$, $q = \frac{P' + \Delta'}{2}$,

and let t be the time from the middle of the general eclipse.

$$\tan \omega = \frac{t}{c} \qquad \Delta = \frac{n}{\cos \omega}$$

$\omega > 90°$ when n is —

---

10. $$S = (-\iota) \mp \omega$$

Use $\begin{Bmatrix}\text{upper}\\\text{under}\end{Bmatrix}$ sign for $\begin{Bmatrix}\text{before}\\\text{after}\end{Bmatrix}$ the time of middle

---

11. $$\sin \frac{m}{2} = \sqrt{\frac{\left(\frac{\Delta}{2} - p\right)\left(q - \frac{\Delta}{2}\right)}{P' . \Delta}}$$

$\frac{m}{2}$ to be less than 90° and positive

---

12. Place following,

$$\sin l = \cos (S - m) \cos \delta \qquad \tan h = -\frac{\tan (S - m)}{\sin \delta}$$

H = Apparent Greenwich time

Longitude E. = h — H

h to be in the same semicircle with S — m

---

13. Place advancing,

$$\sin l = \cos (S + m) \cos \delta \qquad \tan h = -\frac{\tan (S + m)}{\sin \delta}$$

Longitude E. = h — H

h to be in the same semicircle with S + m

---

14. For a more accurate Determination,

Find the values of D, δ, α, for the given time, and $P' = \rho (P - \pi)$ for the latitude: thence

$$(D) = D + (\alpha \text{ corr. from table, page } 63)$$

$$(x) = (D) - \delta \qquad (y) = \alpha \cos (D)$$

$$\tan S = \frac{(y)}{(x)} \qquad \Delta = \frac{(y)}{\sin S} = \frac{(x)}{\cos S}$$

$$p = \frac{P' - \Delta'}{2} \qquad q = \frac{P' + \Delta'}{2}$$

$$\sin\frac{m}{2}=\sqrt{\frac{\left(\frac{\Delta}{2}-p\right)\left(q-\frac{\Delta}{2}\right)}{P'.\Delta}}$$

The quadrant of S to be determined by (x), (y), as co-ordinates.

With these values of S, m, compute the places by Nos. 12 and 13.

III. PLACE WHERE THE RISING AND SETTING LIMITS INTERSECT, WHEN $n > P' - \Delta'$.

---

15. Find $P' = \rho\,(P - \pi)$, for a latitude equal to the complement of δ at ☌.

$$\mu \sin \iota' = D_1$$

$$\mu \cos \iota' = \alpha_1 \cos D \pm [9{\cdot}41796]\, P' \sin \delta$$

$$k = \frac{-\text{diff. dec.} \pm P'}{\cos(\iota' - \iota)} \qquad t \textit{ in seconds} = [3{\cdot}55630]\,\frac{k \sin \iota}{\mu}$$

Use $\left\{\begin{matrix}\text{upper}\\ \text{under}\end{matrix}\right\}$ signs when diff. dec. is $\left\{\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right.$

---

16. At the Place,

When diff. dec. and δ have $\left\{\begin{matrix}\text{the same}\\ \text{different}\end{matrix}\right\}$ signs, app. time of true ☌ $= \left\{\begin{matrix}12^h\\ 0^h\end{matrix}\right\} - t$

which compared with the Greenwich apparent time of the true ☌, will determine the longitude of the place.

17.

$$\cos \omega = \frac{k \cos \iota}{\Delta'} \qquad \tau = \frac{\Delta' \sin \omega}{\mu}$$

$$\tan l = \pm \frac{\cos(\tau . 15^\circ)}{\tan \delta}$$

l to be of the same name as diff. dec.

IV. PLACES WHERE THE MIDDLE OF THE ECLIPSE IS SEEN WITH THE SUN IN THE HORIZON.

18. When $n < P' - (s + \sigma)$, compute

$$\tau_1 = \frac{cP'}{n}$$

$$\cos \omega_2 = \frac{n \pm (s + \sigma)}{P'} \qquad \tau_2 = \left(\frac{cP'}{n}\right) \sin \omega_2$$

using $s + \sigma$ with a sign the same as that of n.

These semidurations give two times of beginning and ending; the phenomenon will take place on the earth between the times of beginning and between the times of ending.

The places of first and last appearance on the earth to be determined thus:—

For first appearance,

$$\sin l = - \sin i \cos \delta \qquad \tan h = - \frac{\cot \iota}{\sin \delta}$$

For last appearance,

change the name of the latitude of the former place, and to the hour angle h apply $\pm 180°$.

For the extreme points compute also

$$\cos \omega_3 = \frac{n \mp (s + \sigma)}{P'} \qquad \tau_3 = \left(\frac{cP'}{n}\right) \sin \omega_3$$

using $s + \sigma$ with a sign contrary to that of n.

Then with the values of ω_2, ω_3, proceed as for the beginning and ending of a phase on the earth.

When diff. dec. is $+$, $\left\{\begin{matrix}\omega_2\\ \omega_3\end{matrix}\right\}$ gives points meeting $\left\{\begin{matrix}\text{northern}\\ \text{southern}\end{matrix}\right\}$ limit

When diff. dec. is $-$, $\left\{\begin{matrix}\omega_2\\ \omega_3\end{matrix}\right\}$ gives points meeting $\left\{\begin{matrix}\text{southern}\\ \text{northern}\end{matrix}\right\}$ limit

The eclipse will be visible on both sides of the equator.

19. When $n > P' - (s + \sigma)$ and $< s + \sigma$, compute

$$\tau_1 = \frac{cP'}{n}$$

The phenomenon will continue throughout the whole of the duration so found.

The two extreme points will be determined as above with the angle ω_3.

The places of first and last appearance also as above.

20. When $n > s + \sigma$, compute ω_3, τ_3, as above.

The phenomenon will continue throughout the whole duration, and the extreme places will be determined by proceeding with this value of ω as for the beginning and ending of a phase.

These places will in this case be also those of first and last appearance.

21. **Places for any time within the Limits.**

Let t be the time from the middle and compute

$$\sin \omega = \left(\frac{n}{c\,P'}\right) t$$

If $n < s + \sigma$, this ω may be taken both greater and less than 90° when t is greater than τ_3 before found; and then four places will be determined. In all other cases whatever ω must be $> 90°$ when diff. dec. is negative.

The places to be determined by proceeding with ω as for the beginning and ending of a phase.

22. **For a more accurate Determination at any Time,**

Find $P' = \rho\,(P - \pi)$ for the latitude before found.

Find (x), (y), S and Δ as in No. 14.

For the time of ☌ form the constants

$$(A) = [0{\cdot}58204]\ \alpha_1 \cos D \qquad\qquad (B) = [0{\cdot}58204]\ D_1$$

Compute ν from the equations

$$\lambda \cos \nu = \frac{(A) + (x) \sin \delta}{P' \cos \delta} \qquad\qquad \lambda \sin \nu = \frac{(B) - (y) \sin \delta}{P' \cos \delta}$$

23. Then

$$g = \frac{\sin (S + \nu)}{P'} \qquad\qquad \sin \theta = g \,.\, \Delta$$

$$\kappa = \frac{\sin (S + \nu - \theta)}{g} \qquad\qquad M = \theta - \nu$$

$$\kappa' = \frac{\sin (S + \nu + \theta)}{g} \qquad\qquad M' = (180° - \theta) - \nu$$

θ to be + or − but less than 90°

24.

$$\sin l = \cos M \cos \delta \qquad\qquad \tan h = -\frac{\tan M}{\sin \delta}$$

If κ, κ' be both less than $s + \sigma$, the angles M, M′ may be both used in these equations, and two places determined. If one of the quantities κ, κ' be greater than $s + \sigma$, the corresponding M will be excluded, and only one place determined with the other value. If κ, κ' be both greater than $s + \sigma$, both computations will be excluded, and the assumed time will be without the limits of the appearance on the earth.

V. NORTHERN AND SOUTHERN LIMITS FOR ANY PHASE.

$$\text{For}\begin{Bmatrix}\text{Partial}\\ \text{Total}\\ \text{Annular}\end{Bmatrix}\text{appearance, } \Delta' = \begin{cases}(s+6'')+\sigma\\ (s+6'')-\sigma\\ \sigma-(s+6'')\end{cases}$$

$6''$ is added as a mean augmentation of s

25. When $n < P' - \Delta'$ both limits will have place.
When $n > P' - \Delta'$ only one limit will have place, viz.

$$\text{a}\begin{Bmatrix}\text{Northern}\\ \text{Southern}\end{Bmatrix}\text{limit when } n \text{ is}\begin{cases}-\\ +\end{cases}$$

26. **First and last Points or Places of Entrance and Departure.**

$$\cos \text{w} = \frac{n \pm \Delta'}{P'} \qquad \tau = \left(\frac{c\,P'}{n}\right)\sin \text{w}$$

$$\begin{matrix}\text{upper}\\ \text{under}\end{matrix}\Bigg\}\text{ sign for }\begin{Bmatrix}\text{northern}\\ \text{southern}\end{Bmatrix}\text{limit}$$

$$\text{Time of}\begin{Bmatrix}\text{entrance}\\ \text{departure}\end{Bmatrix} = \text{time of middle}\begin{Bmatrix}-\\ +\end{Bmatrix}\tau$$

Places of entrance and departure determined as in Nos. 4 and 5, for the beginning and ending of a phase, using $a = (-\iota) - \text{w}$ and $b = (-\iota) + \text{w}$.

For the appearance of external contact these determinations are included in No. 18, and therefore need not be repeated for these limits.

27. **Places for any Times within the Limits.**

Prepare the following constants, using δ at σ,

$$u = \Delta' \cos \iota \qquad D' = \delta \mp u \qquad \alpha' = \pm \frac{\Delta' \sin \iota}{\cos D'}$$

$$E = \frac{n}{c\,(n \pm \Delta')} \qquad \cos \text{w} = \frac{n \pm \Delta'}{P'} \text{ as above}$$

$$\begin{matrix}\text{upper}\\ \text{under}\end{matrix}\Bigg\}\text{ sign for }\begin{Bmatrix}\text{northern}\\ \text{southern}\end{Bmatrix}\text{limit}$$

28. Let t be the time from the middle of the general eclipse

$$\tan w' = t\,.\,E \qquad \sin Z = \frac{\cos \text{w}}{\cos w'}$$

$$M = (-\iota) \mp w'$$

$$\begin{Bmatrix}\text{upper}\\ \text{under}\end{Bmatrix}\text{sign for}\begin{Bmatrix}\text{before}\\ \text{after}\end{Bmatrix}\text{the middle}$$

29. $$\tan \theta = \tan Z \cos M$$

$$\tan (h - \alpha') = \frac{\sin \theta}{\cos (\theta + D')}\tan M \qquad \tan l = \tan (\theta + D') \cos (h - \alpha')$$

$$\text{check - - - - } \frac{\sin \theta}{\cos (\theta + D')} = \frac{\sin Z \cos M}{\cos (h - \alpha') \cos l}$$

$\theta < 90°$ and same sign as $\cos M$; and $h - \alpha'$ to be in the same semicircle with M.

30. FOR A MORE ACCURATE DETERMINATION AT ANY TIME,

Find $P' = \rho\,(P - \pi)$ for the latitude before found.

Also, with Z find the augmented semidiameter $s' = s +$ augmentation, from the table annexed.

Augmentation of the ☽'s Semidiameter.

Argument—*True Zenith Distance Z.*

Z	For P = 54′	Var. for 10′ in P.	Z	For P = 54′	Var. for 10′ in P.	Z	For P = 54′	Var. for 10′ in P.
°	″	″	°	″	″	°	″	″
0	14·0	5·7	30	12·1	4·9	60	6·9	2·9
1	14·0	5·7	31	12·0	4·8	61	6·7	2·8
2	14·0	5·7	32	11·9	4·8	62	6·5	2·7
3	14·0	5·7	33	11·7	4·7	63	6·2	2·6
4	14·0	5·7	34	11·6	4·7	64	6·0	2·5
5	13·9	5·7	35	11·5	4·7	65	5·8	2·4
6	13·9	5·7	36	11·3	4·6	66	5·6	2·3
7	13·9	5·7	37	11·2	4·6	67	5·4	2·2
8	13·8	5·7	38	11·0	4·5	68	5·2	2·1
9	13·8	5·7	39	10·8	4·4	69	4·9	2·0
10	13·8	5·6	40	10·7	4·4	70	4·7	1·9
11	13·7	5·6	41	10·5	4·3	71	4·5	1·8
12	13·7	5·6	42	10·3	4·3	72	4·2	1·7
13	13·6	5·6	43	10·2	4·2	73	4·0	1·6
14	13·6	5·5	44	10·0	4·1	74	3·8	1·5
15	13·5	5·5	45	9·8	4·1	75	3·5	1·4
16	13·4	5·5	46	9·7	4·0	76	3·3	1·3
17	13·4	5·4	47	9·5	3·9	77	3·1	1·2
18	13·3	5·4	48	9·3	3·9	78	2·8	1·1
19	13·2	5·4	49	9·2	3·8	79	2·6	1·1
20	13·1	5·4	50	9·0	3·7	80	2·4	1·0
21	13·0	5·4	51	8·8	3·6	81	2·1	0·9
22	12·9	5·3	52	8·6	3·5	82	1·9	0·8
23	12·8	5·3	53	8·4	3·4	83	1·7	0·7
24	12·7	5·3	54	8·2	3·3	84	1·4	0·6
25	12·6	5·2	55	8·0	3·2	85	1·2	0·5
26	12·5	5·1	56	7·8	3·2	86	1·0	0·4
27	12·4	5·1	57	7·5	3·1	87	0·7	0·3
28	12·3	5·0	58	7·3	3·1	88	0·5	0·2
29	12·2	4·9	59	7·1	3·0	89	0·3	0·1
30	12·1	4·9	60	6·9	2·9	90	0·0	0·0

Then,

$$\text{For}\left\{\begin{array}{l}\text{Partial}\\ \text{Total}\\ \text{Annular}\end{array}\right\}\text{phase, } \Delta' = \left\{\begin{array}{l} s' + \sigma \\ s' - \sigma \\ \sigma - s' \end{array}\right.$$

31. For the time of ☌ form the constants,

$$(A) = [0{\cdot}58204]\ \alpha_1 \cos D \qquad (B) = [0{\cdot}58204]\ D_1$$

Find the values of D, δ, α for the given time.

$$(D) = D + (\alpha \text{ corr. from table, page } 63)$$

$$(x) = (D) - \delta \qquad (y) = \alpha \cos (D)$$

$$\lambda \cos \nu = \frac{(A) + (x) \sin \delta}{P' \cos \delta} \qquad \lambda \sin \nu = \frac{(B) - (y) \sin \delta}{P' \cos \delta}$$

32. (Z from the first computation)

$$\sin \phi = \sqrt{\frac{\cos Z}{2\ \lambda \cos \nu}} \qquad \tan \iota' = \frac{\tan \nu}{\cos 2\ \phi}$$

$$u = \Delta' \cos \iota' \qquad D' = \delta \mp u$$

$$v = \Delta' \sin \iota' \qquad \alpha' = \pm \frac{v}{\cos D'}$$

$$(D) = D + (\alpha - \alpha') \text{ corr.}$$

$$y = (\alpha - \alpha') \cos (D) \qquad x = (D) - D'$$

$$\tan M = \frac{y}{x} \qquad \sin Z = \frac{x}{P' \cos M} = \frac{y}{P' \sin M}$$

$\left\{\begin{matrix}\text{upper}\\ \text{under}\end{matrix}\right\}$ signs for $\left\{\begin{matrix}\text{northern}\\ \text{southern}\end{matrix}\right\}$ limit

Remaining computation the same as in No. 29.

VI. CENTRAL LINE.

33. The computation of the limiting times and places is comprehended under the head, Beginning and Ending of a Phase on the Earth.

34. Places for any Times within the Limits.

t = the time from the middle

$$\tan \omega = \frac{t}{c} \qquad \Delta = \frac{n}{\cos \omega}$$

$\omega > 90°$ when n is negative

35. $$S = (-\iota) \mp \omega$$

$\left\{\begin{matrix}\text{upper}\\ \text{under}\end{matrix}\right\}$ sign for $\left\{\begin{matrix}\text{before}\\ \text{after}\end{matrix}\right\}$ the time of middle

36. (δ at ☌)

$$\sin Z = \frac{\Delta}{P'} \qquad \tan \theta = \tan Z \cos S$$

$$\tan h = \frac{\sin \theta}{\cos (\theta + \delta)} \tan S \qquad \tan l = \tan (\theta + \delta) \cos h$$

$$\text{check - - - -} \frac{\sin \theta}{\cos (\theta + \delta)} = \frac{\sin Z \cos S}{\cos h \cos l}$$

θ, same sign as cos S, and less than 90°.

h, same semicircle with S.

37. For a more accurate determination at any time,

Find P′, S, Δ, as in No. 14, and proceed again with these as in No. 36.

38. Place where the Eclipse will be central at Noon.

(δ at ☌)

$$\sin Z = \frac{\text{diff. dec.}}{P'} \qquad l = \delta + Z$$

Apparent Greenwich time of true ☌ = Longitude W.

Z < 90° and same sign as diff. dec.

39. For a more accurate determination find the horizontal parallax for the latitude, and with it repeat the operation.

*** All latitudes in the preceding formulæ are to be recognized as geocentric, and will therefore need reducing by the table at page 57.

EXAMPLES.

For an elucidation of the practical application of the preceding formulæ we shall take the Solar Eclipse of May 15, 1836. At the time of New Moon, viz. $2^h\ 7^m{\cdot}0$, the Moon's latitude β is $25'\ 43''$, which being less than $1^\circ\ 23'\ 17''$ the eclipse is certain.—See the limits at page 55. The elements of this eclipse, as related to the equator, are

		d	h	m	s
Greenwich Mean Time of ☌ in R. A. - - -	May	15	2	21	22 ·9
☽'s Declination - - - - - - - - - - - - - -	N.		19	25	9 ·8
☉'s Declination - - - - - - - - - - - - -	N.		18	57	58 ·8
☽'s Hourly Motion in R. A. - - - - - - - -				30	8 ·3
☉'s Hourly Motion in R. A. - - - - - - - -				2	28 ·2
☽'s Hourly Motion in Declination - - - -	N.			9	58 ·7
☉'s Hourly Motion in Declination - - - -	N.				35 ·1
☽'s Equatorial Horizontal Parallax - - - -				54	23 ·9
☉'s Equatorial Horizontal Parallax - - - -					8 ·5
☽'s True Semidiameter - - - - - - - - - -				14	49 ·5
☉'s True Semidiameter - - - - - - - - - -				15	49 ·9

from which we prepare the following values:

		°	′	″		′	″		
☽'s Dec. - -	+	19	25	10	☽'s H. M. in R. A.	30	8		
☉'s Dec. - -	+	18	57	59	☉'s H. M. in R. A.	2	28		
Diff. Dec. - - -	+		27	11	α_1 - - - -	27	40		
☽'s H. M. in Dec.	+		9	59	☽'s Eq. Hor. Par.	54	24		
☉'s H. M. in Dec.	+			35	☉'s Eq. Hor. Par.		9		
D_1 - - -	+		9	24	Rel. Eq. Hor. Par.	54	15	log. -	3 ·51255
								const.	9 ·99929
					P′ - -	54	10	log. -	3 ·51184

I.—BEGINNING AND ENDING ON THE EARTH.

D_1 + 9′ 24″	- - - - - - -	2 ·75128	(1)
α_1 27 40	- - - - - - -	3 ·22011	
		9 ·53117	
D + 19° 25′·2	cos - - -	9 ·97456	
ι + 19 49 - -	tan - - -	9 ·55661	(2)
	cos - - -	9 ·97349	(3)
diff. dec. + 27′ 11″	- - - - - - -	3 ·21245	
n + 25 34	- - - - - - -	3 ·18594	
	sin ι - -	9 ·53010	(2) + (3)
	const. - -	3 ·55630	
		6 ·27234	(4)
	c - - - - -	3 ·52106	(4) — (1)
t - - + $19^m\ 56^s$ - -	c tan ι - -	3 ·07767	

	d	h	m	s	
☌ -	15	2	21	23	
	15	2	1	27	- Middle of general eclipse

$$\begin{array}{lll} P' & 54' \ 10'' & = \Delta \text{ for Central Phase} \\ s+\sigma & 30 \ \ 39 & \\ \hline & 84 \ \ 49 & = \Delta \text{ for Partial Phase} \end{array}$$

Partial			Central		
n	+	3·18594	n	+	3·18594
Δ		3·70663	Δ		3·51184 (log. P′)
ω - 72° 27′ cos	+	9·47931	ω - 61° 49′ cos	+	9·67410
tan		0·49999	tan		0·27109
c		3·52106	c		3·52106
τ - 2^d 54^h 57^m ^s		4·02105	τ - 1^d 43^h 17^m ^s		3·79215
15 2 1 27			15 2 1 27		
14 23 6 30 Beginning			15 0 18 10 Beginning		
15 4 56 24 Ending			15 3 44 44 Ending		
$(-\iota)$	—	19° 49′	$(-\iota)$	—	19° 49′
ω		72 27	ω		61 49
a	—	92 16	a'	—	81 38
b	+	52 38	b'	+	42 0

PLACE OF PARTIAL BEGINNING.

cos a	— 8·59715	tan a	+ 1·40251	Greenwich time	23^h 6^m 30^s
cos δ	+ 9·97576	sin δ	+ 9·51191	Equation	3 56
sin l	— 8·57291	tan h	— 1·89060	H in time	23 10 26
l - - S.	2° 9′	h	— 89° 16′	H in space	347° 37′
Reduction	1	H	347 37		
Latitude S.	2 10	Longitude W.	76 53		

In the same manner may the places of Partial Ending, and Central Beginning and Ending, be calculated, which will come out

Partial Ending	Long. E.	28° 51′	Lat. N.	35° 13′
Central Beginning	Long. W.	98 16	Lat. N.	7 58
Central Ending	Long. E.	52 41	Lat. N.	44 50

II.—RISING AND SETTING LIMITS.

$$\begin{array}{lll} P' & 54' \ 10'' & \\ s+\sigma = \Delta' & 30 \ \ 39 & \\ \hline P' - \Delta' & 23 \ \ 31 & p = 11' \ 46'' \\ P' + \Delta' & 84 \ \ 49 & q = 42 \ \ 25 \end{array}$$

Since $n > P' - \Delta'$, these limits will extend throughout the whole duration of the eclipse; and we may therefore calculate the position of a place for any time between the Greenwich times $14^d\ 23^h\ 6^m\ 30^s$ and $15^d\ 4^h\ 56^m\ 24^s$. As an Example take the time $15^d\ 0^h\ 30^m$.

	d	h	m	s
Assumed time - - - - -	15	0	30	
Time of Middle - - - -	15	2	1	27
t - - - - - - - -		1	31	27

	° ′		° ′		
		t		- - - - - - - - -	3 ·73933
				c - - - - - - -	3 ·52106
$-\iota$ - - - - - —	19 49				
ω - - -	58 51	ω - - - - - - - -	58 51	tan - - - - - - -	0 ·21827
S - - - - —	78 40			cos - - - - - - -	9 ·71403
m - - -	34 2			n - - - - - - -	3 ·18594
		Δ - - - - - - -	49′ 24″	log Δ - - - - -	3 ·47191
S $-m$ - - - —	112 42	$\frac{1}{2}\Delta$ - - - - - - - -	24 42	comp. - - - - - -	6 ·52809
S $+m$ - - - - —	44 38				
		$\frac{1}{2}\Delta - p$ - - - -	12 56	- - - - - - - - - -	2 ·88986
		$q - \frac{1}{2}\Delta$ - - - -	17 43	- - - - - - - - - -	3 ·02653
				Comp. log P′ - -	6 ·48816
					2)18 ·93264
		$\frac{1}{2}m$ - - - - - -	17° 0′·9	sin $\frac{1}{2}m$ - - - -	9 ·46632
		m - - - - - -	34 2		

Place Following.

cos (S$-m$) —	9 ·58648	tan (S$-m$) +	0 ·37850	Greenwich time	0h 30m 0s
cos δ - - - +	9 ·97576	sin δ - - - +	9 ·51191	Equation - - +	3 56
sin l - - - - —	9 ·56224	tan h - - - —	0 ·86659	H in time	0 33 56
l - - - S.	21° 24′	h - - - —	82° 15′	H in space	8° 29′
Reduction -	8	H - - -	8 29		
Latitude - - S.	21 32	Longitude - W.	90 44		

Place Advancing.

cos (S$+m$) - - +	9 ·85225	tan (S$+m$) - - —	9 ·99444
cos δ - - - - - +	9 ·97576	sin δ - - - - - - +	9 ·51191
sin l - - - - - +	9 ·82801	tan h - - - - - +	0 ·48253
l - - - - - N.	42 18	h - - - - - —	108 13
Reduction - - -	11	H - - - - -	8 29
Latitude - - - N.	42 29	Longitude - - - W.	116 42

By taking S $= (-\iota) + \omega$ instead of $(-\iota) - \omega$, similar computations will give the places following and advancing for the interval $t = 1^h\ 31^m\ 27^s$ *after* the time of middle, or for the Greenwich time $15^d\ 3^h\ 32^m\ 54^s$. Much time will be saved by taking the computations two and two in this manner.

III.—PLACE WHERE THE RISING AND SETTING LINES INTERSECT.

	° ′
	90 0
δ - -	18 58
l - -	71 2

	ρ - -	9·99872
	P $-\pi$ - -	3·51255
P′ - - 54′ 5″ - - - - - - - - - -		3·51127
	sin δ -	+ 9·51191
	const. - -	9·41796
+ 4 36 - - - - - - - -		+ 2·44114
	α_1 - - - -	3·22011
	cos D - -	9·97456
+ 26 6 - - - - - - - - - -		3·19467
30 42 =	$\mu \cos \iota'$ - -	+ 3·26529
	$\mu \sin \iota'$ - -	+ 2·75128 (D₁)
ι' - - 17° 1′	tan - - - -	+ 9·48599
	cos - - - -	+ 9·98056
ι - - 19 49	μ - - - -	+ 3·28473
$\iota'-\iota$ - 2 48		
— diff. dec. — 27′ 11″		
+ P′ - - + 54 5		
+ 26 54 - - - - - - - -		+ 3·20790
	cos $(\iota'-\iota)$ - -	9·99948

k - - -	+ 3·20842	- - - - - - - -	+ 3·20842
cos ι - -	+ 9·97349	sin ι - - -	+ 9·53010
	+ 3·18191		3·55630
Δ' - - -	3·26458		+ 6·29482
cos ω - -	9·91733	log t - - -	+ 3·01009
sin ω - -	9·75036		h m s
$\Delta' \sin \omega$ -	3·01494	t - - -	+ 0 17 4
μ - - - - -	3·28473		12 0 0
τ - - - - -	9·73021	App. Time true ☌	11 42 56 at the place
15° - - -	2·95424		h m s
			2 21 23
8° 4′ - -	2·68445	Equation - - - -	3 56
cos - - -	9·99568	App. Time true ☌	2 25 19 at Greenwich
tan δ - -	9·53615	Long. in time	9ʰ 17ᵐ 37ˢ E.
tan l - -	0·45953	Long. in space	139° 24′ E.
	° ′		
l - - N.	70 51		
Reduction	7		
Latitude N.	70 58		

Thus we find the required place to be in Longitude E. 139° 24′ and Latitude N. 70° 58′, where simple contact will have place at sunset and again at sunrise; also the middle of the eclipse would be seen at midnight if it were not intercepted by the opacity of the Earth. The duration of the eclipse will correspond with the duration of the night, and therefore no portion of it will be visible.

IV.—PLACES WHERE THE MIDDLE OF THE ECLIPSE HAS THE SUN IN THE HORIZON.

In the present case n is $>P'-(s+\sigma)$ and $<s+\sigma$. We must therefore proceed as in N° 19.

1. *For the extreme Points,*

		c - -	3·52106	
		P' - -	3·51184	
			7·03290	
n - - -	+ 25′ 34″	- - - - - - - - - -	3·18594	
$-(s+\sigma)$	− 30 39	$\frac{cP'}{n}$ -	3·84696	(1)
	− 5 5	- - - - - - - - −	2·48430	
		P' - -	3·51184	
ω_8 - - 95° 23′ - - - - -		cos —	8·97246	
		sin	9·99808	(2)

	° ′		h m s		
$(-\iota)$	− 19 49	τ_3 - -	1 56 39	- - - 3·84504	(1) + (2)
ω_8	95 23		2 1 27	time of middle	
a	−115 12		0 4 48	time of beginning	
b	+ 75 34		3 58 6	time of ending	

Place of Beginning, or First Extreme Place.

cos a	− 9·62918	tan a - -	+ 0·32738	Greenwich time -	0h 4m 48s
cos δ	+ 9·97576	sin δ - -	+ 9·51191	Equation - - - -	3 56
sin l	− 9·60494	tan h - - -	− 0·81547	H in time	0 8 44
l	S. 23° 45′	h - - -	− 81° 18′	H in space	2° 11′
Reduction	8	H - - -	+ 2 11		
Latitude	S. 23 53	Longitude	W. 83 29		

Place of Ending, or Last Extreme Place.

cos b	+ 9·39664	tan b - -	+ 0·58943	Greenwich time -	3h 58m 6s
cos δ	+ 9·97576	sin δ - -	+ 9·51191	Equation - - - -	3 56
sin l	+ 9·37240	tan h - - -	− 1·07752	H in time	4 2 2
l	N. 13° 38′	h - -	+ 94° 47′	H in space	60° 31
Reduction	5	H - -	60 31		
Latitude	N. 13 43	Longitude	E. 34 16		

2. *For the extreme Times,*

the value of τ_1 taken out from the preceding logarithm of $\frac{cP'}{n}$ is $1^h\ 57^m\ 10^s$.

h m s	
2 1 27	time of middle
1 57 10	- - τ_1
0 4 17	first appearance
3 58 37	last appearance

PLACE OF FIRST APPEARANCE.

sin ι	+ 9·53010	cot ι - -	+ 0·44339	Greenwich time -		0h 4m 17s
cos δ	+ 9·97576	sin δ - -	+ 9·51191	Equation - - - -		3 56
sin l	— 9·50586	tan h - -	— 0·93148	H in	time	0 8 13
					space	2° 3′
l	S. 18° 42′	h - - -	— 83° 19′			
Reduction	7	H - -	+ 2 3			
Latitude	S. 18 49	Longitude	W. 85 22			

PLACE OF LAST APPEARANCE.

Latitude	N. 18° 49′		— 83° 19′	Greenwich time -		3h 58m 37s
			180 0	Equation - - - -		3 56
		h - - -	+ 96 41	H in	time	4 2 33
		H - - -	+ 60 38		space	60° 38′
		Longitude	E. 36 3			

For the computation of places in this line, we have therefore the whole range between the Greenwich mean times $0^h\ 4^m\ 17^s$ and $3^h\ 58^h\ 37^s$. As an example, take the time $1^h\ 30^m$.

Time of Middle	2h 1m 27s		
	1 30		
t - - - -	0 31 27	- - - - -	3·27577
$(-\iota)$ - -	— 19° 49′	$\frac{cP'}{n}$ -	3·84696
ω - - - -	15 34	- - sin -	9·42881
a - - - - -	— 35 23		
b - - - - -	— 4 15		

cos a - -	+ 9·91132	tan a - - -	— 9·85140	Greenwich time		1h 30m 0s
cos δ - -	+ 9·97576	sin δ - - -	+ 9·51191	Equation - - -		3 56
sin l - -	+ 9·88708	tan h - -	+ 0·33949	H in	time	1 33 56
					space	23° 29′
l - - -	N. 50° 27′	h - -	— 114° 35′			
Reduction -	11	H - -	+ 23 29			
Latitude	N. 50 38	Longitude	W. 138 4			

By similarly using the angle b we shall find the position for the interval $31^m\ 27^s$ after the time of middle, or for the time $2^h\ 32^m\ 54^s$; thus,

cos b - -	+	9 ·99880
cos δ - -	+	9 ·97576
sin l - -	+	9 ·97456
		° ′
l - - -	N.	70 35
Reduction -		7
Latitude -	N.	70 42

tan b - - -	—	8 ·87106
sin δ - -	+	9 ·51191
tan h - -	+	9 ·35915
		° ′
h - - -	—	167 7
H - -	+	39 13
Longitude	W.	206 20
	E.	153 40

		h m s
Greenwich time		2 32 54
Equation - - -		3 56
H in	time	2 36 50
	space	39° 13′

The places may be computed by two together in this way; and it will perhaps be a little more convenient to assume a value of t in the first instance. We may take any value which does not exceed τ_1 or $1^h\ 57^m\ 10^s$. In the present example we should take $t = 31^m\ 27^s$, and begin as under:

		° ′
$(-\iota)$	—	19 49
ω - -		15 34
a	—	35 23
b	—	4 15

log t - -	3 ·27577
$\dfrac{cP'}{n}$	3 ·84696
sin ω - -	9 ·42881

		h m s
Time of middle - - -		2 1 27
t - - - - -		31 27
Time before middle	-	1 30 0
Time after middle	-	2 32 54

and then proceed for the places as above.

V.—NORTHERN AND SOUTHERN LIMITS.

1. For the partial Phase, *we have only Southern Line of Simple Contact.*

Constants E, cos w, D′, α'.

	′ ″				
$s + 6''$ - - -	14 56				
σ - - - - -	15 50				
Δ' - - - -	30 46				
n - - -	+ 25 34	- - -	+ 3 ·18594		
$n - \Delta'$ -	— 5 12	- - -	— 2 ·49415	- - - - -	— 2 ·49415
			— 0 ·69179		
		c - - -	3 ·52106	P′ - - -	3 ·51184
		E - - —	7 ·17073	cos w —	8 ·98231
		Δ' - - -	3 ·26623	- - - - - -	3 ·26623
		cos ι	+ 9 ·97349	sin ι	+ 9 ·53010
		log u	+ 3 ·23972		+ 2 ·79633
			′ ″		
		u - -	28 57	cos D′	+ 9 ·97448
			°		
		δ	+ 18 57 59	log α'	— 2 ·82185
		D′	+ 19 26 56	α'	— 11′ 4″

The extreme places will be the same as those which have the middle of the eclipse with the Sun in the horizon, page 90; and we may compute for any time between the corresponding times of beginning and ending, viz. $0^h\ 4^m\ 48^s$ and $3^h\ 58^m\ 6^s$; or we may take any value of t less than $1^h\ 56^m\ 39^s$. For an example take $t = 0^h\ 58^m\ 33^s$.

	h m s		° ′		
Time of middle - -	2 1 27	$(-\iota)$	$-$ 19 49	t - -	3·54568
				E	$-$ 7·17073
t - - - - -	0 58 33	ω' -	100 53	tan ω'	$-$ 0·71641
Before middle - - -	1 2 54 - - -	M	$-$120 42	cos ω'	$-$ 9·27571
After middle - - -	3 0 0 - - -	M	+ 81 4	cos w	$-$ 8·98231
		Z	+ 30°35′·3	sin Z	+ 9·70660
				tan Z	+ 9·77167

Remaining calculation for the time $3^h\ 0^m\ 0^s$.

	tan Z - - + 9·77167	sin Z - - + 9·70660	
	cos M - - + 9·19113	cos M - - + 9·19113	
θ + 5° 14′·7	tan θ - - - + 8·96280	+ 8·89773	
D′ + 19 26·9	sin θ - - - + 8·96098	Comp. cos$(h-\alpha')$ + 0·07456	
θ + D′ + 24 41·6 - -	cos - - - + 9·95835	Comp. cos l - - + 0·03035	
	+ 9·00263 - -	check - - - - + 9·00264	
	tan M - - + 0·80357		
$h-\alpha'$ + 32° 37′·2 - -	tan - - - + 9·80620		h m s
α' $-$ 11·1	cos - - - + 9·92544	Greenwich time	3 0 0
h - - - 32 26·1	tan $(\theta+\mathrm{D}')$ + 9·66258	Equation - - -	3 56
	tan l - - - + 9·58802	H in time -	3 3 56
	l - - - N. 21° 10′·2	H in space +	45° 59′
	Reduction - - 7·6	h - - - - - - +	32 26
	Latitude - N. 21 18·-	Longitude - -	W. 13 33

The calculation for the time $1^h\ 2^m\ 54^s$ is to be performed in this manner, with the same values of tan Z, sin Z, only taking the value of $\mathrm{M} = -120°\ 42'$.

A more accurate Calculation for the Time $3^h\ 0^m\ 0^s$.

Constants (A), (B).

α_1 - - - - - - -	+ 3·22011		
cos D - - - -	9·97456	D_1 - - - - - - - - -	+ 2·75128
Const. - - - -	0·58204 - - - - - - - - - - -		0·58204
	3·77671		3·33332
(A) - - - - - -	+ 1° 39′ 40″	(B) - - - - - - -	+ 0° 35′ 54″

These constants may serve for the computations at all times. For the present example the following is the process employed.

D - - -	+	19° 31′ 34″	(D) α - - -	+	17 49″·0			
α corr. -		1						
δ - - -	+	18 58 21	α - - -	+	3·02898	P — π - -		3·51255
(x) - -	+	0 33 14	cos (D)	+	9·97428	ρ - - - -		9·99982
log (x) -	+	3·29973	log (y) -	+	3·00326	P′ - - - -		3·51237
sin δ - -	+	9·51204	- - - - - - - -	+	9·51204	cos δ - - -		9·97574
	+	2·81177		+	2·51530	P′ cos δ - -		3·48811
(x) sin δ	+	0° 10′ 48″	(y) sin δ	+	0° 5′ 28″	cos Z - - -		9·93493
(A) - -	+	1 39 40	(B) - -	+	0 35 54	2 λ cos ν -		0·63430
	+	1 50 28		+	0 30 26	sin² φ - - -		9·30063
log - - -	+	3·82138	log - - -	+	3·26150	sin φ - - -		9·65032
P′ cos δ -		3·48811	- - - - - - - -		3·48811	φ - - - -		26° 33′·1
λ cos ν -	+	0·33327	λ sin ν -	+	9·77339	2 φ - - -		53 6·2
2 - - -		0·30103	λ cos ν -	+	0·33327	cos 2 φ - -		9·77843
2λ cos ν	+	0·63430	tan ν - -	+	9·44012 - - - - - - - -		+	9·44012
						tan ι′ - - -	+	9·66169
			ι - - - -	+	24° 38′·9 -	cos ι′ - - -	+	9·95851
						sin ι′ - - -	+	9·62020

s - - - - -	14′ 50″					
aug. - - - -	12					
s′ - - - - -	15 2					
σ - - - - -	15 50					
Δ′ - - - - -	30 52 - - - - - - -			3·26764 - - - - - - - -		3·26764
		cos ι′ - - -	+	9·95851	sin ι′ - - +	9·62020
			+	3·22615		2·88784
		u - - - -	+	0° 28′ 3″	cos D′ -	9·97451
		δ - - - -	+	18 58 21		2·91333
		D′ - - - -	+	19 26 24	α′ - - - —	0° 13′ 39″
					α - - - +	0 17 49
		D - - - -	+	19 31 34	α—α′ - - +	31 28
		(α—α′) cor.		3	log - - - +	3·27600
		(D) - - -	+	19 31 37 - -	cos - - - +	9·97428
		D′ - - - -	+	19 26 24	y - - - - +	3·25028
		x - - - -	+	0 5 13 - - - - - - -	+	2·49554
					tan - - - +	0·75474
		M - - - -	+	80° 1′·4 -	sin - - - +	9·99338
					P′ - - - +	3·51237
					+	3·50575
		Z - - - -	+	33° 43′·9 -	sin - - - +	9·74453
					cos - - - +	9·91994

		tan Z - - - - - +	9·82459	sin Z - - - - +	9·74453
		cos M - - - - +	9·23864	- - - - - - - - +	9·23864
θ - - - +	6° 35′·9	tan θ - - - - +	9·06323	+	8·98317
D′ - - +	19 26·4	sin θ - - - - +	9·06034	comp. cos$(h-\alpha')$ +	0·09214
θ + D′ - +	26 2·3	- cos - - - - - +	9·95352	comp. cos l - - +	0·03151
		+	9·10682	- - check - - - +	9·10682
		tan M - - - +	0·75474		
$h-\alpha'$ - +	36° 1′·1	$\{$tan - - - - - +	9·86156		
α' - - - —	13·7	$\{$cos - - - - - +	9·90786	Greenwich time	3h 0m 0s
h - - - +	35 47·4	tan (θ + D′) - +	9·68892	Equation - - -	3 56
		tan l - - - - +	9·59678	H in $\{$time - - +	3 3 56
		l - - - - - - N.	21° 33′·7	H in $\{$space - - +	45° 59′·0
		Reduction - -	7·7	h - - - - - - - +	35 47·4
		Latitude - - N.	21 41·4	Longitude - - W.	10 11·6

This result differs materially from the former one; but we are not to infer that the former position is so far wide of the truth. In general the second determination may be considered as an almost accurate point in the limit, and though the first result be some distance apart, yet it will always be very near to the limiting *line*, sufficiently near indeed for the mapping of the lines. By direct calculations of the eclipse for these places the former will have an eclipse of about $\frac{1}{200}$ of the Sun's diameter, and the latter about $\frac{1}{10000}$ of the diameter, which is too small to be perceptible.

2. For the Annular Phase, *we have both Northern and Southern Limits.*

Constants E, cos w, D′, α', for Northern Limit.

$s+6''$ - - -	14′ 56″				
σ - - - - -	15 50				
Δ' - - - -	0 54				
n - - - - - +	25 34	- - - - - - - +	3·18594		
$n+\Delta'$ - - +	26 28	- - - - - - - +	3·20085	- - - - - - - +	3·20085
		+	9·98509		
		c - - - -	3·52106	P′ - - - - +	3·51184
		E - - - - +	6·46403	cos w - - +	9·68901
				w - - - - +	60° 44′·8
				sin w - - +	9·94075
				$\frac{c\,P'}{n}$ - - - +	3·84696
		τ - - - -	1h 42m 14s - - - - - - - - -	+	3·78771
		Δ' - - - -	1·73239 - - - - - - - -		1·73239
		cos ι - - - +	9·97349	sin ι - - - +	9·53010
		log u - - - +	1·70588	+	1·26249
		u - - - - +	0° 0′ 51″	cos D′ - - +	9·97579
		δ - - - - +	18 57 59	+	1·28670
		D′ - - - - +	18 57 8	α' - - - - +	0° 0′ 19″

The semiduration of the Northern limit on the Earth is therefore $1^h\ 42^m\ 14^s$, and we may calculate for any value of t not exceeding this. A calculation of the extreme places on the Earth is to be performed the same as for the beginning and ending of a phase on the Earth, and will be unnecessary here. As an example, for a time within the limits, we shall take $t = 1^h\ 10^m\ 0^s$.

	h m s			° ′	t		3·62325
Time of Middle	2 1 27	$(-\iota)$	−	19 49	E	+	6·46403
t	1 10 0	ω'		50 43	tan ω'	+	0·08728
Before Middle	0 51 27	M	−	70 32	cos ω'	+	9·80147
After Middle	3 11 27	M	+	30 54	cos w	+	9·68901
		Z	+	50 31·3	sin Z	+	9·88754
					tan Z	+	0·08423

Remaining Calculation for the time $3^h\ 11^m\ 27^s$.

		tan Z	+ 0·08423	sin Z	+ 9·88754
		cos M	+ 9·93352	cos M	+ 9·93352
θ	+ 46° 10′·2	tan θ	+ 0·01775		+ 9·82106
D′	+ 18 57·1	sin θ	+ 9·85817	comp. cos $(h-\alpha')$	+ 0·15622
θ + D′	+ 65 7·3	cos	+ 9·62397	comp. cos l	+ 0·25693
			+ 0·23420	check	+ 0·23421
		tan M	+ 9·77706		
$h-\alpha'$	+ 45° 44′·6	tan	+ 0·01126		h m s
α'	+ 0·3	cos	+ 9·84378	Greenwich time	3 11 27
h	+ 45 44·9	tan $(\theta + \mathrm{D}')$	+ 0·33374	Equation	3 56
		tan l	+ 0·17752	H in time	3 15 23
		l	N. 56 23·8	H in space	+ 48° 51′
		Reduction	10·4	h	+ 45 45
		Latitude	N. 56 34·-	Longitude	W. 3 6

The calculation for $0^h\ 51^m\ 27^s$ is to be performed in the same manner, with $\mathrm{M} = -70°\ 32'$.

A more accurate Calculation for the Time $3^h\ 11^m\ 27^s$.

D	+ 19° 33′ 27″ (D)	α	+ 23′ 6″		
α corr.	1				
δ	+ 18 58 28	α	+ 3·14176	P − π	3·51255
(x)	+ 0 35 0	cos (D)	+ 9·97419	ρ	9·99901
log (x)	+ 3·32222	log (y)	+ 3·11595	P′	3·51156
sin δ	+ 9·51208		+ 9·51208	cos δ	9·97574
	+ 2·83430		+ 2·62803	P′ cos δ	3·48730
(x) sin δ	+ 0° 11′ 23″	(y) sin δ	+ 0° 7′ 5″	cos Z	9·80331
(A)	+ 1 39 40	(B)	+ 0 35 54	2 λ cos ν	0·63740
	+ 1 51 3		+ 0 28 49	sin² ϕ	9·16591
log	+ 3·82367	log	+ 3·23779	sin ϕ	9·58296
P′ cos δ	3·48730		3·48730	ϕ	22° 30′·4
λ cos ν	+ 0·33637	λ sin ν	+ 9·75049	2 ϕ	45 0·8

$\lambda \cos \nu$ - + 0·33637	$\lambda \sin \nu$ - + 9·75049				
2 - - - 0·30103	$\lambda \cos \nu$ - + 0·33637		$\cos 2\phi$ - - 9·84939		
$2\lambda \cos \nu$ + 0·63740	$\tan \nu$ - + 9·41412 - - - - - - - - + 9·41412				

ι' - - - + 20° 9′·4 - { $\tan \iota'$ - - - + 9·56473; $\cos \iota'$ - - - + 9·97255; $\sin \iota'$ - - - + 9·53728 }

	′	″
s - - - - - -	14	50
aug. - - - -		9
s' - - - - - -	14	59
σ - - - - - -	15	50
Δ' - - - - -	0	51 - - - - - - - 1·70757 - - - - - - - - - 1·70757

$\cos \iota'$ - - - + 9·97255 $\sin \iota'$ - - + 9·53728

+ 1·68012   + 1·24485

u - - - - - + 0° 0′ 48″ $\cos$ D′ - + 9·97577

δ - - - - +18 58 28   + 1·26908

D′ - - - - +18 57 40 α' - - - - + 0° 0′ 19″

α - - - - + 0 23 6

D - - - - +19 33 27   { $\alpha - \alpha'$ - - + 0 22 47

($\alpha - \alpha'$) cor. 1   log - - - + 3·13577 }

(D) - - - +19 33 28 - - cos - - + 9·97419

D′ - - - - +18 57 40 y - - - + 3·10996

x - - - - + 0 35 48 - - - - - - - + 3·33203

M - - - - + 30° 57′·1 - { tan - - - + 9·77793; cos - - - + 9·93328 }

+ 3·39875

P′ - - - + 3·51156

Z - - - - + 50° 27′·9 - { sin - - - + 9·88719; cos - - - + 9·80383 }

tan Z - - - - - + 0·08336   sin Z - - - - + 9·88719

cos M - - - - + 9·93328   cos M - - - - + 9·93328

θ - - - + 46° 5′·8   tan θ - - - - + 0·01664   + 9·82047

D′ - - + 18 57·7   sin θ - - - - + 9·85764   comp. cos($h - \alpha'$) + 0·15588

θ + D′ + 65 3·5 - cos - - - - - + 9·62499   comp. cos l - - + 0·25630

+ 0·23265 - - check - - - + 0·23265

tan M - - - - + 9·77793

$h - \alpha'$ - + 45° 41′·9 - { tan - - - - - + 0·01058; cos - - - - - - + 9·84412 }   Greenwich time 3ʰ 11ᵐ 27ˢ

α' - - - + 0·4   tan (θ + D′) + 0·33249   Equation - - - 3 56

h - - - + 45 42·3   tan l - - - - - + 0·17661   H in { time - - + 3 15 23

l - - - - - - - N. 56° 20′·5   space - - + 48° 50′·8 }

Reduction - - 10·4 h - - - - - - - + 45 42·3

Latitude - - N. 56 30·9   Longitude - - - W. 3 8·5

VI.—CENTRAL LINE.

We have, at page 87, found the semiduration of the central appearance on the Earth to be $1^h\ 43^m\ 17^s$, which is therefore the greatest value of t for this phase. As an example for a time within the limits take the same value of t as in the two preceding examples.

	h m s		° ′	t	3·62325
Time of Middle	2 1 27	$(-\iota)$	− 19 49	c	3·52106
t	1 10 0	ω	+ 51 41	tan ω	0·10219
Before Middle	0 51 27	S	− 71 30	cos ω	9·79246
After Middle	3 11 27	S	+ 31 52	n	3·18594
				Δ	3·39348
				P′	3·51184
		Z	+ 49 35·6	sin Z	9·88164
				tan Z	0·06994

Remaining computation for the time $3^h\ 11^m\ 27^s$.

		tan Z	+ 0·06994	sin Z	+ 9·88164
	° ′	cos S	+ 9·92905	cos S	+ 9·92905
θ	+ 44 56·0	tan θ	+ 9·99899		+ 9·81069
δ	+ 18 58·0	sin θ	+ 9·84898	comp. cos h	+ 0·15009
$\theta + \delta$	+ 63 54·0	cos	+ 9·64339	comp. cos l	+ 0·24480
			+ 0·20559	check	+ 0·20558
		tan S	+ 9·79354		
h	+ 44 56·6	tan h	+ 9·99913		h m s
		cos h	+ 9·84991	Greenwich time	3 11 27
		tan $(\theta + \delta)$	+ 0·30990	Equation	3 56
		tan l	+ 0·15981	H in time	3 15 23
		l	N. 55° 18′·7	H in space	+ 48° 51
		Reduction	10·6	h	+ 44 57
		Latitude	N. 55 29·-	Longitude	W. 3 54

A more accurate Calculation.

D	+ 19° 33′ 27″ (D)	α	+ 23′ 6″		+ 3·14176
α corr.	1			cos D	+ 9·97419
δ	+ 18 58 28			(y)	+ 3·11595
(x)	+ 0 35 0			(x)	+ 3·32222
		S	+ 31° 52′·7	tan S	+ 9·79373
P − π	3·51255			cos S	+ 9·92899
ρ	9·99903			Δ	+ 3·39323
P′	3·51158				3·51158
		Z	+ 49° 35′·6	sin Z	+ 9·88165
				tan Z	+ 0·06994

		tan Z	+ 0·06994	sin Z	+ 9·88165
		cos S	+ 9·92899	cos S	+ 9·92899
θ	+ 44° 55′·8	tan θ	+ 9·99893		+ 9·81064
δ	+ 18 58 ·5	sin θ	+ 9·84895	comp. cos *h*	+ 0·15020
θ + δ	+ 63 54 ·3	cos	+ 9·64331	comp. cos *l*	+ 0·24480
			+ 0·20564	check	+ 0·20564
		tan S	+ 9·79373		
h	+ 44° 57′·5	tan *h*	+ 9·99937		
		cos *h*	+ 9·84980	Greenwich time	3h 11m 27s
		tan (θ + δ)	+ 0·31000	Equation	3 56
		tan *l*	+ 0·15980	H in time	+ 3 15 23
		l	N. 55° 18′·7	H in space	+ 48° 50′·8
		Reduction	10·6	*h*	+ 44 57 ·5
		Latitude	N. 55 29 ·3	Longitude	W. 3 53 ·3

Central Eclipse at Noon.

Diff. dec.	3·21245	Time of ☌	2h 21m 23s
P	3·51184	Equation	+ 3 56
sin Z	9·70061	Long. in time	2 25 19 W.
Z	+ 30° 8′	Long. in space	36° 20′ W.
δ	+ 18 58		
l	N. 49 6		
Reduction	11		
Latitude	N. 49 17		

By assuming a series of times, and so computing, in conformity with the preceding examples, a series of points on each of the several limits will be determined; and these points being laid down in a geographical map, with respect to latitude and longitude, it will be easy to trace the lines through them. In this manner has the following map been executed, the assumed law of projection being that the parallels of latitude are concentric and equidistant circles. This projection will be found very suitable when an eclipse, as in the present instance, extends completely round one of the poles of the earth. In other cases, any hypothesis whatever may be assumed, with respect to the law of projection, provided the geographical sketching and eclipse-lines be both laid down on the same principle.

REPRESENTATION OF THE PRINCIPAL LINES FOR THE SOLAR ECLIPSE OF MAY 14-15, 1836.

PHENOMENA FOR A PARTICULAR PLACE.

I.—Eclipses of the Sun.

The chief objects of determination for any particular place are—

1. For a partial eclipse, its magnitude, and the times of beginning, greatest phase, and ending.

2. For a total eclipse, the times of external and internal contact of limbs, or the times of partial and total beginning and ending.

3. For an annular eclipse, the times of exterior and interior contact of limbs, or the times of partial and annular beginning and ending.

Also, to secure certainty in the observation, it is necessary to determine, in each case, the particular points on the limb of the sun, as related either to the vertical or a circle of declination, where these contacts take place; and hence the general configuration of the eclipse.

We first proceed to find expressions for calculating, at any time, the apparent relative position of the two bodies, and the augmentation of the semidiameter of the moon. The parallax in altitude depends on the equation (8) or (9) page 57. It will here be necessary to investigate the effects which this parallax will produce in the right ascension and declination of the Moon. These might be accurately determined by the theory of the small variations of spherical triangles, but not quite so simply as in the following manner:—Assume, as before,

l, the geocentric latitude of the place
R.A., the true right ascension of the Moon
D, the true declination of the Moon, + North, — South
h, the true hour angle of the Moon, + West, — East
r, the distance of the centres of the Earth and Moon

Then if, from the Earth's centre, we take

x, on the intersection of the planes of the meridian and equator, + towards upper meridian
y, in the plane of the equator, + West, — East
z, parallel to the Earth's axis, + North, — South

we shall have, for the position of the Moon,

$$x = r \cos \mathrm{D} \cos h \qquad y = r \cos \mathrm{D} \sin h \qquad z = r \sin \mathrm{D}$$

and, for the position of the observer,

$$(x) = \rho \cos l \qquad (y) = 0 \qquad (z) = \rho \sin l$$

Thus the position of the Moon, in relation to the observer as an origin, will be

$$x' = x - (x) = r \cos \mathrm{D} \cos h - \rho \cos l$$
$$y' = y - (y) = r \cos \mathrm{D} \sin h$$
$$z' = z - (z) = r \sin \mathrm{D} - \rho \sin l$$

and hence, D$'$, h' denoting the apparent declination and hour angle, and r' the distance of the Moon from the observer, we shall have

$$x' = r' \cos \mathrm{D}' \cos h' = r \cos \mathrm{D} \cos h - \rho \cos l$$
$$y' = r' \cos \mathrm{D}' \sin h' = r \cos \mathrm{D} \sin h$$
$$z' = r' \sin \mathrm{D}' \qquad = r \sin \mathrm{D} - \rho \sin l$$

Therefore, as $\cot h' = \frac{x'}{y'}$, $\tan \mathrm{D}' = \frac{z'}{y'} \sin h'$, $\frac{1}{r} = \sin \mathrm{P}$, we find

$$\left.\begin{aligned}
&\cot h' = \cot h - \frac{\rho \sin P \cos l}{\cos D \sin h} \\
&\tan D' = \left(1 - \frac{\rho \sin P \sin l}{\sin D}\right)\frac{\sin h'}{\sin h}\tan D \\
\text{Or,}\quad &\cot h - \cot h' = \left(\frac{\rho \sin P}{\cos D \sin h}\right)\cos l \\
&\frac{\tan D}{\sin h} - \frac{\tan D'}{\sin h'} = \left(\frac{\rho \sin P}{\cos D \sin h}\right)\sin l
\end{aligned}\right\} \text{ - - - - - - (1)}$$

which present a direct method of calculating the apparent position of the Moon, at any time, from that of the true. The former of these equations is evidently subservient to the other, and must necessarily be computed first. As the calculation of these expressions will, in general, require seven places of figures, it will be more convenient to determine the simple effects of the parallax, or the small differences A.R.—A.R.$'$, $D - D'$, for which other expressions may be derived from them. Let A.R. — A.R.$'$ $= h' - h = \Delta h$, and $D - D' = \Delta D$; then by multiplying the equation

$$\cot h - \cot h' = \frac{\rho \sin P \cos l}{\cos D \sin h}$$

by $\sin h \sin h'$, the left-hand member will become $\sin (h' - h)$ or $\sin \Delta h$.

$$\therefore \sin \Delta h = \frac{\rho \sin P \cos l}{\cos D} \sin h'$$

Again we have

$$\frac{\tan D}{\sin h} - \frac{\tan D'}{\sin h'} = \frac{\rho \sin P \sin l}{\cos D \sin h}$$

But

$$\frac{\tan D}{\sin h} - \frac{\tan D'}{\sin h'} = \frac{\tan D - \tan D'}{\sin h} + \left(\frac{1}{\sin h} - \frac{1}{\sin h'}\right)\tan D'$$

$$= \frac{\sin (D - D')}{\sin h \cos D \cos D'} + \frac{\sin h' - \sin h}{\sin h \sin h'}\tan D'$$

$$= \frac{\sin \Delta D}{\sin h \cos D \cos D'} + \frac{2 \sin \frac{1}{2} \Delta h \cos (h + \frac{1}{2}\Delta h)}{\sin h \sin h'}\tan D'$$

Equate this with $\dfrac{\rho \sin P \sin l}{\cos D \sin h}$, and we find

$$\frac{\sin \Delta D}{\cos D \cos D'} = \frac{\rho \sin P \sin l}{\cos D} - \frac{2 \sin \frac{1}{2} \Delta h \cos (h + \frac{1}{2}\Delta h)}{\sin h'} \cdot \frac{\sin D'}{\cos D'}$$

But $2 \sin \frac{1}{2} \Delta h = \dfrac{\sin \Delta h}{\cos \frac{1}{2} \Delta h} = \dfrac{\rho \sin P \cos l}{\cos D} \cdot \dfrac{\sin h'}{\cos \frac{1}{2} \Delta h}$

Substitute this value and multiply by $\cos D \cos D'$, and we deduce

$$\sin \Delta D = \rho \sin P \left\{ \sin l \cos D' - \cos l \sin D' \frac{\cos (h + \frac{1}{2}\Delta h)}{\cos \frac{1}{2} \Delta h} \right\}$$

We shall therefore have, for the parallax of the hour angle, and that of the declination,

$$\left.\begin{aligned}\sin \Delta h &= \frac{(\rho \cos l) \sin \mathrm{P}}{\cos \mathrm{D}} \sin h' \\ \sin \Delta \mathrm{D} &= \sin \mathrm{P} \left\{ (\rho \sin l) \cos \mathrm{D}' - (\rho \cos l) \sin \mathrm{D}' \frac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h} \right\}\end{aligned}\right\} \quad (2)$$

These are still however not adapted for direct calculation, since they involve the apparent quantities h', D', which it is our object to determine. The only use that can be made of them is first to use the true quantities, in order to get the parallaxes and apparent values approximately, and then to repeat the operation. To avoid this difficulty, substitute in the former $h + \Delta h$ instead of h', and in the latter put $\mathrm{D} - \Delta \mathrm{D}$ instead of D' and we get, by expansion,

$$\sin \Delta h = \frac{\rho \cos l \sin \mathrm{P}}{\cos \mathrm{D}} (\sin h \cos \Delta h + \cos h \sin \Delta h)$$

$$\sin \Delta \mathrm{D} = \rho \sin \mathrm{P} \cos \Delta \mathrm{D} \left\{ \sin l \cos \mathrm{D} - \cos l \sin \mathrm{D} \frac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h} \right\}$$

$$+ \rho \sin \mathrm{P} \sin \Delta \mathrm{D} \left\{ \sin l \sin \mathrm{D} + \cos l \cos \mathrm{D} \frac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h} \right\}$$

Divide these by $\cos \Delta h$, $\cos \Delta \mathrm{D}$, respectively, and solve for $\tan \Delta h$ and $\tan \Delta \mathrm{D}$, and we find

$$\tan \Delta h = \frac{\left(\frac{\rho \cos l \sin \mathrm{P}}{\cos \mathrm{D}} \right) \sin h}{1 - \left(\frac{\rho \cos l \sin \mathrm{P}}{\cos \mathrm{D}} \right) \cos h} \quad \text{- - - - - - - - - - - - - - - -} \quad (3)$$

$$\left.\begin{aligned}\tan \Delta \mathrm{D} &= \frac{\rho \sin \mathrm{P} \left\{ \sin l \cos \mathrm{D} - \cos l \sin \mathrm{D} \frac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h} \right\}}{1 - \rho \sin \mathrm{P} \left\{ \sin l \sin \mathrm{D} + \cos l \cos \mathrm{D} \frac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h} \right\}} \\ &= \frac{(\rho \sin l \sin \mathrm{P}) \cos \mathrm{D} \left\{ 1 - \frac{\tan \mathrm{D}}{\tan l} \cdot \frac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h} \right\}}{1 - (\rho \sin l \sin \mathrm{P}) \sin \mathrm{D} \left\{ 1 + \frac{1}{\tan l \tan \mathrm{D}} \cdot \frac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h} \right\}}\end{aligned}\right\} \quad (4)$$

These expressions are all of them perfectly rigorous, and better suited to calculation than they would appear at first sight. The process of the calculation, in which five places of figures will be sufficient, is more detailed in the following equations:

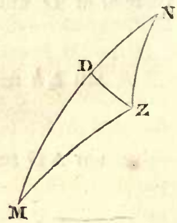
$$\mathrm{n} = \frac{(\rho \cos l) \sin \mathrm{P}}{\cos \mathrm{D}} \qquad \tan \Delta h = \frac{\mathrm{n} \sin h}{1 - \mathrm{n} \cos h} \quad \text{- - - -} \quad (5)$$

$$\left.\begin{array}{ll} c = (\rho \sin l) \sin \mathrm{P} & k = \dfrac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h} \cdot \cot l \\ \mathrm{n}_1 = k \tan \mathrm{D} & \mathrm{n}_2 = \dfrac{k}{\tan \mathrm{D}} \\ \multicolumn{2}{c}{\tan \Delta \mathrm{D} = \dfrac{c \cos \mathrm{D} (1 - \mathrm{n}_1)}{1 - c \sin \mathrm{D} \ (1 + \mathrm{n}_2)}} \end{array}\right\} \ (6)$$

The expression (4) for tan Δ D may, however, be neatly resolved by means of a spherical triangle as follows:

Assume

$$\cos (h) = \frac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h} \text{ - - - - - - } (a)$$



(h) being very nearly equal to $h + \frac{1}{2} \Delta h$. And let N be the North Pole, Z the central zenith, and M the Moon; then $\mathrm{NM} = 90° - \mathrm{D}$, $\mathrm{NZ} = 90° - l$ and the $\angle \mathrm{N} = h$. Without changing these values of NM, N Z, let us suppose the hour angle N to become increased to the value of (h); and with the triangle so constituted suppose the altitude of the Moon to be ε, so that $\mathrm{ZM} = 90° - \varepsilon$; then the spherical relations

$$\sin \mathrm{ZM} \cos \mathrm{M} = \cos \mathrm{NZ} \sin \mathrm{NM} - \sin \mathrm{NZ} \cos \mathrm{NM} \cos \mathrm{N}$$

$$\cos \mathrm{ZM} = \cos \mathrm{NZ} \cos \mathrm{NM} + \sin \mathrm{NZ} \sin \mathrm{NM} \cos \mathrm{N}$$

will give

$$\cos \varepsilon \cos \mathrm{M} = \sin l \cos \mathrm{D} - \cos l \sin \mathrm{D} \cos (h)$$

$$= \sin l \cos \mathrm{D} - \cos l \sin \mathrm{D} \ \frac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h}$$

$$\sin \varepsilon = \sin l \sin \mathrm{D} + \cos l \cos \mathrm{D} \cos (h)$$

$$= \sin l \sin \mathrm{D} + \cos l \cos \mathrm{D} \frac{\cos (h + \frac{1}{2} \Delta h)}{\cos \frac{1}{2} \Delta h}$$

Comparing these with the former expression of (4) we have therefore

$$\tan \Delta \mathrm{D} = \frac{(\rho \sin \mathrm{P}) \cos \varepsilon}{1 - (\rho \sin \mathrm{P}) \sin \varepsilon} \cdot \cos \mathrm{M} \text{ - - - - - - } (f)$$

Before this can be used the angles M and ε must be determined.

Draw ZD perpendicular to MN, and by spherics,

$$\tan \mathrm{ND} = \tan \mathrm{NZ} \cos \mathrm{N} \text{ - - - - - - - - - } (b)$$

$$\sin \mathrm{MD} \tan \mathrm{M} = \tan \mathrm{ZD} = \sin \mathrm{ND} \tan \mathrm{N}$$

$$\therefore \tan \mathrm{M} = \frac{\sin \mathrm{ND}}{\sin \mathrm{MD}} \tan \mathrm{N} \text{ - - - - - - - - - } (c)$$

$$\tan \mathrm{MZ} = \frac{\tan \mathrm{MD}}{\cos \mathrm{M}}, \text{ or } \cot \mathrm{MZ} = \cot \mathrm{MD} \cos \mathrm{M} \text{ - - } (d)$$

Also by (c)

$$\frac{\sin \mathrm{ND}}{\sin \mathrm{MD}} = \frac{\tan \mathrm{M}}{\tan \mathrm{N}} = \frac{\cos \mathrm{N}}{\cos \mathrm{M}} \cdot \frac{\sin \mathrm{M}}{\sin \mathrm{N}} = \frac{\cos \mathrm{N} \sin \mathrm{NZ}}{\cos \mathrm{M} \sin \mathrm{MZ}} \text{ - } (e)$$

Let now $ND = \theta$, and $MD = MN - \theta = 90° - (\theta + D)$; and the equations (a), (b), (c), (d), (e), (f), will give the following

$$\left.\begin{aligned}
\cos(h) &= \frac{\cos(h + \frac{1}{2}\Delta h)}{\cos\frac{1}{2}\Delta h} \text{ - - - - - - - - - - - } (a)\\
\tan\theta &= \cot l \cos(h) \text{ - - - - - - - - - - - - - } (b)\\
\tan M &= \frac{\sin\theta}{\cos(\theta + D)}\tan(h) \text{ - - - - - - - - - } (c)\\
\tan\varepsilon &= \tan(\theta + D)\cos M \text{ - - - - - - - - - - } (d)\\
\frac{\sin\theta}{\cos(\theta + D)} &= \frac{\cos(h)\cos l}{\cos M \cos\varepsilon} \text{ - - - - - - - - - - - - - } (e)\\
\tan\Delta D &= \frac{(\rho\sin P)\cos\varepsilon}{1 - (\rho\sin P)\sin\varepsilon}\cos M \text{ - - - - - - } (f)
\end{aligned}\right\} (7)$$

in which the equation (e) is used as a check on the preceding computations. This check affords a good security to the accuracy of the work, and gives to these equations a decided preference over those of (6), although a trifle more perhaps in point of calculation. They have also another advantage, inasmuch as M may be considered as the parallactic angle and ε the altitude of the Moon; the former of these is useful in determining the position of the line joining the centres of the two bodies in relation to the vertical, and the other is useful in finding the augmentation of the Moon's semidiameter, which we shall now consider.

If s' denote the Moon's apparent semidiameter, and s her true semidiameter as seen from the centre of the Earth, the actual semidiameter of the Moon will be represented by both $r \sin s$ and $r' \sin s'$; also, if a perpendicular be drawn from the centre of the Moon upon the radius ρ produced, this perpendicular will be represented by both $r \sin Z$ and $r' \sin Z'$. We must therefore have $\dfrac{\sin s'}{\sin s} = \dfrac{\sin Z'}{\sin Z}$.

Let M be the true position of the Moon, in the preceding figure, and $\sin ZM \sin\angle NZM = \sin NM \sin N$ will be $\sin Z \sin\angle NZM = \cos D \sin h$; for the apparent position of the Moon the angle NZM will remain the same, and $\sin Z' \sin\angle NZM = \cos D' \sin h'$.

$$\therefore \frac{\sin Z'}{\sin Z} = \frac{\cos D'}{\cos D}\cdot\frac{\sin h'}{\sin h}$$

Also, by means of the equations (8), (9), page 53,

$$\frac{\sin Z'}{\sin Z} = \frac{\rho\sin P\sin Z'}{\rho\sin P\sin Z} = \frac{\sin z}{\rho\sin P\sin Z} = \frac{\cos z}{\rho\sin P\sin Z}\tan z = \frac{\cos z}{1 - \rho\sin P\cos Z}$$

$$\therefore \frac{\sin s'}{\sin s} = \frac{\sin Z'}{\sin Z} = \frac{\cos D'}{\cos D}\cdot\frac{\sin h'}{\sin h} = \frac{\cos z}{1 - \rho\sin P\cos Z} \text{ - } (8)$$

All the preceding formulæ are strict in theory. It now remains to consider what allowances may be made and what facilities given in their actual calculation. In the first place the value of $\cos\frac{1}{2}\Delta h$ may be safely assumed equal to *unity*, and may therefore be rejected in the equations (2), (4), (6), and (7), so that $(h) = h + \frac{1}{2}\Delta h$: it may be shown that this supposition cannot involve an error of more than $0''\cdot03$ in the value of ΔD.

Also, as the arcs P, Δh, Δ D are small, we must have very nearly

$\frac{\sin P}{P} = \sin 1'' = [4 \cdot 68557]$, $\frac{\tan \Delta h}{\Delta h} = \frac{\tan \Delta D}{\Delta D} = \tan 1'' = [4 \cdot 68557]$, where P, Δh, ΔD, denote respectively the numbers of seconds they contain. These equations may be made more exact, for the limits between which the angles are always comprised, by adopting numbers differing a little from sin 1″ and tan 1″; thus, by assuming

$$\frac{\sin P}{P} = [4 \cdot 68555] \qquad \frac{\tan \Delta h}{\Delta h} = [4 \cdot 68561]$$

The first supposition will not in any case involve an error exceeding that of 0″ ·05 in the value of P, nor the second an error of more than 0″ ·1 in the value of Δh, and these are much too small to merit attention; the latter assumption applies equally the same to ΔD.

Thus we shall have $(h) = h + \frac{1}{2}\Delta h$, $\sin P = [4 \cdot 68555]$ P, $\Delta h = [5 \cdot 31439] \tan \Delta h$, $\Delta D = [5 \cdot 31439] \tan \Delta D$; also $\Delta h = \Delta \alpha$, the parallax in right ascension. The equations (3) and (7) may therefore be commodiously arranged as follows:

$$\left.\begin{array}{ccc} & c = [4 \cdot 68555]\rho & \\ A = cP & m = A\cos l & k = \dfrac{m}{\cos D} \\ n = k\cos h & & \Delta\alpha = [5 \cdot 31439]\,\dfrac{k\sin h}{1-n} \end{array}\right\} (9)$$

By taking h less than 180°, positively or negatively, $\Delta\alpha$ will have the same sign as h.

$$\left.\begin{array}{cc} & (h) = h + \frac{1}{2}\Delta\alpha \\ \tan\theta = \cos(h)\cot l & G = \cos(h)\cos l \\ \tan M = \dfrac{\sin\theta}{\cos(\theta + D)}\tan(h) & \tan\varepsilon = \tan(\theta + D)\cos M \\ B = \cos M\cos\varepsilon & \text{check - - } \dfrac{\sin\theta}{\cos(\theta+D)} = \dfrac{G}{B} \\ n_1 = A\sin\varepsilon & \Delta D = [5 \cdot 31439]\,\dfrac{AB}{1-n_1} \end{array}\right\} (10)$$

The auxiliary arc θ may be taken out in the first quadrant, + or —; calling 0° to 180° the first semicircle, and 180° to 360° or 0° to —180° the second semicircle, the parallactic angle M must be taken out in the same semicircle with h; and Δ D will have the same sign as cos M.

It will appear by the preceding investigations that the values of $\Delta\alpha$, ΔD, so deduced, are the quantities to be *subtracted* from the true values of A.R., D, to get the apparent.

As the number n is always very small, the values of comp. log. $(1 - n)$ to the fifth place of figures may be comprised in the following useful Table under the title of *Correction of Log. Parallax*, and conveniently taken out with the nearest third figure of the argument.

Correction of Log. Parallax.

Argument: log. n.

Log n	Corr.	Log n	Corr.	Log n	Corr.	Log n	Corr.	Log n	Corr.
5·00	0	7·100	54	7·400	109	7·700	218	8·000	436
·10	0	·110	55	·410	112	·710	223	·010	447
·20	1	·120	57	·420	114	·720	229	·020	457
·30	1	·130	58	·430	117	·730	234	·030	468
·40	1	·140	60	·440	120	·740	240	·040	479
·50	1	·150	61	·450	123	·750	245	·050	490
·60	2	·160	63	·460	125	·760	251	·060	501
·70	2	·170	64	·470	128	·770	257	·070	513
·80	2	·180	66	·480	131	·780	263	·080	525
·90	3	·190	68	·490	134	·790	269	·090	537
6·00	4	·200	69	·500	137	·800	275	·100	550
·10	6	·210	71	·510	141	·810	281	·110	563
·20	7	·220	72	·520	144	·820	288	·120	576
·30	9	·230	74	·530	148	·830	294	·130	590
·40	11	·240	76	·540	151	·840	302	·140	604
·50	14	·250	77	·550	155	·850	308	·150	618
·60	17	·260	79	·560	158	·860	315	·160	632
·70	22	·270	81	·570	162	·870	323	·170	647
·80	27	·280	83	·580	165	·880	331	·180	663
·90	34	·290	85	·590	169	·890	338	·190	678
7·00	43	·300	87	·600	173	·900	346	·200	694
7·000	43	·310	89	·610	177	·910	355	·210	710
·010	44	·320	91	·620	181	·920	363	·220	727
·020	46	·330	93	·630	186	·930	371	·230	744
·030	47	·340	95	·640	191	·940	379	·240	761
·040	48	·350	98	·650	195	·950	388	·250	779
·050	49	·360	100	·660	199	·960	398	8·260	798
·060	50	·370	102	·670	204	·970	407		
·070	51	·380	104	·680	209	·980	417		
·080	52	·390	107	·690	213	7·990	427		
·090	53	7·400	109	7·700	218	8·000	436		
7·100	54								

This correction is additive when n is positive, and subtractive when n is negative. For the parallax in declination it will always be additive if the Moon be above the horizon.

For the augmentation of the Moon's semidiameter we may assume $\cos z = 1$ and $Z = 90° - \epsilon$, so that

$$\frac{s'}{s} = \frac{1}{1 - \rho \sin P \sin \epsilon} = \frac{1}{1 - n_1}$$

n_1 being the number which enters into the computation of Δ D. Hence

$$s' = \frac{s}{1 - n_1} = \frac{[9 \cdot 43537]\, P}{1 - n_1} \quad \text{- - - -} \quad (11)$$

This and the last formulæ for $\Delta\,\alpha$, Δ D, entirely preclude the necessity of having recourse to a table of the sines and tangents of small arcs, and possess much uniformity and simplicity in their application.

To get the relative parallax of the Moon with respect to the Sun, we must use $P - \pi$, instead of P. If, therefore, P′ denote the value of ρ $(P - \pi)$, or the relative horizontal parallax reduced to the latitude of the place, we must use sin P′, instead of ρ sin P, in the preceding formulæ.

The determination of the apparent relative positions of the centres of the two bodies, as well as the augmentation of the semidiameter of the Moon, at any time, has now been reduced to a practical and expeditious set of formulæ. A series of these apparent positions of the Moon, with respect to that of the Sun, will trace out her apparent relative orbit; and the contact of limbs will evidently take place when the apparent distance of the centres becomes equal to the sum or difference of the semidiameter of the Sun and the augmented semidiameter of the Moon. For a distance equal to the sum of these semidiameters we shall have partial beginning or ending; for a distance equal to their difference we shall have

$$\left\{ \begin{matrix} \text{total} \\ \text{annular} \end{matrix} \right\} \text{ beginning or ending, when } s' \left\{ \begin{matrix} > \\ < \end{matrix} \right\} \sigma$$

Since the hour angle of the bodies is subject to the rapid variation of nearly 15° per hour, the effect produced by parallax will be of so irregular a nature as to give a decided curvature to the apparent relative orbit of the Moon. This curvature will be more strongly characterized when the eclipse takes place at some distance from the meridian or near to the horizon; and the apparent relative hourly motion of the Moon, even during the short interval of the duration of the eclipse, will, through the same irregular influence, experience considerable variation. These circumstances will, in some measure, vitiate any results deduced in the usual manner, by supposing the portion of the orbit described, during the eclipse, to be a straight line, and using the relative motion, at the time of apparent conjunction, as a uniform quantity. The method we are about to pursue is very simple, and consists in assuming any time within the eclipse, and computing for this time the relative positions and motion of the bodies, and thence finding, without any reference whatever, either to the time of the middle of the eclipse, or to the time of conjunction, the times of beginning, greatest phase and ending, and the relative positions of the bodies at these times. The nearer the assumed time is to the time of the greatest phase, the more accurately will the time of that phase be determined; and, similarly, the nearer that time is to the time of beginning or ending, the more certainty will attach to the determination.

To find the apparent relative motion of the Moon, we must first determine the variation which takes in the parallax. For this, take the equations (2), page 103, viz.:

$$\sin \Delta\,\alpha = \sin \Delta\,h = \frac{\sin P' \cos l}{\cos D} \sin h'$$

$$\sin \Delta\,D = \sin P' \left\{ \sin l \cos D' - \cos l \sin D' \frac{\cos\left(h + \frac{1}{2}\Delta\,h\right)}{\cos \frac{1}{2}\Delta\,h} \right\}$$

or, substituting small arcs instead of their sines,

$$\Delta\,\alpha = P' \frac{\cos l}{\cos D} \sin h'$$

$$\Delta\,D = P' \left\{ \sin l \cos D' - \cos l \sin D' \frac{\cos\left(h + \frac{1}{2}\Delta\,h\right)}{\cos \frac{1}{2}\Delta\,h} \right\}$$

Since a portion of the apparent disc of the Moon is projected on that of the Sun, the apparent declination D′ can differ very little from δ. As the hourly variations of these small quantities are only required approximately, we may therefore use δ instead of D′, and neglect Δh, so as to have

$$\Delta \alpha = P' \frac{\cos l}{\cos D} \sin h$$

$$\Delta D = P' (\sin l \cos \delta - \cos l \sin \delta \cos h)$$

which expressions, though rough values of $\Delta \alpha$, ΔD, will give their hourly variations pretty accurately. For these, observing that h is the only quantity which, by its rapid variation, has any sensible influence on these values, we have by differentiation

$$\frac{d(\Delta \alpha)}{dt} = \left(P' \frac{dh}{dt} \sin 1''\right) \frac{\cos l}{\cos D} \cos h$$

$$\frac{d(\Delta D)}{dt} = \left(P' \frac{dh}{dt} \sin 1''\right) \cos l \sin \delta \sin h$$

But by the equations (9)

$$m = [4 \cdot 68555]\, P' \cos l$$

$$n = [4 \cdot 68555]\, P' \frac{\cos l}{\cos D} \cos h$$

Substitute, therefore,

$$P' \frac{\cos l}{\cos D} \cos h = [5 \cdot 31445]\, n$$

$$P' \cos l \qquad = [5 \cdot 31445]\, m$$

and we get

$$\frac{d(\Delta \alpha)}{dt} = [5 \cdot 31445] \left(\frac{dh}{dt} \sin 1''\right) n$$

$$\frac{d(\Delta D)}{dt} = [5 \cdot 31445] \left(\frac{dh}{dt} \sin 1''\right) m \sin \delta \sin h$$

If we adopt 14° 29′ as a mean value of $\frac{dh}{dt}$, we shall have $\frac{dh}{dt} \sin 1'' = [9 \cdot 40274]$, and

$[5 \cdot 31445]\left(\frac{dh}{dt} \sin 1''\right) = [4 \cdot 71719]$ or $[4 \cdot 7172]$. Therefore, if (δ), the value of the Sun's declination at the time of the middle of the eclipse, be adopted in the value of $\frac{d(\Delta D)}{dt}$, we may form the constants

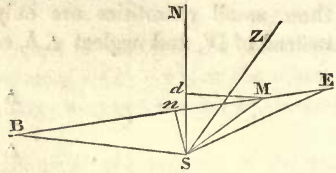
$$\left.\begin{aligned} Q_1 &= [4 \cdot 7172] \\ Q_2 &= [4 \cdot 7172]\, m \sin (\delta) \end{aligned}\right\} \text{- - - - - -} \quad (12)$$

and then, using $\Delta \alpha_1$, ΔD_1 in place of $\frac{d(\Delta \alpha)}{dt}$, $\frac{d(\Delta D)}{dt}$, we shall have

$$\left.\begin{aligned} \Delta \alpha_1 &= Q_1\, n \\ \Delta D_1 &= Q_2 \sin h \end{aligned}\right\} \text{- - - - - - - - -} \quad (13)$$

which offer a simple calculation.

Let now, at any assumed time within the duration of the eclipse, S and M be the apparent positions of the centres of the Sun and Moon; and B M E an arc of a great circle coinciding with the relative direction of the Moon's motion at that time, which arc we shall first adopt in place of the curvilinear orbit actually described. On the circle of declination, S N, demit the great circle perpendicular M d, and suppose B and E to be the positions of the Moon at the respective times of partial beginning and ending of the eclipse, and n the middle point. Assume $SB = SE = s' + \sigma = \Delta'$, $S\,d = x$, $d\,M = y$, $SM = W$, $S\,n = n$, $\angle NSM = S$, $\angle BMd = \angle dSn = \iota$, and the $\angle BSn = \angle ESn = \omega$. Also, for simplicity, let x_1, y_1 denote the hourly variations of x and y.



In determining the value of x, we shall require the α correction, which will reduce the declination of the point M to that of d. This correction is shown in a table at page 63; but, as this small correction may be wanted more accurately than can be obtained from that table, we shall here give some factors for its determination, from which, in fact, the table alluded to has been derived. The correction will resolve as follows:—

$$\tan(D) = \frac{\tan D}{\cos\alpha}$$

$$\therefore \tan\{(D)-D\} = \frac{\dfrac{\tan D}{\cos\alpha} - \tan D}{1 + \dfrac{\tan^2 D}{\cos\alpha}} = \frac{\tan D\,(1-\cos\alpha)}{\cos\alpha + \tan^2 D}$$

Or, supposing $\cos\alpha = 1$ in the denominator,

$$\tan\{(D)-D\} = \frac{\tan D\,(1-\cos\alpha)}{1+\tan^2 D} = \sin 2D \sin^2\frac{\alpha}{2}$$

Suppose, now, α to be expressed numerically in *minutes*, and $(D)-D$ in *seconds* then

$$\tan\{(D)-D\} = \{(D)-D\}\sin 1''$$

$$\sin\frac{\alpha}{2} = \frac{\alpha}{2}\sin 1' = (30\sin 1'')\,\alpha$$

Therefore, by substitution, we find

$$(D)-D = (900\sin 1''\sin 2D)\,\alpha^2$$

Consequently, assuming

$$F = 90000\sin 1''\sin 2D = [9\cdot 63982]\sin 2D$$

we shall have

$$\alpha\text{ corr.} = (D)-D = F\cdot\left(\frac{\alpha}{10}\right)^2$$

The value of F, argument D, is contained in the following small table:—

Factor F for α correction.

D	F	D	F	D	F
°		°		°	
0	·000	10	·149	20	·280
1	·015	11	·164	21	·292
2	·030	12	·178	22	·303
3	·046	13	·191	23	·314
4	·061	14	·205	24	·324
5	·076	15	·218	25	·334
6	·091	16	·231	26	·344
7	·106	17	·244	27	·353
8	·120	18	·256	28	·362
9	·135	19	·268	29	·370
10	·149	20	·280		

$$\alpha \text{ corr. in } \textit{seconds} = \text{F}\,.\left(\frac{\alpha}{10}\right)^2$$

α denoting the number of *minutes* it contains.

From what has preceded, it is evident that $\alpha' = \alpha - \Delta\,\alpha$, is the apparent difference of the right ascensions of the bodies, and that $D' = D - \Delta\,D$, is the apparent declination of the Moon; and that

$$\left.\begin{aligned} x &= \{\,D' + (\alpha - \Delta\,\alpha)\text{ corr.}\,\} - \delta \\ y &= \{\,\alpha \;\; - \Delta\,\alpha\,\}\cos D' \end{aligned}\right\} \text{- - - - - - -} \quad (14)$$

and consequently also

$$\left.\begin{aligned} x_1 &= D_1 - \Delta\,D_1 \\ y_1 &= (\alpha_1 - \Delta\,\alpha_1)\cos D' \end{aligned}\right\} \text{- - - - - - - - - -} \quad (15)$$

Moreover, the figure occupying so small a portion of the sphere, and being composed of arcs of great circles, we may, without any appreciable error, treat these arcs as straight lines; thence we shall obviously have

$$\left.\begin{aligned} &\tan S = \frac{y}{x} \qquad W = \frac{y}{\sin S} = \frac{x}{\cos S} \\ &\cot \iota = \frac{y_1}{x_1} \\ &\text{Hourly motion in the orbit} = \frac{y_1}{\cos \iota} \\ &n = W\cos(S + \iota) \qquad \cos\omega = \frac{n}{\Delta'} \end{aligned}\right\} \text{- - -} \quad (16)$$

Again, in the triangles B S M, E S M,

$$\angle\,BSM = \omega + (S + \iota) \qquad \angle\,ESM = \omega - (S + \iota)$$

and consequently, by plane trigonometry,

$$\mathrm{BM} = \frac{\mathrm{W}}{\cos\omega}\sin\{\omega+(\mathrm{S}+\iota)\} \qquad \mathrm{EM} = \frac{\mathrm{W}}{\cos\omega}\sin\{\omega-(\mathrm{S}+\ )\}$$

$$n\mathrm{M} = \mathrm{W}\sin(\mathrm{S}+\iota)$$

With the above hourly motion in the orbit we shall therefore have

$$\text{Time of describing}\begin{cases}\mathrm{BM} = \dfrac{\mathrm{W}\cos\iota}{y_1\cos\omega}\sin\{\omega+(\mathrm{S}+\iota)\}\\[2ex] n\,\mathrm{M} = \dfrac{\mathrm{W}\cos\iota}{y_1}\sin(\mathrm{S}+\iota)\\[2ex] \mathrm{EM} = \dfrac{\mathrm{W}\cos\iota}{y_1\cos\omega}\sin\{\omega-(\mathrm{S}+\iota)\}\end{cases}$$

Let now, t_1, t_2 be corrections to be applied to the time assumed to get the times of beginning and ending, and (t) the correction for the time of the greatest phase. Then we have evidently

$$\left\{\begin{matrix}t_1\\(t)\\t_2\end{matrix}\right\} = \text{the time of describing}\left\{\begin{matrix}\mathrm{BM}\\n\,\mathrm{M}\\\mathrm{EM}\end{matrix}\right\}\text{with a}\left\{\begin{matrix}\text{negative}\\\text{negative}\\\text{positive}\end{matrix}\right\}\text{sign.}$$

To have these times expressed in seconds, assume

$$c = \frac{\mathrm{W}\cos\iota}{y_1\cos\omega}\times 3600'' = \frac{\mathrm{W}\cos\iota}{y_1}\cdot\frac{[3\cdot55630]}{\cos\omega} \quad - \ - \quad (17)$$

and then we shall derive

$$t_1 = c\sin\{-(\mathrm{S}+\iota)-\omega\} \qquad t_2 = c\sin\{-(\mathrm{S}+\iota)+\omega\}$$

$$(t) = c\cos\omega\sin\{-(\mathrm{S}+\iota)\}$$

and hence

$$\text{The time of}\left\{\begin{matrix}\text{beginning}\\\text{greatest phase}\\\text{ending}\end{matrix}\right\} = \text{Assumed time} + \left\{\begin{matrix}c\sin\{-(\mathrm{S}+\iota)-\omega\}\\c\cos\omega\sin\{-(\mathrm{S}+\iota)\}\\c\sin\{-(\mathrm{S}+\iota)+\omega\}\end{matrix}\right\}\text{-}(18)$$

It has been observed, that any one of these values will be the more to be depended on the more nearly it approximates to the assumed time. Thus, if the assumed time be within ten minutes or so of the end of the eclipse, the point M will approximate so closely to the point E, that no sensible error can arise by supposing the small portion M E of the orbit to be a straight line, and to be passed over by the Moon with an uniform motion. This circumstance renders it advisable, in the first instance, to take the assumed time near to the time of the middle of the eclipse, so as to give a good result for the time of the greatest phase, and results for the times of beginning and ending, which may be nearly equally relied on. Such a computation will be sufficiently exact for the usual purposes of prediction. When the time of beginning or ending is wanted to great minuteness to compare with observation, it will only be necessary to repeat the operation for a time assumed as near as convenient to the first determination, which will mostly give within a fractional part of a second of the true theoretical result; a degree of accuracy, however, seldom wished for, and quite unsupported by the present state of the lunar theory.

To fix on a time near to the middle of the eclipse for the radical computation, one of the most simple expedients will be to determine roughly the time of the apparent conjunction.

We shall now briefly consider the apparent positions of the Moon, as related to the Sun's centre.

It is clear that S is the angle of position of the Moon's centre from the North towards the East, at the time assumed; also, that the angle N S B $= \omega + \iota$ is the similar angle of position from the North towards the West at the time of beginning; and that the angle N S E $= \omega - \iota$ is the angle of position from the North towards the East at the time of ending; and that the angle N S $n = \iota$ is the same angle towards the West at the time of the greatest phase. Therefore, by estimating all these angles towards the East we shall have

$$\text{At}\left\{\begin{array}{c}\text{beginning}\\ \text{greatest phase}\\ \text{ending}\end{array}\right\}\angle \text{ of } ☽\text{'s centre from N. towards E.} = \left\{\begin{array}{l}(-\iota)-\omega\\ (-\iota)\\ (-\iota)+\omega\end{array}\right\} \quad (19)$$

In the computation of the parallax in declination we find an angle M, which in practice may be supposed to be the angle N S Z for the assumed time, the zenith Z being reckoned towards the East; consequently, at this time we shall have S — M for the angle of position of the Moon's centre from the Zenith towards the East. At any other time the parallactic angle M for the latitude of Greenwich may be taken from the following table, arguments the corresponding apparent time and the Sun's declination. This table, for any other place, may be computed by formulæ, such as at page 105, viz.

$$\tan\theta = \cot l \cos h \qquad \tan \mathrm{M} = \frac{\sin\theta}{\cos(\theta+\delta)}\tan h$$

h being the angle answering to the apparent time.

Those who may be engaged in the computation of Eclipses, for any particular places, will find considerable facility in the formation of similar tables.

For an Occultation of a Star by the Moon the argument, instead of the apparent time, will be the star's hour angle, or the sidereal time *minus* the star's right ascension. In this case the required positions will be those of the star with respect to the Moon's centre, which will therefore be different from the angles of position for a solar eclipse, in which the Moon's centre is referred to that of the Sun. The angular positions of the contacts at immersion and emersion will consequently be determined in the same way as for an eclipse of the Sun, and will be estimated in the opposite directions. Thus, for an Occultation,

$$\text{At}\left\{\begin{array}{c}\text{immersion}\\ \text{emersion}\end{array}\right\}\angle \text{ of } * \text{ from N. towards E.} = \left\{\begin{array}{l}(180^\circ-\iota)-\omega\\ (180^\circ-\iota)+\omega\end{array}\right\}$$

And so must 180° be applied to the other angles of position, as expressed for a solar eclipse: this will make the expressions for the direct images of occultations the same as those for the inverted images of eclipses of the Sun, in estimating the contacts either from the north point or from the vertex.

Parallactic Angles for the Latitude of Greenwich.
(same sign as h)
Arguments: Apparent Hour Angle and *Declination.*

Dec. North.	Hour Angle h. 0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0	8	15	22	27	31	35	37	38	39	38	37	35	31	27
1	0	8	15	22	27	32	35	37	38	39	38	37	34	31	27
2	0	8	16	22	28	32	35	37	38	39	38	37	34	31	27
3	0	8	16	22	28	32	35	37	38	39	38	36	34	31	26
4	0	8	16	23	28	32	35	37	38	39	38	36	34	31	26
5	0	9	16	23	28	33	36	38	39	39	38	36	34	30	26
6	0	9	17	23	29	33	36	38	39	39	38	36	34	30	26
7	0	9	17	24	29	33	36	38	39	39	38	36	34	30	26
8	0	9	17	24	29	34	36	38	39	39	38	36	33	30	25
9	0	9	17	24	30	34	37	38	39	39	38	36	33	30	25
10	0	9	18	25	30	34	37	39	39	39	38	36	33	30	25
11	0	9	18	25	31	35	37	39	39	39	38	36	33	29	25
12	0	10	18	25	31	35	38	39	40	39	38	36	33	29	25
13	0	10	19	26	31	35	38	39	40	39	38	36	33	29	25
14	0	10	19	26	32	36	38	40	40	39	38	36	33	29	25
15	0	10	19	27	32	36	39	40	40	39	38	36	33	29	24
16	0	11	20	27	32	37	39	40	40	40	38	36	33	29	24
17	0	11	20	28	33	37	39	40	41	40	38	36	33	29	24
18	0	11	21	28	34	38	40	41	41	40	38	36	33	29	24
19	0	11	21	29	34	38	40	41	41	40	38	36	33	29	24
20	0	12	22	29	35	39	41	41	41	40	38	36	33	29	24
21	0	12	22	30	36	39	41	42	42	40	39	36	33	29	24
22	0	12	23	30	36	40	42	42	42	41	39	36	33	29	24
23	0	13	23	31	37	40	42	43	42	41	39	36	33	29	24
24	0	13	24	32	38	41	43	43	42	41	39	36	33	29	24
25	0	14	25	33	38	42	43	43	43	41	39	36	33	29	24
26	0	14	26	34	39	42	44	44	43	42	39	36	33	29	24
27	0	14	26	35	40	43	44	44	43	42	39	36	33	29	24
28	0	15	27	35	41	43	45	45	44	42	40	37	33	29	24
29	0	16	28	36	41	44	45	45	44	42	40	37	33	29	24

By subtracting the parallactic angle, for the respective times of beginning, greatest phase, and ending, from the foregoing angles of position of the Moon's centre from the North towards the East, we shall evidently obtain the same angles from the Zenith or Vertex towards the East.

If, however, the operation be repeated for the accurate determination of the times of beginning and ending, we shall have in the calculations the angle M, also at these times. Let ι_1, ω_1, M_1 be the angles appertaining to the beginning, and ι_2, ω_2, M_2 those for the ending, and we shall evidently have the following values, which will be more accurate than the preceding:

Parallactic Angles for the Latitude of Greenwich.

(*same sign as h*)

Arguments: *Apparent Hour Angle* and *Declination.*

Dec. South.	Hour Angle h. 0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0	8	15	22	27	31	35	37	38	39	38	37	35	31	27
1	0	8	15	21	27	31	34	37	38	39	38	37	35	32	27
2	0	8	15	21	27	31	34	37	38	39	38	37	35	32	28
3	0	8	15	21	26	31	34	36	38	39	38	37	35	32	28
4	0	7	15	21	26	31	34	36	38	39	38	37	35	32	28
5	0	7	15	21	26	30	34	36	38	39	39	38	36	33	28
6	0	7	14	20	26	30	34	36	38	39	39	38	36	33	29
7	0	7	14	20	26	30	34	36	38	39	39	38	36	33	29
8	0	7	14	20	25	30	33	36	38	39	39	38	36	34	29
9	0	7	14	20	25	30	33	36	38	39	39	38	37	34	30
10	0	7	14	20	25	30	33	36	38	39	39	39	37	34	30
11	0	7	14	20	25	29	33	36	38	39	39	39	37	35	31
12	0	7	14	20	25	29	33	36	38	39	40	39	38	35	31
13	0	7	14	19	25	29	33	36	38	39	40	39	38	35	31
14	0	7	13	19	25	29	33	36	38	39	40	40	38	36	32
15	0	7	13	19	24	29	33	36	38	39	40	40	39	36	32
16	0	7	13	19	24	29	33	36	38	40	40	40	39	37	32
17	0	7	13	19	24	29	33	36	38	40	41	40	39	37	33
18	0	7	13	19	24	29	33	36	38	40	41	41	40	38	34
19	0	7	13	19	24	29	33	36	38	40	41	41	40	38	34
20	0	7	13	19	24	29	33	36	38	40	41	41	41	39	35
21	0	6	13	19	24	29	33	36	39	40	42	42	41	39	36
22	0	6	13	19	24	29	33	36	39	41	42	42	42	40	36
23	0	6	13	18	24	29	33	36	39	41	42	43	42	40	37
24	0	6	13	18	24	29	33	36	39	41	42	43	43	41	38
25	0	6	13	18	24	29	33	36	39	41	43	43	43	42	38
26	0	6	13	18	24	29	33	36	39	42	43	44	44	42	39
27	0	6	13	18	24	29	33	36	39	42	43	44	44	43	40
28	0	6	12	18	24	29	33	37	40	42	44	45	45	43	41
29	0	6	12	18	24	29	33	37	40	42	44	45	45	44	41

$$\text{For}\left\{\begin{array}{c}\text{beginning}\\ \text{greatest phase}\\ \text{ending}\end{array}\right\}\angle\text{ of }☽\text{'s centre from N. towards E.}=\left\{\begin{array}{l}(-\iota_1)-\omega_1\\ (-\iota\,)\\ (-\iota_2)+\omega_2\end{array}\right\}$$

$$\angle\text{ of }☽\text{'s centre from Vertex towards E.}=\left\{\begin{array}{l}(-\iota_1)-\omega_1-M_1\\ (-\iota\,)-M\\ (-\iota_2)+\omega_2-M_2\end{array}\right\}\text{ - (20)}$$

These angles relate to the natural appearance or direct images of the bodies. For the same angles, as they will appear through an inverting telescope, $\pm$ 180° must be applied: this may be simply done by using $(180°-\iota)$ instead of $(-\iota)$.

To find the time when the apparent conjunction takes place, let t denote the interval, in units of an hour, to be applied to the time of the true conjunction, and h the common hour angle of the bodies at the true conjunction. Then the position of the Sun not being supposed to be influenced by parallax, the common apparent hour angle of the bodies, at the time of the apparent conjunction, will be $h' = h + 15° . t$; and therefore, at this time,

$$\alpha = \alpha_1 t \qquad \Delta\alpha = \left(P' \frac{\cos l}{\cos D}\right) \sin(h + 15° . t)$$

so that the condition for apparent conjunction, viz. $\alpha' = \alpha - \Delta\alpha = 0$, gives

$$\alpha_1 t - \left(P' \frac{\cos l}{\cos D}\right) \sin(h + 15° . t) = 0 \quad - - - - - - - - \quad (21)$$

for the determination of the interval t which from this equation will be best found, perhaps, by the usual method of double position. We only want, however, an approximate value, and may therefore avoid much unnecessary labour in estimating this time. Thus, at the time of true conjunction, the same approximate formulæ may be adopted as used at page 109, viz :—

$$\Delta\alpha = P' \frac{\cos l}{\cos D} \sin h$$

$$\Delta\alpha_1 = P' \left(\frac{dh}{dt} \sin 1''\right) \frac{\cos l}{\cos D} \cos h$$

in which $\frac{dh}{dt}$ applies to the Moon. It is evident then, as the true positions of the bodies have no difference of right ascension, that $\Delta\alpha$ is the apparent difference of right ascension; and consequently, as the relative apparent motion in right ascension is $\alpha_1 - \Delta\alpha_1$ or $\alpha_1 - P'\left(\frac{dh}{dt} \sin 1''\right) \frac{\cos l}{\cos D} \cos h$, the correction t to be applied to the time of true conjunction to get that of the apparent, will be

$$t = \frac{P' \frac{\cos l}{\cos D} \sin h}{\alpha_1 - P'\left(\frac{dh}{dt} \sin 1''\right) \frac{\cos l}{\cos D} \cos h} = \frac{\sin h}{\alpha_1 \frac{\cos D}{P' \cos l} - \left(\frac{dh}{dt} \sin 1''\right) \cos h}$$

To facilitate the calculation of this expression, we may use $57'$ as a mean value for P' and $14°$ as a mean value of D. Assume, therefore,

$$\left.\begin{aligned} f &= \frac{100 \cos D}{P' \cos l} = \frac{100}{57} \cdot \frac{\cos 14°}{\cos l} = \frac{[0 \cdot 23103]}{\cos l} \\ \mathfrak{C} &= 100 \left(\frac{dh}{dt} \sin 1''\right) \cos h = [1 \cdot 40274] \cos h \\ \mathfrak{C}^{(1)} &= 100 \sin h \end{aligned}\right\} \quad - - - - \quad (22)$$

for which the nearest whole numbers will suffice, and we shall have

$$t = \frac{\mathfrak{C}^{(1)}}{\alpha_1 . f - \mathfrak{C}} \quad - - - - - - - - - - - - - - \quad (23)$$

The values of the factor f are given for various principal places in the table at page 130: for any place not contained in that table it can be computed from the above expression, and used as a constant factor for all eclipses at that place. The values of $\mathfrak{C}$, $\mathfrak{C}^{(1)}$, are also tabulated at page 129, where, for convenience, the argument h is given in time.

II.—Formulæ of Reduction to different Places.

Before quitting this subject we shall give a method of calculating numerical equations which will serve to determine, with much ease and with sufficient accuracy, the circumstances of an Eclipse of the Sun for any place comprised within a certain range of country. To effect this purpose in the most ample manner, in again proceeding with the general determination of the time of a phase, whose apparent distance of centres is Δ', we shall, in the expressions, separate as much as possible the quantities which involve the position of the place on the earth. The values of the co-ordinates x, y, given at page 111, observing that $\alpha - \Delta\,\alpha = \alpha'$, may be put down as follows:

$$x = \{(\mathrm{D} + \alpha' \text{ corr.}) - \delta\} - \Delta\,\mathrm{D}$$
$$y = \alpha \cos \mathrm{D}' - \Delta\,\alpha \cos \mathrm{D}'$$

and will thus consist of two terms, over the former of which the particular place on the earth has but little influence. If ι denote, as before, the inclination of the apparent relative orbit, these ordinates resolved in the direction of n, perpendicular to the orbit, and in the direction of the orbit, will give $x \cos \iota - y \sin \iota$, and $x \sin \iota + y \cos \iota$. It is evident then, that $x \cos \iota - y \sin \iota$ represents n, the nearest approach, and $x \sin \iota + y \cos \iota$ the distance of the Moon from it, which distance is estimated in the direction of her motion. At the time of the beginning or ending of the phase, the distance of the Moon past the nearest approach, or greatest phase, will be $\mp\,\Delta' \sin \omega$; therefore the Moon precedes this position by a distance equal to $\mp\,\Delta' \sin \omega - (x \sin \iota + y \cos \iota)$, which divided by $\frac{y_1}{\cos \iota}$, the hourly motion in the orbit, gives $\mp\,\frac{\Delta' \cos \iota}{y_1} \sin \omega - \frac{\cos \iota}{y_1}(x \sin \iota + y \cos \iota)$ for the interval, in units of an hour, to be applied to the assumed time T to get the time t when the phase takes place. Assume therefore

$$k = [3\cdot55630]\,\frac{\Delta' \cos \iota}{y_1} \quad \text{- - - - - - - - -} \quad (1)$$

and, the time being counted in seconds,

$$t = \mathrm{T} \mp k \sin \omega - \frac{k}{\Delta'}(x \sin \iota + y \cos \iota) \quad \text{- - - -} \quad (2)$$

Also, $x \cos \iota - y \sin \iota$ expressing the nearest approach, we evidently have

$$\cos \omega = \frac{x \cos \iota - y \sin \iota}{\Delta'} \quad \text{- - - - - - - - - -} \quad (3)$$

Make now the following assumptions:

$$\left.\begin{aligned} p &= \frac{(\mathrm{D} + \alpha' \text{ corr.}) - \delta}{\Delta'} \cos \iota - \frac{\alpha \cos \mathrm{D}'}{\Delta'} \sin \iota \\ q &= \frac{k}{\Delta'}\{(\mathrm{D} + \alpha' \text{ corr.}) - \delta\} \sin \iota + \frac{k}{\Delta'}\,\alpha \cos \mathrm{D}' \cos \iota \end{aligned}\right\} \text{- -} \quad (4)$$

$$\left.\begin{aligned} \Delta p &= \frac{\Delta\,\mathrm{D}}{\Delta'} \cos \iota - \frac{\Delta\,\alpha \cos \mathrm{D}'}{\Delta'} \sin \iota \\ \Delta q &= \frac{k}{\Delta'}\,\Delta\,\mathrm{D} \sin \iota + \frac{k}{\Delta'}\,\Delta\,\alpha \cos \mathrm{D}' \cos \iota \end{aligned}\right\} \text{- - - - - - - -} \quad (5)$$

and, observing the above values of x and y, the equations (2), (3), will become

$$\left.\begin{aligned} \cos \omega &= p - \Delta p \\ t &= \mathrm{T} \mp k \sin \omega - (q - \Delta q) \end{aligned}\right\} \text{- -} \quad (6)$$

Let γ, ψ, be determined by the equations

$$\left.\begin{aligned}\gamma\cos\psi &= \frac{(\mathrm{D}+\alpha'\ \mathrm{corr.})-\delta}{\Delta'}\\ \gamma\sin\psi &= \frac{\alpha\cos \mathrm{D}'}{\Delta'}\end{aligned}\right\}\ \text{- - - - - - (7)}$$

and p, q, will take the following values

$$\left.\begin{aligned}p &= \gamma\cos(\psi+\iota)\\ q &= k\gamma\sin(\psi+\iota)\end{aligned}\right\}\ \text{- - - - - - (8)}$$

It yet remains to determine the values of Δp, Δq, which depend on the position of the place of observation. Adopting the notation used in the equations (3), (4), (9), (10), pages 103 and 106, we shall have

$$\Delta\alpha = \frac{[5\cdot31439]\,\mathrm{A}}{1-n}\cdot\frac{\cos l}{\cos \mathrm{D}}\sin h$$

$$\Delta \mathrm{D} = \frac{[5\cdot31439]\,\mathrm{A}}{1-n_1}\left\{\sin l\cos \mathrm{D} - \cos l\sin \mathrm{D}\frac{\cos(h+\frac{1}{2}\Delta\alpha)}{\cos\frac{1}{2}\Delta\alpha}\right\}$$

To simplify the expressions, let

$$b = \frac{[5\cdot31439]\,\mathrm{A}}{(1-n)\,\Delta'}\cdot\frac{\cos \mathrm{D}'}{\cos \mathrm{D}}$$

$$c = \frac{[5\cdot31439]\,\mathrm{A}}{(1-n_1)\,\Delta'}\cdot\cos \mathrm{D} \qquad a = \frac{[5\cdot31439]\,\mathrm{A}}{(1-n_1)\,\Delta'}\cdot\sin \mathrm{D}$$

and

$$\Delta\alpha = \frac{b\,\Delta'\cos l\sin h}{\cos \mathrm{D}'}$$

$$\Delta \mathrm{D} = c\,\Delta'\sin l - a\,\Delta'\cos l\,\frac{\cos(h+\frac{1}{2}\Delta\alpha)}{\cos\frac{1}{2}\Delta\alpha}$$

$$= c\,\Delta'\sin l - a\,\Delta'\cos l\cos h + a\,\Delta'\tan\frac{\Delta\alpha}{2}\cos l\sin h$$

These substituted in (5) give

$$\Delta p = c\cos\iota\sin l - \cos l\left\{a\cos\iota\cos h - (a\cos\iota\tan\frac{\Delta\alpha}{2} - b\sin\iota)\sin h\right\}$$

$$\Delta q = kc\sin\iota\sin l - \cos l\left\{ka\sin\iota\cos h - (ka\sin\iota\tan\frac{\Delta\alpha}{2} + kb\cos\iota)\sin h\right\}$$

The value of b contains the factor $\dfrac{\cos \mathrm{D}'}{\cos \mathrm{D}}$, for which we have

$$\frac{\cos \mathrm{D}'}{\cos \mathrm{D}} = \cos\Delta \mathrm{D}\,(1+\tan \mathrm{D}\tan\Delta \mathrm{D})$$

Substitute the first value of $\tan\Delta \mathrm{D}$, page 103, and

$$\frac{\cos \mathrm{D}'}{\cos \mathrm{D}} = \cos\Delta \mathrm{D}\cdot\frac{1-\rho\sin \mathrm{P}\,\dfrac{\cos l}{\cos \mathrm{D}}\cos(h)}{1-\rho\sin \mathrm{P}\{\sin l\sin \mathrm{D}+\cos l\cos \mathrm{D}\cos(h)\}}$$

Or, putting h instead of (h) in the numerator, which cannot sensibly affect the value of the fraction,

$$\frac{\cos \mathrm{D}'}{\cos \mathrm{D}} = \cos\Delta \mathrm{D}\cdot\frac{1-n}{1-n_1}$$

This, supposing $\cos \Delta D = 1$, reduces the values of the constants a, b, c, to the following,

$$\left.\begin{array}{l} b = \dfrac{[5\cdot31439]\,A}{(1-n_1)\,\Delta'} \\ c = b\cos D \qquad a = b\sin D \end{array}\right\} \quad \text{- - - - - - (9)}$$

If e be a small arc determined by $g\cos e = b$, $g\sin e = a\tan\frac{\Delta\alpha}{2}$, we shall have

$$a\cos\iota\tan\frac{\Delta\alpha}{2} - b\sin\iota = g\sin(-\iota + e) = g\cos(90^\circ + \iota - e)$$

$$ka\sin\iota\tan\frac{\Delta\alpha}{2} + kb\cos\iota = kg\cos(\iota - e) = kg\sin(90^\circ + \iota - e)$$

However, as e must always be a very small arc, we may suppose $\cos e = 1$; also $g = b$ and, e being expressed in minutes,

$$e = \frac{1}{60}\cdot\frac{a}{b}\cdot\frac{\Delta\alpha}{2} = \frac{a}{120\,b}\cdot\Delta\alpha = [7\cdot9208]\,\Delta\alpha\sin D \quad \text{- - - - (10)}$$

If therefore

$$\chi = (90^\circ + \iota) - e \quad \text{- - - - - - - - - - - - - - - - - - (11)}$$

the values of Δp, Δq, will be

$$\left.\begin{array}{l} \Delta p = c\cos\iota\sin l - \cos l\,(a\cos\iota\cos h - b\cos\chi\sin h) \\ \Delta q = kc\sin\iota\sin l - \cos l\,(ka\sin\iota\cos h - kb\sin\chi\sin h) \end{array}\right\} \quad \text{- - - (12)}$$

Assume now

$\lambda =$ the longitude of the place, + East, — West.

$H =$ the true hour angle of the Moon, for the meridian of Greenwich.

$$\left.\begin{array}{rcl} L' &=& c\cos\iota \\ \gamma'\cos(\psi' - H) &=& a\cos\iota \\ \gamma'\sin(\psi' - H) &=& b\cos\chi \end{array}\right\} \quad \text{- - - - - - - - - - - - (13)}$$

$$\left.\begin{array}{rcl} L'' &=& kc\sin\iota \\ \gamma''\cos(\psi'' - H) &=& ka\sin\iota \\ \gamma''\sin(\psi'' - H) &=& kb\sin\chi \end{array}\right\} \quad \text{- - - - - - - - - - - - (14)}$$

and we shall have

$$\Delta p = L'\sin l - \gamma'\cos l\cos(\psi' + h - H) = L'\sin l - \gamma'\cos l\cos(\psi' + \lambda)$$

$$\Delta q = L''\sin l - \gamma''\cos l\cos(\psi'' + h - H) = L''\sin l - \gamma''\cos l\cos(\psi'' + \lambda)$$

so that the equations (6) will become

$$\left.\begin{array}{l} \cos\omega = p - L'\sin l + \gamma'\cos l\cos(\psi' + \lambda) \\ t = (T - q) \mp k\sin\omega + L''\sin l - \gamma''\cos l\cos(\psi'' + \lambda) \end{array}\right\} \quad \text{- - (15)}$$

After computing the constants k, p, q, L', L'', ψ', ψ'', by means of the equations (1), (7), (8), (9), (10), (11), (13), and (14), we shall thus have two numerical equations for the determination of ω and the Greenwich time t of the phase, for any place whose latitude is l and longitude λ. The accuracy of the determination will principally depend on the proximity of the resulting time t to the assumed time T; and therefore the result will be near the truth for all places where the phase will take place near to this time.

In making these calculations for any particular portion of country, which for the partial phase will be necessary for both the beginning and ending, it will be best in the first instance to fix upon a place near the centre and compute the eclipse for that place, which computation will furnish good mean values for the data D, α, δ, α' corr., ΔD, $\Delta\alpha$, ι, y_1, Δ', A, and comp. log $(1-n_1)$.

By supposing

$$\left.\begin{array}{ll} \xi' \cos l' = \gamma' & \xi'' \cos l'' = \gamma'' \\ \xi' \sin l' = -\mathrm{L}' & \xi'' \sin l'' = -\mathrm{L}'' \end{array}\right\} \text{ - - - - - - - - - } (16)$$

the expressions

$$-\mathrm{L}' \sin l + \gamma' \cos l \cos (\psi' + \lambda)$$
$$-\mathrm{L}'' \sin l + \gamma'' \cos l \cos (\psi'' + \lambda)$$

will take the forms

$$\xi' \{\sin l' \sin l + \cos l' \cos l \cos (\psi' + \lambda)\}$$
$$\xi'' \{\sin l'' \sin l + \cos l'' \cos l \cos (\psi'' + \lambda)\}$$

and, without the factors ξ', ξ'', will represent the cosines of the distances of the proposed place from two other places whose latitudes are l', l'', and west longitudes ψ', ψ''. The former of these two places will be near to the southern pole of the true relative orbit, and the latter will be near to the orbit itself, and will precede the Moon by a distance nearly equal to 90°.

For purposes, which do not require great minuteness, the preceding equations will admit of some simplification by neglecting the small angle e. Add the squares of the equations (13) and (14), observing that $c^2 + a^2 = b^2$, and

$$\mathrm{L}'^2 + \gamma'^2 = b^2 (\cos^2 \iota + \cos^2 \chi)$$
$$\mathrm{L}''^2 + \gamma''^2 = k^2 b^2 (\sin^2 \iota + \sin^2 \chi)$$

which give the general relation

$$\mathrm{L}'^2 + \gamma'^2 + \frac{\mathrm{L}''^2}{k^2} + \frac{\gamma''^2}{k^2} = 2 b^2 \text{ - - - - - - - - - } (17)$$

By neglecting e, $\chi = 90° + \iota$, $\cos \chi = -\sin \iota$, $\sin \chi = \cos \iota$; and then

$$\mathrm{L}'^2 + \gamma'^2 = b^2$$
$$\mathrm{L}''^2 + \gamma''^2 = k^2 b^2$$

which united with the equations (16) give $\xi' = b$, $\xi'' = kb$, and hence

$$\sin l' = -\frac{\mathrm{L}'}{\xi'} = -\frac{c \cos \iota}{b} = -\cos \mathrm{D} \cos \iota$$

$$\gamma' = \xi' \cos l' = b \cos l'$$

$$\sin (\psi' - \mathrm{H}) = \frac{b \cos \chi}{\gamma'} = -\frac{b \sin \iota}{b \cos l'} = -\frac{\sin \iota}{\cos l'}$$

$$\sin l'' = -\frac{\mathrm{L}''}{\xi''} = -\frac{k c \sin \iota}{k b} = -\cos \mathrm{D} \sin \iota$$

$$\gamma'' = \xi'' \cos l'' = k b \cos l''$$

$$\sin (\psi'' - \mathrm{H}) = \frac{k b \sin \chi}{\gamma''} = \frac{k b \cos \iota}{k b \cos l''} = \frac{\cos \iota}{\cos l''}$$

Or,

$$\left.\begin{array}{l} \sin l' = -\cos \mathrm{D} \cos \iota \\ \mathrm{L}' = -b \sin l' \qquad \gamma' = b \cos l' \\ \sin (\psi' - \mathrm{H}) = -\dfrac{\sin \iota}{\cos l'} \end{array}\right\} \text{ - - - - } (18)$$

$$\left.\begin{array}{l} \sin l'' = -\cos \mathrm{D} \sin \iota \\ \mathrm{L}'' = -k b \sin l'' \qquad \gamma'' = k b \cos l'' \\ \sin (\psi'' - \mathrm{H}) = \dfrac{\cos \iota}{\cos l''} \end{array}\right\} \text{ - - - - } (19)$$

These may be employed instead of the equations (13) and (14); or the equations (13) and (14) may be adopted in their reduced form, viz.

$$\left.\begin{aligned} \frac{L'}{b} &= \cos D \cos \iota \\ \frac{\gamma'}{b}\cos(\psi' - H) &= \sin D \cos \iota \\ \frac{\gamma'}{b}\sin(\psi' - H) &= -\sin \iota \end{aligned}\right\} \quad \text{- - - - - -} \quad (20)$$

$$\left.\begin{aligned} \frac{L''}{kb} &= \cos D \sin \iota \\ \frac{\gamma''}{kb}\cos(\psi'' - H) &= \sin D \sin \iota \\ \frac{\gamma''}{kb}\sin(\psi'' - H) &= \cos \iota \end{aligned}\right\} \quad \text{- - - - - -} \quad (21)$$

in which the coefficients c, a, will not be required.

III.—Transits of Mercury and Venus over the Disc of the Sun.

These phenomena are, in many respects, analogous to that of an annular eclipse of the Sun, and admit of a similar calculation; the principal distinction consists in the negative sign of the relative motion of the Planet in right ascension, which will make the inclination of the orbit always obtuse, and therefore render some modifications necessary in the determination of the particular species of the other angles which enter into the computation. To avoid any confusion that might thus arise, we shall adopt the Sun as the moveable body, and refer his positions to that of the Planet which we now suppose to be stationary. Thus,

$\delta =$ the ☉'s declination.
$D =$ the planet's declination.
$\pi =$ the ☉'s equatorial horizontal parallax.
$P =$ the planet's equatorial horizontal parallax.
$\alpha =$ the ☉'s right ascension *minus* that of the planet.
$x = (\delta' + \alpha' \text{ corr.}) - D.$
$y = \alpha' \cos \delta'.$
$x_1 =$ the ☉'s motion in declination *minus* that of the planet.
$y_1 =$ (☉'s motion in right ascension *minus* that of planet)$\cdot \cos \delta'$,

and so we might proceed as with an eclipse of the Sun, only observing that the relative parallax $\rho\,(\pi - P)$ is a negative quantity, and that the positions of the contacts on the limb of the Sun, as in the case of an occultation, will be at points opposite to those which come out in the calculation. However, as the relative parallax is always very small, the ingress and egress of the planet will be seen at all places on the earth at nearly the same absolute time; it will, for this reason, be best to compute first the circumstances for the centre of the earth, and then to ascertain the small variations produced by parallax for any assumed place on the surface, which may be readily deduced from the preceding equations for the reduction of an eclipse of the Sun. Let w, (t), be the values of ω, t, for the centre of the earth, and, by separating the effects of parallax from the equations (6),

$$\cos \mathrm{w} = p$$

$$(t) = (\mathrm{T} - q) \mp k \sin \mathrm{w}$$

$$\Delta \cos \mathrm{w} = \Delta p \qquad \Delta t = -\Delta q \mp k \Delta \sin \mathrm{w}$$

But, as the quantities $\Delta \cos \mathrm{w}$, $\Delta \sin \mathrm{w}$ are very small, $\Delta \sin \mathrm{w} = - \Delta \cos \mathrm{w} \dfrac{\cos \mathrm{w}}{\sin \mathrm{w}}$, that is, $\Delta \sin \mathrm{w} = - \Delta p \dfrac{\cos \mathrm{w}}{\sin \mathrm{w}}$. Therefore,

$$\Delta t = - \Delta q \pm k \Delta p \frac{\cos \mathrm{w}}{\sin \mathrm{w}} = \pm \left(k \Delta p \frac{\cos \mathrm{w}}{\sin \mathrm{w}} \mp \Delta q \right)$$

In this expression substitute the values of Δp, Δq, according to the equations (12), and we find $\Delta t =$

$$\pm \left[kc \frac{\cos\{-\iota \mp \mathrm{w}\}}{\sin \mathrm{w}} \sin l - \cos l \left(ka \frac{\cos\{-\iota \mp \mathrm{w}\}}{\sin \mathrm{w}} \cos h - kb \frac{\cos\{-\chi \mp \mathrm{w}\}}{\sin \mathrm{w}} \sin h \right) \right]$$

in which $b = \dfrac{\rho(\pi - \mathrm{P})}{\Delta} = - \dfrac{\rho(\mathrm{P} - \pi)}{\Delta}$, $c = b \cos \delta$ and $a = b \sin \delta$.

Because of the smallness of the parallax, the angle e will not be appreciable, and consequently $\chi = 90° + \iota$, $\cos\{-\chi \mp \mathrm{w}\} = \sin\{-\iota \mp \mathrm{w}\}$. We shall therefore have for the time of ingress or egress the following general expression, in which the terms within the brackets depend on the position of the place of observation; also the upper signs apply to the ingress, and the under signs to the egress.

$$t = \mathrm{T} - q \mp k \sin \mathrm{w}$$

$$\mp kb \left[\cos\delta \frac{\cos\{-\iota \mp \mathrm{w}\}}{\sin \mathrm{w}} \sin l - \left(\sin\delta \frac{\cos\{-\iota \mp \mathrm{w}\}}{\sin \mathrm{w}} \cos h - \frac{\sin\{-\iota \mp \mathrm{w}\}}{\sin \mathrm{w}} \sin h \right) \cos l \right]$$

Assuming $k'' = \dfrac{-kb}{\rho \sin \mathrm{w}}$, this expression will resolve into the following:

$$\left.\begin{aligned} \tan \iota &= \frac{x_1}{y_1} \\ k &= [3{\cdot}55630] \frac{\Delta \cos \iota}{y_1} \end{aligned}\right\} \text{ - - - - - - - } (a)$$

$$\left.\begin{aligned} \gamma \cos \psi &= \frac{(\delta + \alpha \text{ corr.}) - \mathrm{D}}{\Delta} \\ \gamma \sin \psi &= \frac{\alpha \cos \delta}{\Delta} \end{aligned}\right\} \text{ - - - - - - - } (b)$$

$$\left.\begin{aligned} \cos \mathrm{w} &= \gamma \cos(\psi + \iota) \\ q &= k\gamma \sin(\psi + \iota) \\ (t) &= \mathrm{T} - q \mp k \sin \mathrm{w} \end{aligned}\right\} \text{ - - - - - - - } (c)$$

$$\left.\begin{aligned} k'' &= k \cdot \frac{(\mathrm{P} - \pi)}{\Delta \sin \mathrm{w}} \\ \frac{\mathrm{L}''}{k''} &= \cos\{(-\iota) \mp \mathrm{w}\} \cos\delta \\ \frac{\gamma''}{k''} \cos(\psi'' - \mathrm{H}) &= \cos\{(-\iota) \mp \mathrm{w}\} \sin\delta \\ \frac{\gamma''}{k''} \sin(\psi'' - \mathrm{H}) &= \sin\{(-\iota) \mp \mathrm{w}\} \end{aligned}\right\} \text{ - - - - - - - } (d)$$

$$t = (t) \mp \{\gamma'' \rho \cos l \cos(\psi'' + \lambda) - \mathrm{L}'' \rho \sin l\} \text{ - - - - - - - } (e)$$

In these equations

H = the ☉'s true hour angle from the meridian of Greenwich, at the time (t)

$$\text{For} \left\{ \begin{array}{l} \text{exterior} \\ \text{interior} \end{array} \right\} \text{contact of limbs, } \Delta = \left\{ \begin{array}{l} \sigma + s \\ \sigma - s \end{array} \right\}$$

For contact of centre of planet with ☉'s limb, $\Delta = \sigma$

s denoting the true semidiameter of the Planet, and σ that of the Sun.

The equations (a), (b), (c), (d), will serve to determine the constants (t), γ'', L'', ψ'', for the times of ingress and egress, and then there will result two numerical equations of the form (e) to reduce the phenomena to any place on the earth's surface.

For the points on the limb of the Sun we shall have

$$\text{At} \left\{ \begin{array}{l} \text{ingress} \\ \text{egress} \end{array} \right\}, \text{angle from N. towards E} = \left\{ \begin{array}{l} (180^\circ - \iota) - w \\ (180^\circ - \iota) + w \end{array} \right\} \text{for } \textit{direct} \text{ image;}$$

$$\text{or} \left\{ \begin{array}{l} (-\iota) - w \\ (-\iota) + w \end{array} \right\} \text{for } \textit{inverted} \text{ image;}$$

which will be sufficiently accurate for all places on the earth.

The time T may be assumed near to the time of conjunction in longitude, or right ascension, as it may suit convenience. For Mercury, if very minute accuracy is wanted, it may be necessary, for more correct values of (t), to assume two times T near to the times of ingress and egress; but it is very questionable whether such a precarious extent of accuracy would sufficiently recompense the time expended on the calculation.

IV.—Occultations of Stars by the Moon.

These may be calculated in the same manner as Eclipses of the Sun, the only difference in the operation consisting in the star having neither motion, parallax, nor semidiameter. But, where great minuteness is not wanted, these particular circumstances will afford some degree of simplification to the expressions, if that parallax of the Moon be adopted which would answer to the star as an apparent place, since this parallax, at the times of immersion and emersion, will then be precisely that of the respective points of the Moon's limb which come in contact with the star; and thus the augmentation of the Moon's semidiameter will be evaded, so that the true semidiameter may be employed. For this novel and judicious expedient we are indebted to Carlini.—See *Zach's Correspondance*, vol. xviii. page 528.

As in the case of the Sun, let δ denote the declination, and h the hour angle of the star; and let P represent the equatorial horizontal parallax of the Moon. Then, for the effects of parallax in right ascension and declination, we must substitute δ for D$'$, and h for h' in the formulæ (2) at page 103, which thus become, disregarding $\frac{1}{2}\Delta h$,

$$\Delta \alpha = \rho P \frac{\cos l}{\cos D} \sin h$$

$$\Delta D = \rho P (\sin l \cos \delta - \cos l \sin \delta \cos h)$$

As soon as the immersion takes place, these expressions will represent the parallax of that point of the Moon's limb which is in contact with the star; and, therefore, the application of this parallax to the centre of the Moon will produce an apparent distance Δ', of the centres, equal to the *true* semidiameter s of the Moon. Also as the star, in the course of the occultation, is only affected with its apparent diurnal motion, the hourly variations of the above values will be

$$\Delta\,\alpha_1 = \rho\,\mathrm{P}\left(\frac{dh}{dt}\sin 1''\right)\frac{\cos l}{\cos \mathrm{D}}\cos h$$

$$\Delta\,\mathrm{D}_1 = \rho\,\mathrm{P}\left(\frac{dh}{dt}\sin 1''\right)\cos l\sin\delta\sin h$$

in which $\frac{dh}{dt}$ is $15°\ 2'\ 28''$, the hourly diurnal motion of the Earth, and therefore $\frac{dh}{dt}\sin 1'' = [9\cdot41916]$.

Assume

$$\left.\begin{aligned}
\phi^{(1)} &= \rho\cos l = \frac{\cos l'}{\sqrt{(1-e^2\sin^2 l')}}\\
\phi^{(2)} &= \rho\sin l = \frac{(1-e^2)\sin l'}{\sqrt{(1-e^2\sin^2 l')}}\\
\phi^{(3)} &= \rho\cos l\,\frac{dh}{dt}\sin 1'' = [9\cdot41916]\,\phi^{(1)}
\end{aligned}\right\}\ \text{- - - -}\ (1)$$

which are constant coefficients depending on the latitude of the place; then

$$\Delta\,\alpha = \frac{\phi^{(1)}.\mathrm{P}}{\cos \mathrm{D}}\sin h \qquad \Delta\,\alpha_1 = \frac{\phi^{(3)}.\mathrm{P}}{\cos \mathrm{D}}\cos h$$

$$\Delta\,\mathrm{D} = (\phi^{(2)}\cos\delta - \phi^{(1)}\sin\delta\cos h)\,.\,\mathrm{P} \qquad \Delta\,\mathrm{D}_1 = \phi^{(3)}\,.\,\mathrm{P}\sin\delta\sin h$$

If, in the values of $\Delta\alpha$, $\Delta\alpha_1$, we use $\cos\delta$ instead of $\cos \mathrm{D}$, the values of x, y, x_1, y_1, page 111, will become

$$\left.\begin{aligned}
x &= (\mathrm{D}-\delta) - (\phi^{(2)}.\,\mathrm{P}\cos\delta - \phi^{(1)}\,.\,\mathrm{P}\sin\delta\cos h)\\
y &= \alpha\cos\delta - \phi^{(1)}\,.\,\mathrm{P}\sin h\\
x_1 &= \mathrm{D}_1 - \phi^{(3)}\,.\,\mathrm{P}\sin\delta\sin h\\
y_1 &= \alpha_1\cos\delta - \phi^{(3)}\,.\,\mathrm{P}\cos h
\end{aligned}\right\}\ \text{- -}\ (2)$$

in which we have disregarded the α correction.

With the values of x, y, x_1, y_1, so found, we may then proceed with the equations (16) and (18), pages 111 and 112, as in the case of a Solar Eclipse.

This method is similar, and, as far as accuracy goes, the same as the recent method of Professor Bessel, who divides all the quantities by the equatorial horizontal parallax of the Moon. He assumes

$$\left.\begin{aligned}
p &= \frac{\alpha\cos\delta}{\mathrm{P}} & p' &= \frac{\alpha_1\cos\delta}{\mathrm{P}}\\
q &= \frac{\mathrm{D}-\delta}{\mathrm{P}} & q' &= \frac{\mathrm{D}_1}{\mathrm{P}}
\end{aligned}\right\}\ \text{- - - -}\ (3)$$

$$\left.\begin{aligned}
u &= \phi^{(1)}\sin h & u' &= \phi^{(3)}\cos h\\
v &= \phi^{(2)}\cos\delta - \phi^{(1)}\sin\delta\cos h & v' &= \phi^{(3)}\sin\delta\sin h
\end{aligned}\right\}\ \text{- -}\ (4)$$

so that if we change the signification of the symbols x, y, x_1, y_1, and suppose them now to represent the preceding values divided by P, we shall have

$$\left.\begin{aligned}
x &= q - v & x_1 &= q' - v'\\
y &= p - u & y_1 &= p' - u'
\end{aligned}\right\}\ \text{- - - - - -}\ (5)$$

These values being adopted, in proceeding with the equations (16) and (18) we must use $\Delta' = \frac{s}{\mathrm{P}}$, the value of which, according to Burckhardt's *Tables de la Lune*, (Paris, 1812), page 73, is $[9\cdot43537]$. Much facility is thus given to the calculation

of Occultations, for different places, if the values of p, q, p', q', which are independent of geographical position, are published; but if these quantities are to be prepared by the computer, the equations (2) will be more simple and advantageous.

The chief difficulty in the calculation of Occultations, for any particular place, rests in the selection of the list of stars: in the course of any year a great number will be liable to Occultation on the earth generally, though the majority of them will not be occulted at the particular place for which the special calculations are to be made. It will therefore be expedient to reject such stars as may at different stages of the calculation be shown to violate any conditions necessary for the existence of the Occultation, its appearance above the horizon, or its exemption from the glare of sun-light. For the general list we may observe, that the difference of declination at the time of conjunction must be within the limit of about 1° 30′, and that all stars, whose conjunctions with the Moon occur within two days of New Moon, may be omitted. In the process of exclusion for the particular place, the first and most palpable condition is, that at the time of conjunction the Sun must be below, or near to, the horizon; if more than half an hour above the horizon, the occultation will surely be useless; another condition is, that the star must be above the horizon; and, to satisfy this, the hour angles at the times of immersion and emersion must be less than its semidiurnal arc. The value of the hour angle at the time of apparent conjunction may be determined by increasing that at the time of true conjunction by the quantity $\dfrac{\zeta^{(1)}}{\alpha_1 \cdot f - \zeta}$, according to the tables on pages 129 and 130; and it may be observed, that this hour angle must not exceed the semidiurnal arc by more than half an hour. For the latitude of Greenwich, the semidiurnal arcs, allowing 33′ for refraction in the horizon, are shown in the annexed table.

Dec. of Star.	Semidiurnal Arcs, for the Latitude of Greenwich.			
	Dec. North.		*Dec. South.*	
°	h m		h m	
0	6 4		6 4	
1	6 9	+5	5 59	−5
2	6 14	5	5 54	5
3	6 19	5	5 49	5
4	6 24	5	5 43	6
5	6 29	5	5 38	5
6	6 34	5	5 33	5
7	6 39	5	5 28	5
8	6 44	5	5 23	5
9	6 50	6	5 18	5
10	6 55	5	5 13	5
11	7 0	5	5 7	6
12	7 6	6	5 2	5
13	7 11	5	4 56	6
14	7 17	6	4 51	5
15	7 23	6	4 45	6
16	7 28	5	4 40	5
17	7 34	6	4 34	6
18	7 40	6	4 28	6
19	7 47	7	4 22	6
20	7 53	6	4 15	7
21	8 0	7	4 9	6
22	8 6	6	4 2	7
23	8 13	7	3 56	6
24	8 21	8	3 49	7
25	8 28	7	3 41	8
26	8 36	8	3 34	7
27	8 44	8	3 26	8
28	8 53	9	3 18	8
29	9 2	9	3 9	9
30	9 12	+10	3 0	−9

As a final test for the exclusion of unnecessary stars, it is useful to calculate the extreme limits of latitude between which the star will be visibly occulted on the earth. These will evidently appertain to the extreme northern and southern points of the northern and southern limits of contact, determined as for a solar eclipse: a point in the northern or southern limit will depend on the formulæ Nos. 27, 28, page 82. Thus,

$$\cos w = \frac{n \pm \Delta'}{P'}$$

$$\sin Z = \frac{\cos w}{\cos \omega'} \qquad M = -\iota \pm \omega'$$

and thence,

$$\sin l = \sin D' \cos Z + \cos D' \sin Z \cos M$$

It is now our object to ascertain what value of ω' will render the value of l, so deduced, a maximum or a minimum, and what will be the corresponding value of l.

Let ϕ be an arc determined by the equation,

$$\cos Z = \cos\phi \sin w \quad - - - - - - \quad (6)$$

Then by uniting with it the equation

$$\cos\omega' \sin Z = \cos w \quad - - - - - - \quad (7)$$

we infer that

$$\sin\omega' \sin Z = \sin\phi \sin w \quad - - - - - \quad (8)$$

because the squares of these three equations added together will give *unity* on each side. By these equations we shall hence have

$$\begin{aligned} \sin D' \cos Z &= \sin D' \cos\phi \sin w \\ \sin Z \cos M &= \sin Z\,(\cos\iota \cos\omega' \mp \sin\iota \sin\omega') \\ &= (\cos\omega' \sin Z) \cos\iota \mp (\sin\omega' \sin Z) \sin\iota \\ &= \cos\iota \cos w \mp \sin\iota \sin\phi \sin w \end{aligned}$$

and consequently

$$\sin l = \cos D' \cos\iota \cos w + \sin w\,(\sin D' \cos\phi \mp \cos D' \sin\iota \sin\phi)$$

which now involves only one variable ϕ. Again, assume two arcs θ, ψ, which will fulfil the equations

$$\cos\theta \cos\psi = \sin D' \quad - - - - - - - \quad (9)$$

$$\cos\theta \sin\psi = \pm \cos D' \sin\iota \quad - - - \quad (10)$$

A third equation will follow from these, viz.

$$\sin\theta = \cos D' \cos\iota \quad - - - - - - - \quad (11)$$

because, as before, the squares of these three equations will together make *unity*. The value of $\sin l$ will now become

$$\sin l = \cos w \sin\theta + \sin w \cos\theta \cos(\phi + \psi)$$

The angle $\phi + \psi$ being the only variable in this expression, it is evident that the greatest value of l will have $\phi + \psi = 0$, and the least $\phi + \psi = 180°$. Therefore,

$$\left\{\begin{matrix}\text{greatest}\\ \text{least}\end{matrix}\right\} \text{ value of } l = \left\{\begin{matrix}\theta + w\\ \theta - w\end{matrix}\right\}, \text{ using w for } \left\{\begin{matrix}\text{northern}\\ \text{southern}\end{matrix}\right\} \text{ limit.}$$

These would be the extreme latitudes for the appearance of the occultation if the Earth were a transparent body; as this, however, is not the case, it will be necessary that the star should be above the horizon, a condition not included in the preceding equations. The zenith distance Z must not exceed 90°, and therefore $\cos Z$ must necessarily be a positive quantity.

By the equation (6), $\cos Z$ must have the same sign as $\cos\phi$, and this must be the same as $+\cos\psi$ for northern limit, or $-\cos\psi$ for southern limit, because in the former case $\phi + \psi = 0$, and in the latter $\phi + \psi = 180°$. But, by (9), $\cos\psi$ must have the same sign as D′. Consequently

$$\text{For} \left\{\begin{matrix}\text{northern}\\ \text{southern}\end{matrix}\right\} \text{ limit, } \cos Z \text{ has the same sign as } \left\{\begin{matrix}+D'\\ -D'\end{matrix}\right.$$

It is evident, therefore, that the extreme northern limit will have the star below the horizon and be excluded when D′ is negative, and that for the same reason the southern limit will be excluded when D′ is positive. Thus the only admissible extreme limit will be determined by the equations

$$\cos w = \frac{n \pm \Delta'}{p'} \qquad l_1 = \theta \pm w \quad - - - - - - - - \quad (12)$$

using upper signs when D′ is positive, and under signs when D′ is negative.

The other limit for the actual appearance of the occultation will evidently be one of

the two places where the other limiting line meets the rising and setting limits, and will be determined by

$$\cos w = \frac{n \mp \Delta'}{P'} \qquad \sin l_2 = \cos D' \cos \left\{ (-\iota) \mp w \right\} \quad - - - \quad (13)$$

using, as before, upper signs when D′ is positive, and under signs when D′ is negative.

The equations (11), (12), (13), for convenience in determining the species of the angles, may be put in the following form,

$$\left.\begin{array}{l} \cos w_1 = \dfrac{\mp n - \Delta'}{P'} \qquad \cos w_2 = \dfrac{\mp n + \Delta'}{P'} \\ \sin \theta = \cos D' \cos \iota \\ l_1 = w_1 - \theta \\ \sin l_2 = \mp \cos D' \cos (w_2 - \iota) \end{array}\right\} \quad - - \quad (14)$$

observing that w_1, w_2, θ, and ι, must here take the same sign as D′; also

$$\left\{\begin{array}{l} \text{upper} \\ \text{under} \end{array}\right\} \text{signs when } D' \text{ is } \left\{\begin{array}{l} \text{positive.} \\ \text{negative.} \end{array}\right.$$

These formulæ are applicable to a solar eclipse. For an occultation of a star by the Moon, P′ will be the Moon's horizontal parallax, and Δ' her semidiameter, which, as these limits are not wanted very accurately, may be regarded as true quantities; also we may neglect u and so take δ instead of D′. Since $\frac{s}{P} = [9{\cdot}43537] = {\cdot}2725$, the formulæ for an occultation will hence be

$$\left.\begin{array}{l} \tan \iota = \dfrac{D_1}{\alpha_1 \cos \delta} \qquad n = (\text{diff. dec.}) \cos \iota \\ \cos w_1 = \mp \dfrac{n}{P} - {\cdot}2725 \qquad \cos w_2 = \mp \dfrac{n}{P} + {\cdot}2725 \\ \sin \theta = \cos \delta \cos \iota \\ l_1 = w_1 - \theta \qquad \sin l_2 = \mp \cos \delta \cos (w_2 - \iota) \end{array}\right\} \quad - - \quad (15)$$

in which we also give to the angles w_1, w_2, ι, θ, the same sign as δ, and use upper signs when δ is positive, and under signs when δ is negative. We may also observe, that,

1. When δ is North, l_1 is the most northern limit; and when δ is South, l_1 is the most southern limit.

2. When w_1 is imaginary, l_1 will be 90°, and of the same name as δ. In this case the occultation will be visible about the pole of the Earth which is presented to the star; the visibility will extend beyond the extremity of the disc of the Earth as it would be seen from the star.

3. When w_2 is imaginary, l_2 will be the complement of δ and of a different name from δ. In this case, if we consider the disc of the Earth as seen from the star, the visibility of the occultation will extend beyond that extremity of the disc which has the pole on the other side of it.

After an occultation is computed for any particular place, if we deduct the star's right ascension from the sidereal times of immersion and emersion we shall get the hour angles of the star, + West, — East. By comparing these hour angles with the semidiurnal arc of the star, we can distinctly ascertain the positions of the star with respect to the horizon.

V.—Eclipses of the Moon by the Earth's Shadow.

These may be also resolved in the same way as those of the Sun. The absolute positions of the Moon and Shadow being independent of the position of the spectator on the Earth, the determination of parallaxes will be here unnecessary, which much simplifies the calculation of these eclipses. The considerations requisite to be attended to, by way of distinction, are the following:

$$\text{Semidiameter of the Shadow} = \frac{61}{60}(P' + \pi - \sigma)$$

$$\text{Semidiameter of the Penumbra} = \frac{61}{60}(P' + \pi - \sigma) + 2\sigma$$

Right Ascension of centre of Shadow = that of the Sun $\pm 12^h$

Declination of centre of Shadow = that of the Sun *with a contrary name.*

The figure of the Earth being spheroidal, that of the shadow will deviate a little from a circle, so that, to have a mean radius, the horizontal parallax of the Moon must be reduced to a mean latitude of 45°. This will give

$$P' = [9{\cdot}99929]\ P$$

P denoting the Moon's equatorial horizontal parallax.

Also,

α = right ascens. Moon *minus* right ascens. centre of shadow

x = (dec. Moon + α corr.) *minus* dec. centre of shadow

$y = \alpha \cos D$

With these we compute according to the equations (16) and (18), pages 111 and 112, observing the following values of Δ'

For $\left\{\begin{matrix}\text{external}\\ \text{internal}\end{matrix}\right\}$ contact with shadow, Δ' = semidiam. shadow $\pm s$

For $\left\{\begin{matrix}\text{external}\\ \text{internal}\end{matrix}\right\}$ contact with penumbra, Δ' = semidiam. penumbra $\pm s$

The angular positions of the points where the contacts take place will be estimated on the circumference of the shadow or penumbra the same as they were before on the limb of the Sun. These angles will therefore be in a reversed position on the disc of the Moon, and consequently as they come out from the computation will have reference in the first instance to the inverted appearance of the phase.

The relative orbit of the Moon, not being affected with parallax, will not sensibly deviate from a great circle in the course of the eclipse; and hence the assumption of the particular time, on which to found the calculation, will be but of little importance: any convenient time may be assumed near the time of opposition.

It will be unnecessary to add any further remarks. We shall conclude this paper with a tabular recapitulation of the formulæ which relate to the phenomena for a particular place, in which eclipses of the Moon, for the sake of clearness, are given separately. The object of this table, like the former one for the general eclipse, is to simplify and expedite, by an easy reference, the actual operations of the computer.

I.—ECLIPSE OF THE SUN FOR A PARTICULAR PLACE.

1. $h =$ Apparent Time of true ☌ in R. A. to nearest minute.

With this as an argument take out the numbers ζ, $\zeta^{(1)}$, from the following Table:

Table for reducing the *true* to the *app.* ☌ in R. A.

Hour Angle h at true ☌.		ζ + −	$\zeta^{(1)}$ same sign as h.	Hour Angle h at true ☌.		ζ + −	$\zeta^{(1)}$ same sign as h.
h m	h m			h m	h m		
0 0	12 0	25	0	3 0	9 0	18	71
10	11 50	25	4	10	8 50	17	74
20	40	25	9	20	40	16	77
30	30	25	13	30	30	15	79
40	20	25	17	40	20	14	82
50	10	25	22	50	10	14	84
1 0	11 0	24	26	4 0	8 0	13	87
10	10 50	24	30	10	7 50	12	89
20	40	24	34	20	40	11	91
30	30	23	38	30	30	10	92
40	20	23	42	40	20	9	94
50	10	22	46	50	10	8	95
2 0	10 0	22	50	5 0	7 0	7	97
10	9 50	21	54	10	6 50	5	98
20	40	21	57	20	40	4	98
30	30	20	61	30	30	3	99
40	20	19	64	40	20	2	100
50	10	19	68	50	10	1	100
3 0	9 0	18	71	6 0	6 0	0	100

Then, T denoting the approximate mean time of app. ☌, in units of an hour,

$$T = \text{mean time true ☌} + \frac{\zeta^{(1)}}{\alpha_1 \cdot f - \zeta}$$

in which α_1 must be used in minutes of arc; also $f = \dfrac{[0{\cdot}2310]}{\cos l}$, is a factor depending on the latitude, which for several principal Observatories is, for convenience, included in the following Table:

Auxiliary Quantities depending on Geographical Position.

Place.	ρ	cot l	cos l	f	Longitude. h m s
Aberdeen	9·99900	+9·81289	9·73637	3·12	W. 0 8 23
Altona	9·99908	+9·87133	9·77576	2·85	E. 0 39 47
Berlin	9·99910	+9·88751	9·78603	2·79	E. 0 53 36
Bedford	9·99911	+9·89345	9·78974	2·76	W. 0 1 52
Cambridge	9·99911	+9·89231	9·78903	2·77	E. 0 0 24
Cape of Good Hope	9·99956	—0·17494	9·91980	2·05	E. 1 13 55
Dublin	9·99909	+9·87385	9·77737	2·84	W. 0 25 22
Edinburgh	9·99902	+9·83256	9·75001	3·03	W. 0 12 44
Greenwich	9·99913	+9·90381	9·79610	2·72	0 0 0
Ormskirk	9·99908	+9·87092	9·77549	2·85	W. 0 11 36
Oxford	9·99912	+9·89939	9·79340	2·74	W. 0 5 2
Kensington	9·99913	+9·90340	9·79586	2·72	W. 0 0 47
Milan	9·99928	+9·99577	9·84736	2·42	E. 0 36 47
Paris	9·99920	+9·94451	9·81997	2·58	E. 0 9 22
Slough	9·99913	+9·90337	9·79584	2·72	W. 0 2 24

2. The time T being computed to the nearest minute, take out the corresponding values of P, π, σ, δ, from the Ephemeris; and prepare the constants

$$c = [4 \cdot 68555]\,\rho$$

$$A = c(P - \pi) \qquad m = A \cos l$$

$$Q_1 = [4 \cdot 7172] \qquad Q_2 = m\,Q_1 \sin \delta$$

$$s = [9 \cdot 43537]\,P$$

3. Take out D, δ, α, D_1, α_1, for the time T.

h = sidereal time at place *minus* ☽'s right ascension, to the tenth of a minute, *in arc.*

$$k = \frac{m}{\cos D} \qquad n = k \cos h$$

$$\Delta\,\alpha = [5 \cdot 31439]\,k \sin h \text{ [corr. for } n]$$

$$\Delta\,\alpha_1 = Q_1 n \qquad \Delta\,D_1 = Q_2 \sin h$$

Correction for n to be taken from the table on page 107

4. $$(h) = h + \tfrac{1}{2}\Delta\,\alpha$$

$$\tan\theta = \cos(h)\cot l \qquad G = \cos(h)\cos l$$

$$\tan M = \frac{\sin\theta}{\cos(\theta+D)}\tan(h) \qquad \tan\varepsilon = \tan(\theta+D)\cos M$$

$$B = \cos M \cos\varepsilon$$

$$\text{check} - - \frac{\sin\theta}{\cos(\theta+D)} = \frac{G}{B}$$

M to be in the same semicircle with h

$n_1 = \text{A} \sin \epsilon \qquad \Delta \text{D} = [5 \cdot 31439] \text{ A B [corr. for } n_1]$

$s' = s \text{ [corr. for } n_1]$

$$\text{For} \left\{ \begin{matrix} \text{partial} \\ \text{total or annular} \end{matrix} \right\} \text{phase, } \Delta' = \left\{ \begin{matrix} s' + \sigma \\ s' - \sigma \end{matrix} \right\}$$

Correction for n_1 to be taken from the table on page 107

5. $\text{D}' = \text{D} - \Delta \text{D} \qquad \alpha' = \alpha - \Delta \alpha$

$y = (\alpha - \Delta \alpha) \cos \text{D}' \qquad y_1 = (\alpha_1 - \Delta \alpha_1) \cos \text{D}'$

$x = (\text{D}' + \alpha' \text{ corr.}) - \delta \qquad x_1 = \text{D}_1 - \Delta \text{D}_1$

6. $$\tan \text{S} = \frac{y}{x} \qquad \cot \iota = \frac{y_1}{x_1}$$

$$\text{W} = \frac{y}{\sin \text{S}} = \frac{x}{\cos \text{S}}$$

$$n = \text{W} \cos \{-(\text{S} + \iota)\} \qquad \text{H} = \frac{\text{W} \cos \iota \, [3 \cdot 55630]}{y_1}$$

7. $$\cos \omega = \frac{n}{\Delta'} \qquad c = \frac{\text{H}}{\cos \omega}$$

$$a = \{-(\text{S} + \iota)\} - \omega \qquad b = \{-(\text{S} + \iota)\} + \omega$$

$$t_1 = c \sin a \qquad t_2 = c \sin b$$

$$\text{Time of} \left\{ \begin{matrix} \text{beginning} \\ \text{ending} \end{matrix} \right\} = \text{T} + \left\{ \begin{matrix} t_1 \\ t_2 \end{matrix} \right\}$$

Time of greatest phase $= \frac{1}{2}$ sum of times of beginning and ending

When $n < s' - \sigma$, the eclipse will be total if $s' > \sigma$, or annular if $s' < \sigma$: in this case these last equations No. 7 must be repeated for this phase with $\Delta' = s' - \sigma$, the results of which ought to give the same time for the greatest phase.

Take Δ' for partial phase, and

Portion of Sun's disc eclipsed $= \Delta' - n$

Magnitude of Eclipse $= \dfrac{\Delta' - n}{2\sigma}$, the Sun's diameter being *unity*.

8. For the positions of the points of contact on the limb of the Sun,

At $\left\{ \begin{matrix} \text{beginning} \\ \text{ending} \end{matrix} \right\}$, angle from North towards East $= \left\{ \begin{matrix} (-\iota) - \omega \\ (-\iota) + \omega \end{matrix} \right\}$ for *direct* image.

At $\left\{ \begin{matrix} \text{beginning} \\ \text{ending} \end{matrix} \right\}$, angle from North towards East $= \left\{ \begin{matrix} (180^\circ - \iota) - \omega \\ (180^\circ - \iota) + \omega \end{matrix} \right\}$ for *inverted* image.

For the position of the Moon's centre at greatest phase,

Angle from $\left\{ \begin{matrix} \text{North} \\ \text{Vertex} \end{matrix} \right\}$ towards East $= \left\{ \begin{matrix} (-\iota) \\ (-\iota) - \text{M} \end{matrix} \right\}$ for *direct* image.

Angle from $\left\{ \begin{matrix} \text{North} \\ \text{Vertex} \end{matrix} \right\}$ towards East $= \left\{ \begin{matrix} (180^\circ - \iota) \\ (180^\circ - \iota) - \text{M} \end{matrix} \right\}$ for *inverted* image.

9. For a more accurate Calculation of the Time, &c., of Beginning of the Partial Phase, assume a convenient time near to the preceding determination. For this time, take out the quantities D, D_1, δ, α, α_1, from the Ephemeris; and proceed as in Nos. 3, 4, 5, 6, 7, omitting b, t_2, and the times of greatest phase and ending.

Let M_1, ι_1, ω_1, be the values of the angles in this computation; then, for the position of the point of contact on the limb of the Sun,

$$\text{Angle from } \begin{Bmatrix}\text{North}\\ \text{Vertex}\end{Bmatrix} \text{ towards the East} = \begin{Bmatrix}(-\iota_1)-\omega_1\\ (-\iota_1)-\omega_1-M_1\end{Bmatrix} \text{ for } \textit{direct} \text{ image.}$$

$$\text{Angle from } \begin{Bmatrix}\text{North}\\ \text{Vertex}\end{Bmatrix} \text{ towards the East} = \begin{Bmatrix}(180^\circ-\iota_1)-\omega_1\\ (180^\circ-\iota_1)-\omega_1-M_1\end{Bmatrix} \text{ for } \textit{inverted} \text{ image.}$$

10. For a more accurate Calculation of the Time, &c., of Ending of the Partial Phase, assume a convenient time near to the first determination. For this time, take out the values of D, D_1, δ, α, α_1; and proceed as in Nos. 3, 4, 5, 6, 7, omitting a, t_1, and the times of beginning and greatest phase.

Let M_2, ι_2, ω_2, be the angles in this computation; then, for the position of the point of contact on the limb of the Sun,

$$\text{Angle from } \begin{Bmatrix}\text{North}\\ \text{Vertex}\end{Bmatrix} \text{ towards the East} = \begin{Bmatrix}(-\iota_2)+\omega_2\\ (-\iota_2)+\omega_2-M_2\end{Bmatrix} \text{ for } \textit{direct} \text{ image.}$$

$$\text{Angle from } \begin{Bmatrix}\text{North}\\ \text{Vertex}\end{Bmatrix} \text{ towards the East} = \begin{Bmatrix}(180^\circ-\iota_2)+\omega_2\\ (180^\circ-\iota_2)+\omega_2-M_2\end{Bmatrix} \text{ for } \textit{inverted} \text{ image.}$$

II.—FORMULÆ FOR REDUCTION TO DIFFERENT PLACES.

11. Instead of Nos. 5, 6, 7, substitute the following:—

$$D' = D - \Delta D \qquad \alpha' = \alpha - \Delta\alpha$$

$$x_1 = D_1 - \Delta D_1 \qquad y_1 = (\alpha_1 - \Delta\alpha_1)\cos D'$$

$$\tan\iota = \frac{x_1}{y_1} \qquad k = [3{\cdot}55630]\,\frac{\Delta'\cos\iota}{y_1}$$

$$\gamma\cos\psi = \frac{(D+\alpha'\text{ corr.})-\delta}{\Delta'} \qquad \gamma\sin\psi = \frac{\alpha\cos D'}{\Delta'}$$

$$p = \gamma\cos(\psi+\iota) \qquad q = k\gamma\sin(\psi+\iota)$$

$$T - q = T'$$

12.
$$b = \frac{[5{\cdot}31439]\,A}{\Delta'}\ [\text{corr. for } n_1]$$

$$e \textit{ in minutes} = [7{\cdot}9208]\,\Delta\alpha\sin D \qquad \chi = (90^\circ+\iota)-e$$

13. H = the true Greenwich hour angle of ☽ at the time T

$$\frac{L'}{b} = \cos D\cos\iota \qquad \frac{L''}{kb} = \cos D\sin\iota$$

$$\frac{\gamma'}{b}\cos(\psi'-H) = \sin D\cos\iota \qquad \frac{\gamma''}{kb}\cos(\psi''-H) = \sin D\sin\iota$$

$$\frac{\gamma'}{b}\sin(\psi'-H) = \cos\chi \qquad \frac{\gamma''}{kb}\sin(\psi''-H) = \sin\chi$$

14. The constants T', k, p, L', L'', γ', γ'', being so computed, the angle ω and the time t of the phase for any place whose North latitude is l and East longitude λ, will be determined by the two following equations, in which the upper sign relates to the beginning and the under sign to the ending.

$$\cos \omega = p - L' \sin l + \gamma' \cos l \cos (\lambda + \psi')$$
$$t = T' \mp k \sin \omega + L'' \sin l - \gamma'' \cos l \cos (\lambda + \psi'')$$

The result will be the most accurate when the place is near to that on which the previous part of the calculation is founded.

III.—TRANSIT OF MERCURY OR VENUS OVER THE DISC OF THE SUN.

(Same notation for the Planet as for the Moon.)

15. Assume the time T near to the time of conjunction in longitude, or right ascension.

$\alpha =$ Sun's right ascension — Planet's right ascension *in arc*
$\alpha_1 =$ hourly variation of α
$D_1 =$ Sun's hourly motion in declination *minus* that of the Planet

$$\text{For} \left\{ \begin{matrix} \text{exterior} \\ \text{interior} \end{matrix} \right\} \text{contact of limbs, } \Delta = \left\{ \begin{matrix} \sigma + s \\ \sigma - s \end{matrix} \right.$$

For contact of Planet's centre with Sun's limb, $\Delta = \sigma$

$$\tan \iota = \frac{D_1}{\alpha_1 \cos \delta}$$

$$k = [3{\cdot}55630] \frac{\Delta \cos \iota}{\alpha_1 \cos \delta} \qquad b = \frac{P - \pi}{\Delta}$$

$$\gamma \cos \psi = \frac{(\delta + \alpha \text{ corr.}) - D}{\Delta} \qquad \gamma \sin \psi = \frac{\alpha \cos \delta}{\Delta}$$

$$\cos \mathrm{w} = \gamma \cos (\psi + \iota) \qquad q = k\gamma \sin (\psi + \iota)$$

16. H = the true Greenwich hour angle of ☉ at the time T

$$k'' = \frac{kb}{\sin \mathrm{w}}$$

$$\frac{L''}{k''} = \cos \delta \cos \{(-\iota) \mp \mathrm{w}\}$$

$$\frac{\gamma''}{k''} \cos (\psi'' - H) = \sin \delta \cos \{(-\iota) \mp \mathrm{w}\}$$

$$\frac{\gamma''}{k''} \sin (\psi'' - H) = \sin \{(-\iota) \mp \mathrm{w}\}$$

17. Then, for the centre of the earth,

$$(t) = (T - q) \mp k \sin \mathrm{w}$$

and, for any place whose latitude is l and east longitude λ,

$$t = (t) \mp \{\gamma'' \rho \cos l \cos (\lambda + \psi'') - L'' \rho \sin l\}$$

using the upper signs for the ingress, and the under signs for the egress.

The positions of the points of ingress and egress, estimated from the North point of the Sun's limb towards the East, as the transit would be seen from the centre of the earth, will be determined in the same manner as for the immersion and emersion of an Occultation, No. 19, using w for ω. These angles may be assumed to be the same for any place on the surface, the effect of parallax being so very minute.

IV.—OCCULTATION OF A STAR BY THE MOON.

General Limits of Latitude.

18. (α_1 and D_1 at true ☌)

$$\tan \iota = \frac{D_1}{\alpha_1 \cos \delta} \qquad n = (\text{diff. dec.}) \cos \iota$$

$$\cos w_1 = \mp \frac{n}{P} - \cdot 2725 \qquad \cos w_2 = \mp \frac{n}{P} + \cdot 2725$$

$$\sin \theta = \cos \delta \cos \iota$$

$$l_1 = w_1 - \theta \qquad \sin l_2 = \mp \cos \delta \cos (w_2 - \iota)$$

w_1, w_2, ι, θ, same sign as δ

$\left\{\begin{matrix}\text{upper}\\ \text{under}\end{matrix}\right\}$ signs when δ is $\left\{\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right\}$

When w_1 is impossible, $l_1 = 90°$, with the same name as δ.
When w_2 is impossible, $l_2 =$ complement of δ, with different name from δ.

Calculation for particular Place.

19. For the latitude of the place prepare the constants

$$\phi^{(1)} = \rho \cos l \qquad \phi^{(2)} = \rho \sin l = \frac{\phi^{(1)}}{\cot l} \qquad \phi^{(3)} = [9 \cdot 41916]\, \phi^{(1)}$$

which will serve for all Occultations at that place.

For the time of true ☌ find

$$h = \text{sidereal time at place} - \text{right ascension of Star}$$

and thence determine the time T, as in No. 1. For this time take out the quantities P, s, D, D_1, α, α_1; and compute

$$x = (D - \delta) - (\phi^{(2)} . P \cos \delta - \phi^{(1)} . P \sin \delta \cos h)$$
$$y = \alpha \cos \delta - \phi^{(1)}\, P \sin h$$
$$x_1 = D_1 - \phi^{(3)} . P \sin \delta \sin h$$
$$y_1 = \alpha_1 \cos \delta - \phi^{(3)} . P \cos h$$

With these proceed as in Nos. 6 and 7, using $\Delta' = s = [9 \cdot 43537]$ P.

20. For the positions of the points of immersion and emersion on the limb of the Moon,

At $\left\{\begin{matrix}\text{immersion}\\ \text{emersion}\end{matrix}\right\}$, angle from North towards East $= \left\{\begin{matrix}(180° - \iota) - \omega\\ (180° - \iota) + \omega\end{matrix}\right\}$ for *direct* image.

At $\left\{\begin{matrix}\text{immersion}\\ \text{emersion}\end{matrix}\right\}$, angle from North towards East $= \left\{\begin{matrix}(-\iota) - \omega\\ (-\iota) + \omega\end{matrix}\right\}$ for *inverted* image.

For the same angles from the Vertex we must deduct the parallactic angle for each time.

21. If an accurate calculation is wanted, proceed as with a Solar Eclipse.

V.—ECLIPSE OF THE MOON.

22. Fix on a convenient time near to the time of opposition in longitude, or full moon; and for this time find P, s, π, σ,

α = ☽'s right ascension *minus* (☉'s right ascension $\pm 12^h$), *in arc.*

α_1 = hourly motion of α

x = (☽'s dec. + α corr.) *plus* ☉'s dec.

x_1 = hourly motion of x

y = α cos ☽'s dec.

y_1 = α_1 cos ☽'s dec.

$$P' = [9{\cdot}99929]\,P$$

23. $$\text{Semid. shadow} = \frac{61}{60}(P' + \pi - \sigma)$$

$$\text{Semid. penumbra} = \frac{61}{60}(P' + \pi - \sigma) + 2\sigma$$

For $\left\{\begin{matrix}\text{external}\\ \text{internal}\end{matrix}\right\}$ contact with shadow, Δ' = simid. shadow $\left\{\begin{matrix}+\\ -\end{matrix}\right\} s$

For $\left\{\begin{matrix}\text{external}\\ \text{internal}\end{matrix}\right\}$ contact with penumbra, Δ' = semid. penumbra $\left\{\begin{matrix}+\\ -\end{matrix}\right\} s$

The remaining computation as in Nos. 6 and 7.

24. For the positions of the points of contact on the limb of the Moon,

At $\left\{\begin{matrix}\text{immersion}\\ \text{emersion}\end{matrix}\right\}$, angle from North towards East $= \left\{\begin{matrix}(180^\circ - \iota) - \omega\\ (180^\circ - \iota) + \omega\end{matrix}\right\}$ for *direct* image.

At $\left\{\begin{matrix}\text{immersion}\\ \text{emersion}\end{matrix}\right\}$, angle from North towards East $= \left\{\begin{matrix}(-\iota) - \omega\\ (-\iota) + \omega\end{matrix}\right\}$ for *inverted* image.

At the middle of the eclipse,

$\angle$ cent. shadow from North towards East $= \left\{\begin{matrix}(180^\circ - \iota)\\ (-\iota)\end{matrix}\right\}$ for $\left\{\begin{matrix}\textit{direct}\\ \textit{inverted}\end{matrix}\right\}$ image.

To get the same angles from the Vertex, the parallactic angle must be deducted for the respective times.

EXAMPLES.

I.—ECLIPSE OF THE SUN.

Let it be required to calculate the circumstances of the Solar Eclipse of May 15, 1836, as it will be seen at the Observatory of Edinburgh.

The Elements of this Eclipse are stated at page 86.

	h m s		
Greenwich sidereal time at Greenwich mean noon	3 32 58 ·0		
Longitude	12 43 ·6 W.		
Edinburgh sidereal time at Greenwich mean noon	3 20 14 ·4	α_1 -	27′·7
Sun's right ascension at ☌	3 29 25 ·2	f -	3 ·03
Hour angle h at Greenwich mean noon	— 0 9 10 ·8		83 ·1
Greenwich mean time of ☌	2 21 22 ·9		·8
Acceleration	23 ·2	$\alpha_1 . f$	84 ·—
h at ☌	+ 2 13	- - ζ +	21 $\zeta^{(1)}$ +55 (+ ·87
		$\alpha_1 . f - \zeta$ +	63 50 ·4
			4 ·6

	h m
Greenwich mean time of true ☌	2 21
$\zeta^{(1)} \div (\alpha_1 . f - \zeta)$	+ 52
T	3 13

Constants.

		ρ -	9 ·99902
P -	54′ 23″ ·4	const. -	4 ·68555
π -	8 ·5	c -	4 ·68457
P—π -	54 14 ·9		3 ·51254
log P -	3 ·51367	A - -	8 ·19711
const. -	9 ·43537	cos l -	9 ·75001
log s - -	2 ·94904	m -	7 ·94712
σ - -	15′ 49″·9	Q₁ -	4 ·7172
δ +	18° 58′·5	sin δ +	9 ·5121
		Q_2 +	2 ·1764

Computation for 3h 13m, Greenwich time.

D + 19° 33′ 43″	δ + 18° 58′ 29″	α + 23′ 49″
D_1 + 9 19		α_1 + 27 43

	h m s
Edinburgh sidereal time at Greenwich mean noon	3 20 14 ·4
Sidereal equivalent for 3h 0m	3 0 29 ·6
Sidereal equivalent for 13	13 2 ·1
	6 33 46 ·1
Moon's right ascension	3 31 9 ·0
h in time	3 2 37 ·1
h in arc	+ 45° 39′·3

m - -	7·94712				
cos D - -	9·97418	const. -	5·31439		
k - -	7·97294	- - - - -	7·97294		
cos h	+ 9·84446	sin h	+ 9·85440	- - - - -	+ 9·8544
n	+ 7·81740	corr. for n	286		
Q_1 - -	4·7172	log	+ 3·14459	Q_2 - -	+ 2·1764
log -	+ 2·5346	$\Delta\alpha$	+ 23′ 15″	log - -	+ 2·0308
$\Delta\alpha_1$	+ 5′ 42″			ΔD_1 -	+ 1′ 47″
h - -	+ 45° 39′·3				
$\frac{1}{2}\Delta\alpha$	+ 11·6				
(h)	+ 45 50·9	cos - - -	+ 9·84296	- - - - - - -	+ 9·84296
		cot l - -	+ 9·83256	cos l - -	+ 9·75001
θ	+ 25° 20′·9	tan θ - -	+ 9·67552	G - -	+ 9·59297
D	+ 19 33·7	sin θ - -	+ 9·63156		
θ+D	+ 44 54·6	cos - - -	+ 9·85017	B - -	+ 9·81158
			+ 9·78139	check - -	+ 9·78139
		tan (h) -	+ 0·01286		
M	+ 31° 54′·5	tan M -	+ 9·79425		
		cos M -	+ 9·92885	- - - - - -	+ 9·92885
		tan $(\theta$+D$)$	+ 9·99864	cos ϵ - -	+ 9·88273
ϵ	+ 40° 14′·3	tan ϵ - -	+ 9·92749	B - -	+ 9·81158
		cos ϵ - -	+ 9·88273	const. - -	+ 5·31439
		sin ϵ - -	+ 9·81022		5·12597
		A - -	8·19711	- - - - - -	8·19711
		n_1 - -	+ 8·00733	corr. for n_1	444
				log - - - -	3·32752
		log s - - -	2·94904	ΔD - -	+ 35′ 26″
			444		
		log - - - -	2·95348		
		s' - - - -	14′ 58″·4	D_1 - -	+ 9′ 19″
		σ - - - - -	15 49·9	ΔD_1 - -	+ 1 47
		Partial Δ' - - - -	30 48·3	x_1 - -	+ 7 32
		Annular Δ' - - - -	0 51·5		

D	+ 19° 33′ 43″	α	+ 23′ 49″	α_1	+ 27′ 43″
ΔD	+ 35 26	$\Delta\alpha$	+ 23 15	$\Delta\alpha_1$	+ 5 42
D′	+ 18 58 17	α'	+ 0 34		+ 22 1
α' corr.	0	log	+ 1·53148	log	+ 3·12090
δ	+ 18 58 29	cos D′	+ 9·97574	- - -	+ 9·97574
x - —	0 12	y -	+ 1·50722	y_1 -	+ 3·09664 (1)

	y -	+ 1·50722	y_1 -	+ 3·09664	(1)
	x -	— 1·07918	x_1 -	+ 2·65514	
S + 110° 28′·0	tan S	— 0·42804	cot ι	+ 0·44150	
	cos S	— 9·54364	cos ι	+ 9·97328	
ι + 19 53·5	W -	+ 1·53554	- - -	+ 1·53554	
—(S+ι) — 130 21·5	cos -	— 9·81129	const.	3·55630	
	n	— 1·34683		+ 5·06512	(2)
Partial - -	log Δ'	3·26677	H	+ 1·96848	(2) — (1)
ω + 90 41·1	cos ω	— 8·08006	- - -	— 8·08006	
a — 221 2·6	c	— 3·88842	c	— 3·88842	
b — 39 40·4	sin a	+ 9·81732	sin b	— 9·80510	
		— 3·70574		+ 3·69352	
	t_1	— 1h 24m 39s	t_2	+ 1h 22m 18s	
	Assumed time	3 13	- - - - -	3 13	
	Beginning - -	1 48 21	Ending	4 35 18	Greenwich mean times.
	Longitude - -	12 44 W.	- - -	12 44 W.	
PARTIAL.	Beginning - -	1 35 37	Ending	4 22 34	Edinburgh mean times.

	n - -	— 1·34683			
Annular - -	log Δ'	1·71181	H	+ 1·96848	
ω + 115° 33′·9	cos ω	— 9·63502	- - -	— 9·63502	
—(S+ι) — 130 21·5	c	— 2·33346	c	— 2·33346	
a — 245 55·4	sin a	+ 9·96047	sin b	— 9·40711	
b — 14 47·6		— 2·29393		+ 1·74057	
	t_1	— 0h 3m 17s	t_2	+ 0h 0m 55s	
	Assumed time	3 13	- - - - -	3 13	
	Beginning -	3 9 43	Ending	3 13 55	Greenwich mean times.
	Longitude -	12 44 W.	- - -	12 44 W.	
ANNULAR.	Beginning -	2 56 59	Ending	3 1 11	Edinburgh mean times.

POSITIONS OF CONTACTS FOR DIRECT IMAGE.

	($-\iota$)	— 19°·9	
	ω	+ 90·7	
Partial contact at	beginning - - -	110·6	from North towards West.
	ending - - - -	70·8	from North towards East.
	($-\iota$)	— 19°·9	
	ω	+ 115·6	
Annular contact at	beginning - - -	135·5	from North towards West.
	ending - - - -	95·7	from North towards East.

For the same angles from Vertex we must estimate them towards the East, and deduct the angle M, thus

Beginning	$-\ 135^\circ\!\cdot5$	Ending	$+\ 95^\circ\!\cdot7$
M	$+\ 31\cdot9$	M	$+\ 31\cdot9$
	$167\cdot4$ towards West.		$63\cdot8$ towards East.

Computation for $1^h\ 48^m$, for an accurate Determination of Partial Beginning.

D $+\ 19^\circ\ 19'\ 35''\!\cdot9$	$\delta\ +\ 18^\circ\ 57'\ 39''\!\cdot3$		$\alpha\ -\ 15'\ 23''\!\cdot2$
$D_1\ +\ 9\ 26$			$\alpha_1\ +\ 27\ 38$

		h m s
Edinburgh Sid. Time at Greenwich Mean Noon - -		3 20 14 ·4
Sidereal Equivalent for	$1^h\ 0^m$ - - - - - - - - -	1 0 9 ·9
	48 - - - - - - - - -	48 7 ·9
		5 8 32 ·2
☽'s R. A. - - - - - - - -		3 28 18 ·2
h in	time - - - - - - - - +	1 40 14 ·0
	arc - - - - - - - - +	25° 3′ ·5

m - - - -	7 ·94712				
cos D - -	9 ·97481	Const. - - -	5 ·31439		
k - - - -	7 ·97231	- - - - - - - -	7 ·97231		
cos h - -	+ 9 ·95707	sin h - - - - -	+ 9 ·62690	- - - - -	+ 9 ·6269
n - - - -	7 ·92938	corr. for n - -	370		
Q_1 - - -	4 ·7172	log - - - - -	+ 2 ·91730	Q_2 -	2 ·1764
log - - -	+ 2 ·6466	$\Delta\ \alpha$ - - - -	+ 13′ 46″·6	log -	+ 1 ·8033
$\Delta\ \alpha_1$ - - -	+ 7′ 23″			$\Delta\ D_1$	+ 1′ 4″
h - - - - -	+ 25° 3′·5				
$\frac{1}{2}\Delta\alpha$ - - -	+ 6 ·9				
(h) - - - -	+ 25 10 ·4	- cos - - - -	+ 9 ·95666	- - - - - - -	+ 9 ·95666
		cot l - - - - -	+ 9 ·83256	cos l - - -	+ 9 ·75001
θ - - - - -	+ 31° 36′·7	tan θ - - - - -	+ 9 ·78922	G - - - - -	+ 9 ·70667
D - - - - -	+ 19 19 ·6	sin θ - - - - -	+ 9 ·71946		
θ + D - - -	+ 50 56 ·3	- cos - - - -	+ 9 ·79945	B - - - - -	+ 9 ·78665
			+ 9 ·92001	- - check -	+ 9 ·92002
		tan (h) - - -	+ 9 ·67209		
M_1 - - - -	+ 21° 21′·1	tan M_1 - - -	+ 9 ·59210		
		cos M_1 - - -	+ 9 ·96911	- - - - - - -	+ 9 ·96911
		tan $(\theta + D)$ -	+ 0 ·09068	cos ε - - -	+ 9 ·81754
ε - - - - -	+ 48° 55′·9	tan - - - - - -	+ 0 ·05979	B - - - - -	+ 9 ·78665
		cos - - - - -	+ 9 ·81754	const. - -	5 ·31439
		sin - - - - - -	+ 9 ·87733		5 ·10104
		A - - - - - -	+ 8 ·19711	- - - - - - -	8 ·19711
		n - - - - - -	+ 8 ·07444	corr. for n_1	518
				log - - -	3 ·30333
				Δ D - - -	+ 33′ 30″·6

log s - - - -	2·94904				
	518				
log - - - - -	2·95422	D_1 - -	+	9′ 26″	
s' - - - - -	15 0·0	ΔD_1 - -	+	1 4	
σ - - - - -	15 49·9	x_1 - - -	+	8 22	
Δ' - - - - -	30 49·9				

D - - -	+ 19° 19′ 35″·9	α	− 15′ 23″·2	α_1	+ 27′ 38″	
ΔD - -	+ 33 30·6	$\Delta\alpha$	+ 13 46·6	$\Delta\alpha_1$	+ 7 23	
D′ - - -	+ 18 46 5·3	α'	− 29 9·8		+ 20 15	
α' corr. -	2·2	log -	3·24299	log -	+ 3·08458	
δ - - -	+ 18 57 39·3	cos D′	+ 9·97627	- - -	+ 9·97627	
x - - -	− 11 31·8	y -	− 3·21926	y_1 -	+ 3·06085	(1)
		x -	− 2·83998	x_1 -	+ 2·70070	
S - - -	− 112° 39′·81	tan S	+ 0·37928	cot ι_1	+ 0·36015	
		sin S	− 9·96510	cos ι_1	+ 9·96215	
ι - - -	+ 23 34·49	W -	+ 3·25416	- - -	+ 3·25416	
−(S+ι_1)	+ 89 5·32	cos -	+ 8·20168	Const.	3·55630	
					6·77261	(2)
		n -	+ 1·45584			
		log Δ'	+ 3·26715	H	+ 3·71176	(2)—(1)
ω_1 - - -	+ 89 6·93	cos ω_1	+ 8·18869	- - -	+ 8·18869	
a - - -	− 0 1·61			c - -	+ 5·52307	
		sin a	sin 1′		6·46373	
			—1·61	—	0·20683	
				—	2·19363	
			t_1 -	—	0^h 2^m 36^s	
			Assumed time		1 48 0	
			Beginning		1 45 24	Greenh. M. time.
			Long. - -		12 44	W.
		PARTIAL.	Beginning		1 32 40	Edin. M. time.

If the calculation be repeated for the Greenwich time 1^h 45^m, it will lead to exactly the same result, which is therefore to the accurate second, according to the data employed.

Position of Contact for direct Image.

$(-\iota_1)$ - - - - - -	—	23°·6
ω_1 - - - - - -	+	89·1
$(-\iota_1)-\omega_1$ - - - -	—	112·7
M_1 - - -	+	21·4
$(-\iota_1)-\omega_1-M_1$ - -	—	134·1

The point of contact is therefore $\left\{\begin{matrix}113^\circ\\134\end{matrix}\right\}$ from $\left\{\begin{matrix}\text{North}\\\text{Vertex}\end{matrix}\right\}$ towards West.

II.—EQUATIONS FOR REDUCTION OF PARTIAL BEGINNING.

The data for this computation are taken from the preceding one.

	3·55630		5·31439		7·9208
Δ'	3·26715	A	8·19711	$\Delta\alpha$	+ 2·9173
$\cos\iota$	9·96215	Corr. for n_1	518	sin D	+ 9·5198
	6·78560		3·51668		+ 0·3579 - - e + 0° 2′·3
y_1	3·06085	Δ'	3·26715		90° + ι - 113 34·5
k	+ 3·72475	b	+ 0·24953	χ	- 113 32·2
		k	+ 3·72475		
		kb	+ 3·97428		

D	+ 19° 19′ 35″·9	α	− 2·96530
α' corr.	2·2	cos D′	+ 9·97627
δ	+ 18 57 39·3	$\Delta'\gamma\sin\psi$	− 2·94157
	+ 21 58·8 - - - - - - -	$\Delta'\gamma\cos\psi$	+ 3·12018
ψ	− 33° 32′·2 - - - - - - - - -	$\tan\psi$	− 9·82139
ι	+ 23 34·5	$\cos\psi$	+ 9·92092
$\psi+\iota$	− 9 57·7	$\Delta'\gamma$	- - + 3·19926
		Δ'	- - - - - 3·26715
γ	9·93211 - - - - - - - - - -	γ	- - - 9·93211
$\cos(\psi+\iota)$	+ 9·99341	$\sin(\psi+\iota)$	− 9·23802
		k	- - - - 3·72475
	+ 9·92552		− 2·89488
p	+ 0·84240	q	- — − $0^h\ 13^m\ 5^s$
h	+ 25° 3′·5	T	- - + 1 48
Long.	3 10·9 W.		
H	+ 28 14·4	T′	- - + 2 1 5

cos D	+ 9·97481	cos D	+ 9·97481
$\cos\iota$	+ 9·96215	$\sin\iota$	+ 9·60200
b	- - + 0·24953	kb	+ 3·97428
L′	- - + 0·18649	L″	+ 3·55109

sin D	+ 9·51977	sin D	+ 9·51977
$\cos\iota$	+ 9·96215	$\sin\iota$	+ 9·60200
	+ 9·48192		+ 9·12177
$\cos\chi$	− 9·60134	$\sin\chi$	+ 9·96228

ψ'—H — 52° 46′·8	tan	− 0·11942		ψ''—H + 81° 47′·1	tan	+ 0·84051	
	sin	− 9·90109			sin	+ 9·99552	
H + 28 14·4		+ 9·70025		H + 28 14·4		+ 9·96676	
ψ' - - — 24 32·4	b - -	0·24953		ψ'' + 110 1·5	kb - -	3·97428	
	γ' -	+ 9·94978			γ''	+ 3·94104	

We have hence, for the Greenwich time t of beginning, at any place whose latitude is l, $+$ North, $-$ South, and longitude λ, $+$ East, $-$ West, the two following equations, which may be safely depended on for any place in Scotland or the North of England.

$$\cos\omega = 0{\cdot}84240 - [0{\cdot}18649]\sin l + [9{\cdot}94978]\cos l\cos(\lambda - 24^\circ\ 32'{\cdot}4)$$
$$t = 2^h\ 1^m\ 5^s - [3{\cdot}72475]\sin\omega + [3{\cdot}55109]\sin l - [3{\cdot}94104]\cos l\cos(\lambda + 110^\circ\ 1'{\cdot}5)$$

Contact on ☉'s limb, $\omega + 23^\circ\ 34'{\cdot}5$ from the North towards the West.

As a check on this calculation take the assumed radical place, Edinburgh, and $l = +55^\circ\ 46'{\cdot}9$, $\lambda = -3^\circ\ 10'{\cdot}9$, giving $\omega = 89^\circ\ 6'{\cdot}9$ and $t = 1^h\ 45^m\ 24^s$, which perfectly coincide with the results of the original calculation.

Similar calculations for the ending of the Eclipse give the equations,

$$\cos\omega = 0{\cdot}93848 - [0{\cdot}20291]\sin l + [9{\cdot}88677]\cos l\cos(\lambda + 27^\circ\ 6'{\cdot}7)$$
$$t = 1^h\ 38^m\ 33^s + [3{\cdot}66890]\sin\omega + [3{\cdot}35544]\sin l - [3{\cdot}90073]\cos l\cos(\lambda + 153^\circ\ 3'{\cdot}8)$$

Contact on ☉'s limb, $\omega - 16^\circ\ 56'{\cdot}2$ from the North towards the East.

Also by calculating with $T = 3^h\ 13^m$ for the annular phase there will result

$$\cos\omega = 29{\cdot}66600 - [1{\cdot}75159]\sin l + [1{\cdot}46950]\cos l\cos(\lambda + 1^\circ\ 42'{\cdot}4)$$
$$t = 1^h\ 43^m\ 7^s \mp [2{\cdot}14475]\sin\omega + [3{\cdot}45484]\sin l - [3{\cdot}92550]\cos l\cos(\lambda + 131^\circ\ 55'{\cdot}9)$$

Contact on ☉'s limb, $-19^\circ\ 53'{\cdot}5 \mp \omega$ from the North towards the East,

the upper sign appertaining to the beginning and the under sign to the ending. If $\cos\omega > 1$ the place will be without the limits, and the eclipse will not be annular.

By taking $l = +55^\circ\ 46'{\cdot}9$, $\lambda = -3^\circ\ 10'{\cdot}9$, the results will exactly correspond with the special calculation.

Note.—The expression of $\cos\omega$ for the annular phase, as the appearance of this phase is comprised within narrow limits on the surface of the Earth, will afford a very convenient and simple determination of the places which range in those limits as well as those which range in the central line; and we may expect very accurate results throughout the portion of country originally taken into consideration. Thus for the Southern limit we must obviously have $\cos\omega = +1$, for the Central line $\cos\omega = 0$, and for the Northern limit $\cos\omega = -1$; and hence the following conditions:

$$p - L'\sin l + \gamma'\cos l\cos(\lambda + \psi') = \left\{\begin{array}{r} +1 \\ 0 \\ -1 \end{array}\right\} \text{ for } \left\{\begin{array}{l} \text{southern limit.} \\ \text{central eclipse.} \\ \text{northern limit.} \end{array}\right.$$

By making the assumptions

$$\left.\begin{array}{l} n'\cos N' = \gamma'\cos(\lambda + \psi') \\ n'\sin N' = L' \end{array}\right\} \text{- - - - - - - - - - - - - -} \quad (r)$$

they will give

$$n'\cos(N' + l) = \left\{\begin{array}{l} -p + 1 \\ -p \\ -p - 1 \end{array}\right\} \text{ for } \left\{\begin{array}{l} \text{southern limit} \\ \text{central eclipse} \\ \text{northern limit} \end{array}\right\} \text{- - - -} \quad (s)$$

If we therefore take any meridian whose East longitude is λ, these two equations (r), (s) will serve to determine the extreme latitudes l, on this meridian, between which the eclipse will be annular as well as that where it will be central.

For the preceding eclipse, these equations will be

$$n'\cos N' = [1{\cdot}46950]\cos(\lambda + 1^\circ\ 42'{\cdot}4)$$
$$n'\sin N' = [1{\cdot}75159]$$

$$n'\cos(N' + l) = \left\{\begin{array}{l} -[1{\cdot}45737] \\ -[1{\cdot}47226] \\ -[1{\cdot}48665] \end{array}\right\} \text{ for } \left\{\begin{array}{l} \text{southern limit.} \\ \text{central eclipse.} \\ \text{northern limit.} \end{array}\right.$$

If we take, for example, the meridian of Edinburgh, and use $\lambda = -3^\circ\ 10'\cdot9$, there will result,

Extreme Southern Point of annular appearance, N. $54^\circ\ 19'\cdot7$
Point of Central appearance, N. $55\ \ 20\cdot4$
Extreme Northern Point of annular appearance, N. $56\ \ 21\cdot7$

which are geocentric latitudes.

III.—CALCULATION OF THE TRANSIT OF MERCURY, Nov. 7, 1835.

The conjunction in right ascension takes place about $7^h\ 38^m$; take therefore $T = 7^h\ 40^m$, and we readily find from the ephemeris the following data,

$$\delta - 16^\circ\ 15'\ 58''\cdot2$$

D	$-\ 16^\circ\ 22'\ 4''\cdot2$	α	$+\ 0'\ 10''\cdot95$
D_1	$-\ 2\ 32\cdot6$	α_1	$+\ 5\ 32\cdot7$
s	$4\cdot8$	σ	$16\ 10\cdot4$
P	$12\cdot66$	π	$8\cdot66$

With these quantities the calculation, for external contact of limbs, is as follows:

$\sigma\ \ 16'\ 10''\cdot4$ P $12''\cdot66$
$s\ \ 4\cdot8$ $\pi\ \ 8\cdot66$
$\Delta\ \ 16\ 15\cdot2$ $4\cdot00$ - - - - $0\cdot60206$
$\Delta\ \ 2\cdot98909$
$b + 7\cdot61297$

$\alpha + 1\cdot03941$ $\alpha_1 + 2\cdot52205$
$\cos\delta + 9\cdot98226$ - - - - - - $+ 9\cdot98226$
$\alpha\cos\delta + 1\cdot02167$ $\alpha_1\cos\delta + 2\cdot50431$

$\delta - 16^\circ\ 15'\ 58''\cdot2$ } $D_1 - 2'\ 32''\cdot6$ - - $- 2\cdot18355$
α corr. $\cdot0$ } $\alpha_1\cos\delta + 2\cdot50431$
$D - 16\ 22\ 4\cdot2$ $\alpha\cos\delta + 1\cdot02167$ { tan $- 9\cdot67924$
$+\ 6\ 6\cdot0$ - - - - $+ 2\cdot56348$ $\iota - 25^\circ\ 32'\cdot3$ { cos $+ 9\cdot95535$
$\psi + 1^\circ\ 38'\cdot7$ - - { $\tan\psi + 8\cdot45819$ $\Delta\ \ 2\cdot98909$
{ $\cos\psi + 9\cdot99982$ const. $3\cdot55630$
$\iota - 25\ 32\cdot3$ $+ 2\cdot56366$ $+ 6\cdot50074$
$\psi + \iota - 23\ 53\cdot6$ $\Delta\ \ 2\cdot98909$ $k + 3\cdot99643$
$\gamma + 9\cdot57457$ $b + 7\cdot61297$
$\gamma + 9\cdot57457$ $k + 3\cdot99643$ $kb + 1\cdot60940$
$\cos(\psi+\iota) + 9\cdot96109$ $\sin(\psi+\iota) - 9\cdot60749$ $\sin w + 9\cdot97278$
$\cos w + 9\cdot53566$ $- 3\cdot17849$ $k'' + 1\cdot63662$
$w + 69^\circ\ 55'\cdot4$ $q - 0^h\ 25^m\ 8^s\cdot3$ $\cos\delta + 9\cdot98226$
$T + 7\ 40$ $k''\cos\delta + 1\cdot61888$
$\sin w + 9\cdot97278$ $T - q + 8\ 5\ 8\cdot3$
$k + 3\cdot99643$
$k\sin w + 3\cdot96921$ - - - - - - $2\ 35\ 15\cdot6$

Mean time of { ingress $5\ 29\ 52\cdot7$ / egress $10\ 40\ 23\cdot9$ } for the centre of the Earth.

Constants for Reduction of Ingress.

	h m s			
	5 29 52 ·7			
Equa. +	16 10 ·0			
H in time +	5 46 2 ·7			
H in arc +	86° 30′·7			
$-\iota$ +	25 32 ·3			
w	69 55 ·4			
$-\iota-$w $-$	44 23 ·1	cos + 9 ·85410		sin − 9 ·84477
		sin δ − 9 ·44733		
		− 9 ·30143		− 9 ·30143
$\psi''-$H −	105 58 ·3		tan	+ 0 ·54334
			sin	− 9 ·98290
ψ'' −	19 27 ·6			+ 9 ·86187
		k'' cos δ + 1 ·61888	k''	+ 1 ·63662
		L″ + 1 ·47298	γ''	+ 1 ·49849

Constants for Reduction of Egress.

	h m s			
	10 40 23 ·9			
Equa. +	16 9 ·2			
H in time +	10 56 33 ·1			
H in arc +	164° 8′·3			
$-\iota+$w +	95 27 ·7	cos − 8 ·97854		sin + 9 ·99802
		sin δ − 9 ·44733		
		+ 8 ·42587		+ 8 ·42587
$\psi''-$H +	88 28 ·0		tan	+ 1 ·57215
			sin	+ 9 ·99984
ψ'' +	252 36 ·3			+ 9 ·99818
ψ'' −	107 23 ·7	k'' cos δ + 1 ·61888	k''	+ 1 ·63662
		L″ − 0 ·59742	γ''	+ 1 ·63480

The former part of the calculation repeated for the times $5^h\ 30^m$ and $10^h\ 40^m$ we shall find more accurate times of ingress and egress, for the centre of the earth, to be $5^h\ 29^m\ 56^s$ and $10^h\ 40^m\ 31^s$, which however still cannot be depended on within a few seconds. More reliance can be placed in the amount of reduction for parallax. The times reduced for any place whose North latitude is l, and East longitude λ, viz.:

Ingress, Nov. 7^d $5^h\ 29^m\ 56^s + [1{\cdot}4730]\ \rho \sin l - [1{\cdot}4985]\ \rho \cos l \cos(\lambda - 19°\ 28')$

Egress, ——— $10\ 40\ 31 + [0{\cdot}5974]\ \rho \sin l + [1{\cdot}6348]\ \rho \cos l \cos(\lambda - 107°\ 24')$

will indicate, with considerable accuracy, the difference between the times at any two places.

The positions of the contacts on the Sun's limb, for an *inverted* image, will be

Contact at { ingress - - 44° 23′ / egress - - 95 28 } from the North towards the { West. / East.

IV.—OCCULTATION OF A STAR.

On January 7, 1836, the star ι Leonis, whose right ascension is $10^h\ 23^m\ 26^s{\cdot}4$ and declination N. $14°\ 58'\ 39''$, will be Occulted by the Moon.

Limits of Latitude.

At the time of true ☌ in right ascension, viz. $12^h\ 12^m\ 17^s$, we have the following data,

	° ′ ″		′ ″
D +	15 33 2	D_1 —	11 47
δ +	14 58 39	α_1 +	30 41
D — δ +	0 34 23	P +	56 4

with which we proceed thus:

D_1 — 11′ 47″		— 2 ·84942		$-\frac{n}{P}$	— ·5699
α_1 + 30 41		+ 3 ·26505		const.	+ ·2725
		— 9 ·58437	w_1 + 147° 24′	nat. cos	— ·8424
δ + 14° 59′	cos	+ 9 ·98498	w_2 + 107 18	nat. cos	— ·2974
ι — 21 41	tan	— 9 ·59939	ι + 21 41	log. cos	+ 9 ·9681 (1)
	cos	+ 9 ·96813			
diff. dec. + 34′ 23″		+ 3 ·31450	+ 86 37	log. cos	+ 8 ·8833 (2)
	n	+ 3 ·28263		log. cos δ	+ 9 ·9850 (3)
P + 56′ 4″		+ 3 ·52686	θ + 63 51	log. cos	+ 9 ·9531 (1)+(3)
$\frac{n}{P}$ + ·5699		+ 9 ·75577	l_1 + 83 33		
			l_2 — 4 14	log. sin l_2	+ 8 ·8683 (2)+(3)

The star may therefore be occulted between the parallels of latitude N. 83° 33′ and S. 4° 14′. The parallel of Greenwich is within these limits; and if the hour angle of the star be computed roughly for the meridian of Greenwich, the star will be found to be considerably elevated above the horizon. A special calculation for the Observatory of Greenwich will consequently serve as an example of the circumstances for a particular place.

Calculation for Greenwich Observatory.

Constants $\phi^{(1)}, \phi^{(2)}, \phi^{(3)}$.

ρ	9 ·99913			
cos l	+ 9 ·79610			
$\phi^{(1)}$	+ 9 ·79523			+ 9 ·79523
cot l	+ 9 ·90381		Const.	9 ·41916
$\phi^{(2)}$	+ 9 ·89142		$\phi^{(3)}$	+ 9 ·21439

These will be constant for all Occultations at Greenwich.

	h m s		h m s
Sidereal time at mean noon	19 4 22 ·4		
Star's right ascension	10 23 26 ·4		
h at mean noon	— 15 19 4 ·0		— 15 19 4 ·0
Mean time of true ☌	+ 12 12	T	+ 11 6
Acceleration	+ 2	Acceleration	+ 1 49 ·4
h at true ☌	— 3 5	h in time	— 4 11 14 ·6
		h in arc	— 62° 48′·7

With this and $\alpha_1 = 30'{\cdot}7$ we find, by the table at page 129, T = $11^h\ 6^m$.

h at mean noon is put down negatively, in order to have more readily the other values of h less than 12^h or 180°.

P 56′ 4″	+ 3·52686		+ 3·52686		+ 3·52686
$\phi^{(2)}$	+ 9·89142	cos h	+ 9·65983	sin h	− 9·94915
	+ 3·41828		+ 3·18669		− 3·47601
cos δ	+ 9·98499	sin δ	+ 9·41236	sin δ	+ 9·41236
	+ 3·40327		+ 2·59905		− 2·88837
	+ 42′ 11″	$\phi^{(1)}$	+ 9·79523	$\phi^{(3)}$	+ 9·21439
	+ 4 8		+ 2·39428		− 2·10276
	+ 38 3				− 2′ 7″
D − δ	+ 47 22			D_1	− 11 42
x	+ 9 19			x_1	− 9 35
α	− 33′ 54″			α_1	+ 30′ 44″
	− 3·30835				+ 3·26576
cos δ	+ 9·98499			cos δ	+ 9·98499
	− 3·29334				+ 3·25075
	− 32′ 45″				+ 29′ 41″
P sin h	− 3·47601	P	3·52686	P cos h	+ 3·18669
$\phi^{(1)}$	+ 9·79523	Const.	9·43537	$\phi^{(3)}$	+ 9·21439
	− 3·27124	Δ'	2·96223		+ 2·40108
	− 31′ 7″				+ 4′ 12″
y	− 1′ 38″			y_1	+ 25 29
	− 1·99123				+ 3·18441
x	+ 2·74741			x_1	− 2·75967
tan S	− 9·24382	S	− 9° 56′·6	cot ι	− 0·42474
cos S	+ 9·99343	ι	− 20 36·6	cos ι	+ 9·97128
W	+ 2·75398			W	+ 2·75398
cos − (S + ι)	+ 9·93508	− (S + ι)	+ 30 33·2		3·55630
n	+ 2·68906				+ 6·28156
Δ'	2·96223			H	+ 3·09715
cos ω	+ 9·72683	ω	+ 57 47·0	cos ω	+ 9·72683
c	+ 3·37032	a	− 27 13·8	c	+ 3·37032
sin a	− 9·66045	b	+ 88 20·2	sin b	+ 9·99982
	− 3·03077				+ 3·37014

t_1	− $0^h\ 17^m·9$	t_2	+ $0^h\ 39^m·1$	
T	11 6	T	11 6	
Immersion	10 48·1	Emersion	11 45·1	Mean times.
Acceleration	1·8	Acceleration	2·0	
S. T. mean noon	19 4·4	S. T. mean noon	19 4·4	
Immersion	5 54·3	Emersion	6 51·5	Sid. times.
Star's R. A.	10 23·4	Star's R. A.	10 23·4	
Im. h	− 4 29·1 = − 67°	Em. h =	− 3 31·9 = − 53°	
Parallactic ∠	− 39°·7	Parallactic ∠	− 36°·9	
(− ι)	+ 20·6	(− ι)	+ 20·6	
ω	+ 57·8	ω	+ 57·8	
From North	− 37·2 to the East.	From North	+ 78·4 to the East.	
From Vertex	+ 2·5 to the East.	From Vertex	+ 115·3 to the East.	

These angles are for the *inverted* image; and, being estimated towards the East, the negative values must be considered as towards the West. The declination of the Star gives for the latitude of Greenwich a semidiurnal arc of $7^h\ 23^m$; as this exceeds the value of h both at Immersion and Emersion, the Immersion and Emersion will both occur above the horizon.

V.—CALCULATION OF THE ECLIPSE OF THE MOON, April 30, 1836.

The Opposition or Full Moon takes place at $19^h\ 58^m$. For the computation assume the time $20^h\ 0^m$.

	19^h	20^h	21^h
	h m s	h m s	h m s
☽'s R.A.	14 32 51·35	14 35 11·19	14 37 31·43
☉'s R.A. + 12^h	14 33 52·38	14 34 1·91	14 34 11·45
α in time	− 1 1·03	+ 1 9·28	+ 3 19·98
α in space	− 15′ 15″	+ 17′ 19″	+ 50′ 0″

$$\alpha = \begin{matrix} -\ 15'\ 15'' \\ +\ 17\ \ 19 \\ +\ 50\ \ \ 0 \end{matrix} \quad \begin{matrix} +\ 32'\ 34'' \\ +\ 32\ \ 41 \end{matrix} \qquad \alpha_1 = +\ 32'\ 38''$$

	19^h	20^h	21^h
	° ′ ″	° ′ ″	° ′ ″
☽'s Dec.	− 14 5 19	− 14 19 58	− 14 34 32
α cor.	0	1	5
☉'s Dec.	+ 15 6 35	+ 15 7 20	+ 15 8 6
x	+ 1 1 16	+ 47 21	+ 33 29

$$x = \begin{matrix} +\ 61'\ 16'' \\ +\ 47\ \ 21 \\ +\ 33\ \ 29 \end{matrix} \quad \begin{matrix} -\ 13'\ 55'' \\ -\ 13\ \ 52 \end{matrix} \qquad x_1 = -\ 13'\ 54''$$

		α	+3·01662	α_1 +3·29181	P = 60′ 19″
		cos D	+9·98627	+9·98627	
		y	+3·00289	y_1 +3·27808	P 3·55859
		x	+3·45347	x_1 −2·92117	9·99929
					3·55788
S	+19° 30′·7	tan S	+9·54942	cot ι −0·35691	P′ 60′ 13″
		cos S	+9·97431	cos ι +9·96163	π 9
ι	−23 43·9	W	+3·47916	+3·47916	σ 15 53
−(S+ι)	+ 4 13·2	cos	+9·99882	3·55630	44 29
		n	+3·47798	+6·99709	$\frac{1}{60}$ 44
External		Δ'	3·56808	H +3·71901	45 13 SHADOW
					s 16 26
ω	+35 38·7	cos ω	+9·90990	+9·90990	
a	−31 25·5	c	+3·80911	c +3·80911	Δ' 61 39 external
b	+39 51·9	sin a	−9·71716	sin b +9·80685	28 47 internal
			−3·52627	+3·61596	
		t_1	− 56m·0	t_2 + 1h 8m·8	
Assumed time			20h 0	20 0	
Beginning			19 4·0	Ending 21 8·8	Greenwich mean times.

For the times at any other place, it will only be necessary to take into account the difference of longitude.

The positions of the points of contact on the limb of the Moon may be determined in the same manner as those of an Occultation, and will here be unnecessary.

As Δ' for internal contact with Shadow is less than n, no internal contact can take place, and therefore the Eclipse is only partial.

The contacts with the Penumbra are to be determined in a similar manner from the same values of n, H, and will also be unnecessary here.

Δ' for external contact with Shadow	61′ 39″
n - - - - - - - - - -	50 6
Eclipsed - - - - - - -	11 33

which divided by $2s = 32' \, 52''$, gives 0·351 for the Magnitude of the Eclipse, the Moon's diameter being *unity*.

ON THE CALCULATION

OF

THE PERTURBATIONS

OF THE

SMALL PLANETS AND THE COMETS OF SHORT PERIOD.

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[The Author of the following paper wishes it to be understood that he lays no claim to novelty for any part of it. He has been induced to offer it to the Superintendent of the Nautical Almanac, by the hope that it may be useful to those who wish to study the theory of these remarkable bodies, or to effect the calculations connected with the prediction of their places; and by the belief that there does not exist in the English language any equivalent treatise.]

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(1) The smallness of the excentricities and inclinations of the orbits of the old planets, including Uranus, allows us to expand the expressions, on which their perturbations depend, in series proceeding by powers of the excentricities and inclinations, and therefore converging with considerable rapidity. The magnitude of the excentricities and inclinations of the four new planets, and more especially of the comets of short period, makes it difficult to form such expressions for the former and wholly impracticable for the latter. We are obliged here, therefore, to use a method which requires no expansion. While we thus give up all the elegance of mathematical theorems and all the facilities of tabulation, which mark so strikingly the usual planetary investigations, we are on the other hand perfectly secure against the loss of sensible quantities from imperfect development of the series, and even against the omission of terms depending on the second and all higher orders of the disturbing force.

Section I.

METHOD OF VARIATION OF ELEMENTS.

(2) The method which it is convenient to adopt here exclusively, is the method of *variation of elements*. Whatever the motion of a planet, under the action of dis-

turbing forces, may be, still it is evident that its place and its motion at any given time may be represented by supposing it to be moving at that time, undisturbed, in an ellipse with certain elements. This ellipse we shall call the *instantaneous ellipse.* The dimensions and position of the instantaneous ellipse may be imagined by conceiving the disturbing force to cease entirely, and then conceiving the curve in which the body will proceed to move to be observed or calculated from the place, velocity, and direction of motion, at the instant when the disturbing force ceases. The epoch, or mean longitude at a given previous time, will be found, by calculating backwards from the instant when the force ceases with the mean motion due to the orbit so determined. It is our object now to show how these elements may be determined for any given time, so that the calculation of the place and motion of the planet near that time may have precisely the same form as if the planet were moving in an undisturbed ellipse.

(3) The equations of motion give us only certain relations between the co-ordinates and the differential coefficients of the velocities in different directions. But from what has been said above, it will be readily inferred that the elements of the orbit can at every time be expressed by means of these co-ordinates and velocities at that time. Thus, by proper treatment, the equations of motion may be made to express the differential coefficients of the elements of the ellipse in which (as above) the planet's place and motion near that time are to be determined by the usual formulæ. This gives only the momentary changes of the elements: a process equivalent to integration will then give us the total change that has taken place between any one time and any other time.

(4) Let μ be the Sun's mass, m_1, m_2, m_3, &c. the masses of the disturbing planets (the masses being represented by the number of units of velocity which their action at the unit of distance would give in the unit of time), and for brevity let the letter m be used for the disturbed planet; also let x, x_1, x_2, &c., y, y_1, y_2, &c., z, z_1, z_2, &c., be the co-ordinates of the disturbed and disturbing planets at the time t: x being measured from the Sun towards the first point of Aries (supposed invariable), y being measured towards the first point of Cancer (the plane of the ecliptic being supposed invariable), and z perpendicular to the plane of the ecliptic: let r, r_1, r_2, &c. be the true radii vectores: also let a, e, ϖ, i, ν, be the mean distance, excentricity, longitude of perihelion, inclination, and longitude of node, of m at that time; n the mean angular motion (in the instantaneous ellipse) in one unit of time, measured in parts of radius: and let ε be the epoch of mean longitude of m, or the angle that must be added to nt, to form in the instantaneous ellipse what in an invariable ellipse is called the mean longitude; so that the place of m may be calculated at the time t, by supposing it to move in the ellipse whose elements are a, e, ϖ, i, ν, and supposing its mean longitude in that ellipse $= nt + \varepsilon$. Then ε will be a variable quantity, like all the other elements. Let θ be the true longitude of m. The true longitude is supposed to be measured from the first point of Aries, along the ecliptic to the node, and then along the orbit to the place of m.

(5) The co-ordinates in (4) are all referred to the Sun. If the Sun's co-ordinates referred to a fixed point are X, Y, Z, those of m referred to the same point are $X+x$, $Y+y$, and $Z+z$. The consideration of the attractions produced by the different bodies on m and the Sun, (neglecting the attractions which m itself produces) gives us these equations—

$$\frac{d^2(X+x)}{dt^2} = -\frac{\mu x}{r^3} - \frac{m_1(x-x_1)}{\{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2\}^{\frac{3}{2}}}$$

$$-\frac{m_2(x-x_2)}{\{(x-x_2)^2+(y-y_2)^2+(z-z_2)^2\}^{\frac{3}{2}}} - \&c.$$

$$\frac{d^2X}{dt^2} = +\frac{m_1 x_1}{r_1^3} + \frac{m_2 x_2}{r_2^3} + \&c.$$

Taking the difference, and putting A for the sum of such quantities as

$$\frac{m_1}{\mu}\left\{\frac{x-x_1}{\{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2\}^{\frac{3}{2}}} + \frac{x_1}{r_1^3}\right\}$$

for all the different disturbing planets, we have

$$\frac{d^2x}{dt^2} = -\frac{\mu x}{r^3} - \mu A.$$

Similarly, putting B for the sum of such quantities as

$$\frac{m_1}{\mu}\left\{\frac{y-y_1}{\{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2\}^{\frac{3}{2}}} + \frac{y_1}{r_1^3}\right\}$$

and C for the sum of such quantities as

$$\frac{m_1}{\mu}\left\{\frac{z-z_1}{\{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2\}^{\frac{3}{2}}} + \frac{z_1}{r_1^3}\right\}$$

for all the different disturbing planets, we have

$$\frac{d^2y}{dt^2} = -\frac{\mu y}{r^3} - \mu B$$

$$\frac{d^2z}{dt^2} = -\frac{\mu z}{r^3} - \mu C$$

These are our fundamental equations.

(6) To discover the best method of combining these, we must, in conformity with the considerations of (3), express some of the elements in elliptic motion by means of the co-ordinates and velocities of the planet. Now in elliptic motion $\frac{1}{a} = \frac{2}{r} - \frac{(\text{velocity})^2}{\mu} = \frac{2}{r} - \frac{1}{\mu}\left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right\}$: consequently by inferring from the equations of (5) the variation of $\frac{2}{r} - \frac{1}{\mu}\left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right\}$ we shall have the variation of $\frac{1}{a}$. Also putting h for twice the area passed over in the unit of time by the radius vector, in undisturbed elliptic motion, the projections of this double area on the planes of xy, yz, and xz, are $h \cos i$, $h \sin i \sin \nu$, $h \sin i \cos \nu$;

but these projections are respectively $x\frac{dy}{dt}-y\frac{dx}{dt}$, $y\frac{dz}{dt}-z\frac{dy}{dt}$, $x\frac{dz}{dt}-z\frac{dx}{dt}$; therefore inferring from the equations of (5) the variations of these latter quantities we shall obtain the variations of $h\cos i$, $h\sin i\sin\nu$, and $h\sin i\cos\nu$. From these we shall obtain the variations of i, ν, and h: and since $h=\sqrt{\{\mu a(1-e^2)\}}$, and the variation of a is already found, the variation of e will be found. The mean distance and excentricity being known, the place of perihelion is easily found, as there is but one place of perihelion which can give the proper values of r and $\frac{dr}{dt}$: and in like manner the epoch is found. This is the general outline of the method which we shall follow.

(7) From the equations of (5) we obtain

$$2\left(\frac{dx}{dt}\cdot\frac{d^2x}{dt^2}+\frac{dy}{dt}\cdot\frac{d^2y}{dt^2}+\frac{dz}{dt}\cdot\frac{d^2z}{dt^2}\right)=-\frac{2\mu}{r^3}\left(x\frac{dx}{dt}+y\frac{dy}{dt}+z\frac{dz}{dt}\right)$$

$$-2\mu\left(\mathrm{A}\frac{dx}{dt}+\mathrm{B}\frac{dy}{dt}+\mathrm{C}\frac{dz}{dt}\right)$$

$$\text{or } \frac{d}{dt}\left\{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right\}=2\mu\frac{d}{dt}\cdot\frac{1}{r}-2\mu\left(\mathrm{A}\frac{dx}{dt}+\mathrm{B}\frac{dy}{dt}+\mathrm{C}\frac{dz}{dt}\right)$$

$$\text{therefore } \frac{d}{dt}\left\{\frac{2}{r}-\frac{1}{\mu}\left\{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right\}\right\}=2\mathrm{A}\frac{dx}{dt}+2\mathrm{B}\frac{dy}{dt}+2\mathrm{C}\frac{dz}{dt}$$

or, as the quantity under the differential sign on the first side is $=\frac{1}{a}$,

$$-\frac{1}{a^2}\cdot\frac{da}{dt}=2\mathrm{A}\frac{dx}{dt}+2\mathrm{B}\frac{dy}{dt}+2\mathrm{C}\frac{dz}{dt}$$

$$\text{whence } \frac{da}{dt}=-2a^2\left(\mathrm{A}\frac{dx}{dt}+\mathrm{B}\frac{dy}{dt}+\mathrm{C}\frac{dz}{dt}\right)$$

from which the variation of the semi-major axis of the orbit is found.

(8) Now the longitude of m from the node is $\theta-\nu$ (θ being measured as mentioned in (4)), and the co-ordinates of m parallel to the line of nodes, perpendicular to the line of nodes in the plane of the ecliptic, and perpendicular to the ecliptic, are therefore $r\cos(\theta-\nu)$, $r\sin(\theta-\nu)\cos i$, and $r\sin(\theta-\nu)\sin i$. From these we readily obtain

$$x=r\{\cos(\theta-\nu)\cos\nu-\sin(\theta-\nu)\cos i\sin\nu\}$$

$$y=r\{\sin(\theta-\nu)\cos i\cos\nu+\cos(\theta-\nu)\sin\nu\}$$

$$z=r\sin(\theta-\nu)\sin i$$

and hence

$$\frac{dx}{dt} = \frac{dr}{dt}\cdot\frac{x}{r} + r\frac{d\theta}{dt}\left\{-\sin(\theta-\nu)\cos\nu - \cos(\theta-\nu)\cos i\sin\nu\right\}$$

$$\frac{dy}{dt} = \frac{dr}{dt}\cdot\frac{y}{r} + r\frac{d\theta}{dt}\left\{\cos(\theta-\nu)\cos i\cos\nu - \sin(\theta-\nu)\sin\nu\right\}$$

$$\frac{dz}{dt} = \frac{dr}{dt}\cdot\frac{z}{r} + r\frac{d\theta}{dt}\cos(\theta-\nu)\sin i$$

These expressions, it is to be observed, are formed on the supposition that the elements are invariable; which is correct, because the motion in the actual part of the real orbit is the same as it would be in the instantaneous ellipse of that instant, supposing that ellipse to remain unvaried.

(9) Substituting these in the expression for $\frac{da}{dt}$,

$$\frac{da}{dt} = -\frac{2a^2}{r}\cdot\frac{dr}{dt}(\mathrm{A}x + \mathrm{B}y + \mathrm{C}z)$$

$$+ 2a^2 r\frac{d\theta}{dt}\left\{\begin{array}{l}\mathrm{A}\Big(\sin(\theta-\nu)\cos\nu + \cos(\theta-\nu)\cos i\sin\nu\Big)\\ + \mathrm{B}\Big(\sin(\theta-\nu)\sin\nu - \cos(\theta-\nu)\cos i\cos\nu\Big)\\ - \mathrm{C}\cos(\theta-\nu)\sin i\end{array}\right\}$$

which for brevity we shall write

$$-\frac{2a^2}{r}\cdot\frac{dr}{dt}\mathrm{A}' + 2a^2 r\frac{d\theta}{dt}\mathrm{B}'.$$

The calculation of A' involves no difficulty; that of B' is rendered very easy by the use of two subsidiary constant angles, ψ and χ, where $\tan\psi = \tan\nu\cos i$, and $\tan\chi = \cot\nu\cos i$; whence

$$\mathrm{B}' = \mathrm{A}\frac{\cos\nu}{\cos\psi}\sin(\theta-\nu+\psi) + \mathrm{B}\frac{\sin\nu}{\cos\chi}\sin(\theta-\nu-\chi) - \mathrm{C}\sin i\cos(\theta-\nu).$$

(10) Since $r^2\frac{d\theta}{dt} = h = \sqrt{(\mu)}\sqrt{\Big(a(1-e^2)\Big)}$, and $\sqrt{\mu} = na^{\frac{3}{2}}$,

$$\text{we have } r^2\frac{d\theta}{dt} = na^2\sqrt{(1-e^2)}$$

$$\text{or } r\frac{d\theta}{dt} = \frac{na^2}{r}\sqrt{(1-e^2)}$$

$$\text{Also } \frac{a(1-e^2)}{r} = 1 + e\cos(\theta-\varpi)$$

$$\text{therefore } \frac{a(1-e^2)}{r^2}\cdot\frac{dr}{dt} = e\sin(\theta-\varpi)\frac{d\theta}{dt}$$

$$\text{and } \frac{a^2}{r}\cdot\frac{dr}{dt} = \frac{aer}{1-e^2}\sin(\theta-\varpi)\frac{d\theta}{dt} = \frac{na^3e}{r\sqrt{(1-e^2)}}\sin(\theta-\varpi)$$

Substituting these values in the expression of (9),

$$\frac{da}{dt} = -2\,\frac{na^3 e}{\sqrt{(1-e^2)}}\cdot\frac{\sin(\theta-\varpi)}{r}\,A' + 2\,na^4\sqrt{(1-e^2)}\,\frac{B'}{r}$$

(11) From the equations of (5) we find

$$x\,\frac{d^2y}{dt^2} - y\,\frac{d^2x}{dt^2} = \mu\,(Ay - Bx)$$

$$y\,\frac{d^2z}{dt^2} - z\,\frac{d^2y}{dt^2} = \mu\,(Bz - Cy)$$

$$x\cdot\frac{d^2z}{dt^2} - z\,\frac{d^2x}{dt^2} = \mu\,(Az - Cx)$$

or, by (6), since $x\,\dfrac{d^2y}{dt^2} - y\,\dfrac{d^2x}{dt} = \dfrac{d}{dt}\left(x\,\dfrac{dy}{dt} - y\,\dfrac{dx}{dt}\right)$, &c.

$$\frac{d}{dt}\,(h\cos i) = \mu\,(Ay - Bx)$$

$$\frac{d}{dt}\,(h\sin i\sin\nu) = \mu\,(Bz - Cy)$$

$$\frac{d}{dt}\,(h\sin i\cos\nu) = \mu\,(Az - Cx)$$

(12) $$\frac{d\nu}{dt} = \cos^2\nu\,\frac{d\tan\nu}{dt}$$

$$= \cos^2\nu\,\frac{d}{dt}\cdot\frac{h\sin i\sin\nu}{h\sin i\cos\nu}$$

$$= \frac{1}{h^2\sin^2 i}\left\{h\sin i\cos\nu\,\frac{d}{dt}\,(h\sin i\sin\nu) - h\sin i\sin\nu\,\frac{d}{dt}\,(h\sin i\cos\nu)\right\}$$

$$= \frac{1}{h\sin i}\left\{\cos\nu\,\frac{d}{dt}\,(h\sin i\sin\nu) - \sin\nu\,\frac{d}{dt}\,(h\sin i\cos\nu)\right\}$$

$$= \frac{\mu}{h\sin i}\left\{\cos\nu\,(Bz - Cy) - \sin\nu\,(Az - Cx)\right\}$$

$$= \frac{\mu}{h\sin i}\left\{A\,(-z\sin\nu) + B\,(z\cos\nu) + C\,(x\sin\nu - y\cos\nu)\right\}$$

If we substitute for x and y the values in (8), we find

$$x\sin\nu - y\cos\nu = -r\sin(\theta-\nu)\cos i = -z\cot i$$

Therefore $$\frac{d\nu}{dt} = \frac{\mu z}{h\sin^2 i}\left\{-A\sin\nu\sin i + B\cos\nu\sin i - C\cos i\right\}$$

Let $-A\sin\nu\sin i + B\cos\nu\sin i - C\cos i = C'$

Then, since $h = \surd\mu\,\surd\{a(1-e^2)\}$, and $n = \surd\mu\, a^{-\frac{3}{2}}$,

$$hn = \frac{\mu\surd(1-e^2)}{a}, \text{ and } \frac{\mu}{h} = \frac{an}{\surd(1-e^2)}$$

therefore
$$\frac{d\nu}{dt} = \frac{an}{\surd(1-e^2)\sin^2 i}\, z\,\mathrm{C}'.$$

$$(13)\quad \frac{di}{dt} = \frac{1}{2}\cot i\cos^2 i\,\frac{d\tan^2 i}{dt}$$

$$= \frac{1}{2}\cot i\cos^2 i\,\frac{d}{dt}\cdot\frac{(h\sin i\,\sin\nu)^2 + (h\sin i\cos\nu)^2}{(h\cos i)^2}$$

$$= \mu\cot i\cos^2 i\left\{\frac{h\sin i\,\sin\nu}{h^2\cos^2 i}(\mathrm{B}z-\mathrm{C}y) + \frac{h\sin i\,\cos\nu}{h^2\cos^2 i}(\mathrm{A}z-\mathrm{C}x)\right.$$
$$\left. - \frac{(h\sin i\,\sin\nu)^2 + (h\sin i\cos\nu)^2}{(h\cos i)^3}(\mathrm{A}y-\mathrm{B}x)\right\}$$

$$= \frac{\mu}{h}\left\{\cos i\sin\nu\,(\mathrm{B}z-\mathrm{C}y) + \cos i\cos\nu\,(\mathrm{A}z-\mathrm{C}x) - \sin i\,(\mathrm{A}y-\mathrm{B}x)\right\}$$

$$= \frac{\mu}{h}\left\{\mathrm{A}\,(z\cos i\cos\nu - y\sin i) + \mathrm{B}\,(z\cos i\sin\nu + x\sin i)\right.$$
$$\left. - \mathrm{C}\cos i\,(x\cos\nu + y\sin\nu)\right\}.$$

On substituting the values of x, y, and z, from (8) this is changed to

$$\frac{an}{\surd(1-e^2)}\left\{\mathrm{A}\Big(-z\cot(\theta-\nu)\sin\nu\Big) + \mathrm{B}\Big(z\cot(\theta-\nu)\cos\nu\Big)\right.$$
$$\left. - \mathrm{C}\,z\cot(\theta-\nu)\cot i\right\}$$

$$= \frac{an}{\surd(1-e^2)\sin i}\,z\cot(\theta-\nu)\,\mathrm{C}'$$

$$= \frac{an}{\surd(1-e^2)}\,r\cos(\theta-\nu)\,\mathrm{C}'$$

$$(14)\qquad \frac{1}{h}\cdot\frac{dh}{dt} = \frac{1}{2h^2}\cdot\frac{d(h^2)}{dt}$$

$$= \frac{1}{2h^2}\cdot\frac{d}{dt}\left\{(h\cos i)^2 + (h\sin i\sin\nu)^2 + (h\sin i\cos\nu)^2\right\}$$

$$= \frac{\mu}{h^2}\left\{h\cos i\,(\mathrm{A}y-\mathrm{B}x) + h\sin i\sin\nu\,(\mathrm{B}z-\mathrm{C}y) + h\sin i\cos\nu\,(\mathrm{A}z-\mathrm{C}x)\right\}$$

$$= \frac{an}{\surd(1-e^2)}\left\{\mathrm{A}\,(y\cos i + z\sin i\cos\nu) + \mathrm{B}\,(z\sin i\sin\nu - x\cos i)\right.$$
$$\left. - \mathrm{C}\,(x\sin i\cos\nu + y\sin i\sin\nu)\right\}$$

Substituting the values of x, y, and z, from (8)

$$\frac{1}{h}\cdot\frac{dh}{dt}=\frac{anr}{\sqrt{(1-e^2)}}\left\{\mathrm{A}\left(\sin(\theta-\nu)\cos\nu+\cos(\theta-\nu)\cos i\sin\nu\right)\right.$$

$$\left.+\mathrm{B}\left(-\cos(\theta-\nu)\cos i\cos\nu+\sin(\theta-\nu)\sin\nu\right)-\mathrm{C}\sin i\cos(\theta-\nu)\right\}$$

$$=\frac{an}{\sqrt{(1-e^2)}}\,r\,\mathrm{B}'$$

And as $h=\sqrt{\mu}\,\sqrt{\{a\,(1-e^2)\}}$, we get

$$\frac{1}{h}\cdot\frac{dh}{dt}=\frac{1}{2a}\cdot\frac{da}{dt}-\frac{e}{1-e^2}\cdot\frac{de}{dt}$$

$$\text{or }\frac{an}{\sqrt{(1-e^2)}}\,r\,\mathrm{B}'=-\frac{na^2e}{\sqrt{(1-e^2)}}\,\mathrm{A}'\cdot\frac{\sin(\theta-\varpi)}{r}$$

$$+\,na^3\sqrt{(1-e^2)}\cdot\frac{\mathrm{B}'}{r}-\frac{e}{1-e^2}\cdot\frac{de}{dt}$$

whence

$$\frac{e}{1-e^2}\cdot\frac{de}{dt}=-\frac{na^2e}{\sqrt{(1-e^2)}}\,\mathrm{A}'\,\frac{\sin(\theta-\varpi)}{r}+\frac{na}{\sqrt{(1-e^2)}}\left(a^2\,(1-e^2)-r^2\right)\frac{\mathrm{B}'}{r}$$

$$\text{and }\frac{de}{dt}=-na^2\sqrt{(1-e^2)}\,\frac{\sin(\theta-\varpi)}{r}\,\mathrm{A}'+\frac{na\sqrt{(1-e^2)}}{e}\left(a^2\,(1-e^2)-r^2\right)\frac{\mathrm{B}'}{r}$$

(15) Now $\frac{d\,(\log r)}{dt}$ is found by differentiating

$$\log r=\log a+\log(1-e^2)-\log\{1+e\cos(\theta-\varpi)\},$$

which represents the correct value of $\log r$, because by (2) and (4) the same expressions are to be taken to represent the place of m (using the elements of the instantaneous ellipse), as those which are employed in undisturbed elliptic motion (using the elements of the permanent ellipse). Still it is to be borne in mind that the elements vary from one instant to another; and therefore their variation must be taken into account in forming $\frac{d\,(\log r)}{dt}$. Thus we have for $\frac{d\,(\log r)}{dt}$ the rigorous expression

$$\frac{d\,(\log a)}{dt}+\frac{d\,\{\log(1-e^2)\}}{dt}-\frac{d\,\{\log(1+e\cos(\theta-\varpi))\}}{de}\cdot\frac{de}{dt}$$

$$-\frac{d\,\{\log(1+e\cos(\theta-\varpi))\}}{d\varpi}\cdot\frac{d\varpi}{dt}-\frac{d\,\{\log(1+e\cos(\theta-\varpi))\}}{d\theta}\cdot\frac{d\theta}{dt}.$$

This expression, it is evident, has been obtained merely by considering that the *place* of m is always represented truly by the elliptic formulæ applied to the variable elements. But by (2) the *motion* of m is also to be represented truly by the elliptic formulæ for motion applied to the variable elements: and therefore generally the first differential coefficient with respect to t, of r or θ, or of any function of r or θ, must be

represented truly by the elliptic formulæ. Now the elliptic formula for $\frac{d \log r}{dt}$ is

$$-\frac{d\{\log(1+e\cos(\theta-\varpi))\}}{d\theta}\cdot\frac{d\theta}{dt}$$

where $\frac{d\theta}{dt}$ is the same as in the former expression. Making the two expressions equal,

$$\frac{d(\log a)}{dt}+\frac{d\{\log(1-e^2)\}}{dt}-\frac{d\{\log(1+e\cos(\theta-\varpi))\}}{de}\cdot\frac{de}{dt}$$
$$-\frac{d\{\log(1+e\cos(\theta-\varpi))\}}{d\varpi}\cdot\frac{d\varpi}{dt}=0.$$

The reasoning of this article is general for any expression in terms of the co-ordinates of the place of m; and its general result may be stated thus: The differential coefficient of any function of the *co-ordinates* (including polar co-ordinates) of m, taking those parts only which depend on the elements, is equal to zero. This would not be true if the *motion* of m entered into the function; for then the differential coefficient of the function would involve second differential coefficients of the co-ordinates, which, as in (5), have not the same form for undisturbed and for disturbed motion.

(16) The last equation is

$$\frac{1}{a}\cdot\frac{da}{dt}-\frac{2e}{1-e^2}\cdot\frac{de}{dt}-\frac{\cos(\theta-\varpi)}{1+e\cos(\theta-\varpi)}\cdot\frac{de}{dt}-\frac{e\sin(\theta-\varpi)}{1+e\cos(\theta-\varpi)}\cdot\frac{d\varpi}{dt}=0$$

whence

$$\frac{d\varpi}{dt}=\frac{1+e\cos(\theta-\varpi)}{e\sin(\theta-\varpi)}\left(\frac{1}{a}\cdot\frac{da}{dt}-\frac{2e}{1-e^2}\cdot\frac{de}{dt}\right)-\frac{\cos(\theta-\varpi)}{e\sin(\theta-\varpi)}\cdot\frac{de}{dt}$$

$$=\frac{1+e\cos(\theta-\varpi)}{e\sin(\theta-\varpi)}\cdot\frac{2an}{\sqrt{(1-e^2)}}r\mathrm{B}'+\frac{na^2\sqrt{(1-e^2)}}{e}\mathrm{A}'\frac{\cos(\theta-\varpi)}{r}$$

$$-\frac{na\sqrt{(1-e^2)}}{e^2}\left(a^2(1-e^2)-r^2\right)\frac{\cos(\theta-\varpi)}{\sin(\theta-\varpi)}\cdot\frac{\mathrm{B}'}{r}$$

The factor of B′ in this expression

$$=\frac{nar}{e^2\sqrt{(1-e^2)}\sin(\theta-\varpi)}\left\{2e+2e^2\cos(\theta-\varpi)-\left\{\left(\frac{a(1-e^2)}{r}\right)^2-(1-e^2)\right\}\cos(\theta-\varpi)\right\}$$

$$=\frac{nar}{e^2\sqrt{(1-e^2)}\sin(\theta-\varpi)}\left\{2e+2e^2\cos(\theta-\varpi)\right.$$
$$\left.-\cos(\theta-\varpi)\{1+2e\cos(\theta-\varpi)+e^2\cos^2(\theta-\varpi)-1+e^2\}\right\}$$

$$=\frac{nar}{e^2\sqrt{(1-e^2)}\sin(\theta-\varpi)}\left\{2e\sin^2(\theta-\varpi)+e^2\cos(\theta-\varpi)-e^2\cos^3(\theta-\varpi)\right\}$$

$$=\frac{nar}{e^2\sqrt{(1-e^2)}\sin(\theta-\varpi)}\left\{2e\sin^2(\theta-\varpi)+e^2\sin^2(\theta-\varpi)\cos(\theta-\varpi)\right\}$$

$$= \frac{nar}{e\sqrt{(1-e^2)}} \sin(\theta-\varpi)\left(2+e\cos(\theta-\varpi)\right)$$

and therefore

$$\frac{d\varpi}{dt} = \frac{na^2\sqrt{(1-e^2)}}{e} \cdot \frac{\cos(\theta-\varpi)}{r} \mathrm{A}' + \frac{na}{e\sqrt{(1-e^2)}} r\sin(\theta-\varpi)\left(2+e\cos(\theta-\varpi)\right)\mathrm{B}'$$

(17) We have now determined the variation of the five elements by which the form and position of the orbit are defined: it remains to determine the variation of the epoch, the element by which the body's place in the orbit is found. For this purpose let us consider that θ, the longitude at the time t, depends upon the elements a, e, ϖ and ϵ, inasmuch as it is expressed by the series

$$nt + \epsilon + (2e + \&c.) \sin(nt + \epsilon - \varpi) + \&c.$$

where $n = \sqrt{\mu}\, a^{-\frac{3}{2}}$. The reasoning of (15), applied to this instance, shows that we must have

$$\frac{d\theta}{da} \cdot \frac{da}{dt} + \frac{d\theta}{de} \cdot \frac{de}{dt} + \frac{d\theta}{d\varpi} \cdot \frac{d\varpi}{dt} + \frac{d\theta}{d\epsilon} \cdot \frac{d\epsilon}{dt} = 0$$

The values of $\frac{da}{dt}$, $\frac{de}{dt}$, $\frac{d\varpi}{dt}$, have been found; we have therefore only to determine the values of $\frac{d\theta}{da}$, $\frac{d\theta}{de}$, $\frac{d\theta}{d\varpi}$, and $\frac{d\theta}{d\epsilon}$, and then the equation above will give us $\frac{d\epsilon}{dt}$.

(18) For $\frac{d\theta}{da}$. Whatever be the definition of a differential coefficient, the practical rule for finding it is this: give to a the increment δa, and leave the other elements and the time unaltered; find the increment $\delta\theta$ which this causes to θ, and then take the value to which $\frac{\delta\theta}{\delta a}$ approaches when δa is made indefinitely small. Now θ is found in terms of t by integrating this expression

$$\frac{dt}{d\theta} = \frac{r^2}{h}$$

$$= \frac{a^2(1-e^2)^2}{\{1+e\cos(\theta-\varpi)\}^2} \times \frac{1}{\sqrt{\mu}\sqrt{\{a(1-e^2)\}}}$$

$$= a^{\frac{3}{2}} \frac{(1-e^2)^{\frac{3}{2}}}{\sqrt{\mu}\{1+e\cos(\theta-\varpi)\}^2}$$

from which

$$t = \frac{a^{\frac{3}{2}}}{\sqrt{\mu}} f(\theta)$$

where

$$\frac{d.f(\theta)}{d\theta} = \frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}$$

If now we put $a+\delta a$ instead of a, $\frac{dt}{d\theta}$ becomes

$$\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}+\frac{3}{2}\cdot\frac{a^{\frac{1}{2}}}{\sqrt{\mu}}\,\delta a\,\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}+\text{higher powers of } \delta a:$$

and integrating,

$$t=\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}f(\theta)+\frac{3}{2}\cdot\frac{a^{\frac{1}{2}}}{\sqrt{\mu}}\,\delta a\left(f(\theta)-\text{constant}\right)+\text{higher powers of } \delta a.$$

Putting $\theta+\delta\theta$ in the place of θ,

$$t=\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}f(\theta)+\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}\cdot\frac{d.f(\theta)}{d\theta}\delta\theta+\frac{3}{2}\cdot\frac{a^{\frac{1}{2}}}{\sqrt{\mu}}\delta a\left(f(\theta)-\text{constant}\right)$$
$$+\text{higher powers and combinations of } \delta a \text{ and } \delta\theta.$$

Making this value of t equal to the former,

$$\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}\cdot\frac{d.f(\theta)}{d\theta}\delta\theta+\frac{3}{2}\cdot\frac{a^{\frac{1}{2}}}{\sqrt{\mu}}\,\delta a\left(f(\theta)-\text{constant}\right)+\&c.=0;$$

whence the limit of the value of $\frac{\delta\theta}{\delta a}$ is

$$=-\frac{3}{2a}\cdot\frac{f(\theta)-\text{constant}}{\frac{df(\theta)}{d\theta}}$$

$$=-\frac{3}{2a}\cdot\frac{\{1+e\cos(\theta-\varpi)\}^2}{(1-e^2)^{\frac{3}{2}}}\left(f(\theta)-\text{constant.}\right)$$

To determine the constant, it is to be remarked, that from the beginning we have assumed that in the instantaneous ellipse, whatever changes it may undergo, $nt+\epsilon$ is to represent the mean longitude: and therefore the variation of n (and consequently the variation of a on which n depends) must have t for a factor; and therefore its effect in $nt+\epsilon$, and in θ which depends on $nt+\epsilon$, must vanish when $t=0$, or when $f(\theta)=0$. From this we find,

$$\text{constant}=0;$$

$$\frac{d\theta}{da}\text{ or the limit of }\frac{\delta\theta}{\delta a}=-\frac{3}{2a}\cdot\frac{\{1+e\cos(\theta-\varpi)\}^2}{(1-e^2)^{\frac{3}{2}}}\cdot\frac{\sqrt{\mu}}{a^{\frac{3}{2}}}\,t$$

$$=-\frac{3}{2}na\sqrt{(1-e^2)}\,\frac{\{1+e\cos(\theta-\varpi)\}^2}{a^2(1-e^2)^2}\,t$$

$$=-\frac{3}{2}na\sqrt{(1-e^2)}\,\frac{t}{r^2}.$$

The reader's attention is particularly invited to the circumstance, that in this investigation distinct reference is made to the assumption, that in the instantaneous ellipse the mean longitude is found by adding to the epoch the mean motion corresponding to that ellipse since $t=0$.

19. For $\frac{d\theta}{de}$. Since, when the excentricity is e, t is found in terms of θ by integrating (with respect to θ)

$$\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}$$

or

$$\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}$$

t will be found, when the excentricity is $e+\delta e$, by integrating

$$\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}-\frac{\sqrt{(1-e^2)}}{n}\cdot\frac{3e+2\cos(\theta-\varpi)+e^2\cos(\theta-\varpi)}{\{1+e\cos(\theta-\varpi)\}^3}\delta e + \text{higher}$$

powers of δe

(where the coefficient of δe is found merely by differentiating the preceding term with respect to e.)

Performing the integration, then, as far as possible,

$$t=\frac{1}{n}f(\theta)-\frac{\sqrt{(1-e^2)}}{n}\cdot\frac{\sin(\theta-\varpi)\{2+e\cos(\theta-\varpi)\}}{\{1+e\cos(\theta-\varpi)\}^2}\delta e$$

$$+\text{ higher powers of }\delta e+\text{constant}\times\delta e.$$

Now putting $\theta+\delta\theta$ instead of θ we have the corresponding value of $t=$

$$\frac{1}{n}f(\theta)+\frac{1}{n}\cdot\frac{d.f(\theta)}{d\theta}\delta\theta-\frac{\sqrt{(1-e^2)}}{n}\cdot\frac{\sin(\theta-\varpi)\{2+e\cos(\theta-\varpi)\}}{\{1+e\cos(\theta-\varpi)\}^2}\delta e$$

$$+\text{ higher powers and combinations of }\delta\theta\text{ and }\delta e+\text{constant}\times\delta e.$$

Supposing t and all the elements except e unaltered, this value of t must be the same as if θ and e had no variations, that is it must be $\frac{1}{n}f(\theta)$. Consequently

$$0=\frac{1}{n}\cdot\frac{d.f(\theta)}{d\theta}\delta\theta-\frac{\sqrt{(1-e^2)}}{n}\cdot\frac{\sin(\theta-\varpi)\{2+e\cos(\theta-\varpi)\}}{\{1+e\cos(\theta-\varpi)\}^2}\delta e$$

$$+\text{ higher powers, \&c.}+\text{constant}\times\delta e$$

whence $$\frac{d\theta}{de}=\frac{\sqrt{(1-e^2)}}{\frac{d.f(\theta)}{d\theta}}\cdot\frac{\sin(\theta-\varpi)\{2+e\cos(\theta-\varpi)\}}{\{1+e\cos(\theta-\varpi)\}^2}+\text{constant}\times\frac{n}{\frac{d.f(\theta)}{d\theta}}$$

$$=\frac{\sin(\theta-\varpi)\{2+e\cos(\theta-\varpi)\}}{1-e^2}+\text{constant}\times\frac{n\{1+e\cos(\theta-\varpi)\}^2}{(1-e^2)^{\frac{3}{2}}}.$$

To determine the constant we must observe that in the elliptic expressions the only part of θ which depends on e is the equation of the centre; and that this is 0, and its variation produced by a variation of e is 0, when $\theta-\varpi=0$; and therefore we must have $0+\text{constant}\times\frac{n(1+e)^2}{(1-e^2)^{\frac{3}{2}}}=0,$

whence

$$\text{constant} = 0,$$

and therefore

$$\frac{d\theta}{de} = \frac{\sin(\theta-\varpi)\,\{2+e\cos(\theta-\varpi)\}}{1-e^2}.$$

(20) For $\frac{d\theta}{d\varpi}$. As before, t is found by integrating (with respect to θ)

$$\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}$$

when the longitude of perihelion is ϖ: and therefore when the longitude of perihelion is $\varpi+\delta\varpi$, t will be found by integrating

$$\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2} + \delta\varpi\,\frac{d}{d\varpi}\left(\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\right) + \text{higher powers of } \delta\varpi.$$

From the manner in which θ and ϖ enter into the last term, it is evident that

$$\frac{d}{d\varpi}\left(\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\right) = -\frac{d}{d\theta}\left(\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\right);$$

and therefore t will now be found by integrating, with respect to θ,

$$\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2} - \delta\varpi\,\frac{d}{d\theta}\left(\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\right)$$

or the value of t is

$$\frac{1}{n}f(\theta) - \delta\varpi\,\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2} + \text{higher powers of } \delta\varpi + \text{constant}\times\delta\varpi.$$

Put $\theta+\delta\theta$ in the place of θ, as before; this becomes

$$\frac{1}{n}f(\theta) + \frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\,\delta\theta - \frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\,\delta\varpi$$
$$+ \text{constant}\times\delta\varpi$$
$$+ \text{higher powers and combinations of } \delta\theta \text{ and } \delta\varpi.$$

From this as in the former cases,

$$\frac{d\theta}{d\varpi} = 1 - \text{constant}\times\frac{n\{1+e\cos(\theta-\varpi)\}^2}{(1-e^2)^{\frac{3}{2}}}$$

$$= 1 - \text{constant}\times\frac{na^2\sqrt{(1-e^2)}}{r^2}.$$

To determine the constant, we have to observe that the only part of the expression for θ which depends on ϖ is the equation of the centre: and that (as is well known)

the equation of the centre is maximum, or its variation produced by a variation of ϖ is nothing, when $r^2 = a^2\sqrt{(1-e^2)}$; therefore

$$1 - \text{constant} \times n = 0$$

$$\text{or constant} = \frac{1}{n};$$

and therefore

$$\frac{d\theta}{d\varpi} = 1 - \frac{a^2\sqrt{(1-e^2)}}{r^2}$$

(21) For $\frac{d\theta}{d\varepsilon}$. Since $nt+\varepsilon$ is the corrected quantity which results from the integration of $\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}$ we have

$$nt + \varepsilon = f(\theta);$$

and putting $\varepsilon + \delta\varepsilon$ for ε, and $\theta + \delta\theta$ for θ,

$$nt + \varepsilon + \delta\varepsilon = f(\theta) + \frac{d.f(\theta)}{d\theta} \cdot \delta\theta + \&\text{c}.$$

$$= f(\theta) + \frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\delta\theta + \&\text{c}.;$$

whence, as before,

$$\frac{d\theta}{d\varepsilon} = \frac{\{1+e\cos(\theta-\varpi)\}^2}{(1-e^2)^{\frac{3}{2}}} = \frac{a^2\sqrt{(1-e^2)}}{r^2}$$

(22) Now substituting all the values in the equation of (17),

$$-\frac{3}{2}na\sqrt{(1-e^2)}\frac{t}{r^2} \times \left\{-2\frac{na^3e}{\sqrt{(1-e^2)}}\text{A}'\frac{\sin(\theta-\varpi)}{r} + 2na^4\sqrt{(1-e^2)}\frac{\text{B}'}{r}\right\}$$

$$+\frac{\sin(\theta-\varpi)\{2+e\cos(\theta-\varpi)\}}{1-e^2} \times \left\{-na^2\sqrt{(1-e^2)}\frac{\sin(\theta-\varpi)}{r}\text{A}' + \frac{na\sqrt{(1-e^2)}}{e}\{a^2(1-e^2)-r^2\}\frac{\text{B}'}{r}\right\}$$

$$+\left(1-\frac{a^2\sqrt{(1-e^2)}}{r^2}\right) \times \left\{\frac{na^2\sqrt{(1-e^2)}}{e}\cdot\frac{\cos(\theta-\varpi)}{r}\text{A}' + \frac{na}{e\sqrt{(1-e^2)}}r\sin(\theta-\varpi)\{2+e\cos(\theta-\varpi)\}\text{B}'\right\}$$

$$+\frac{a^2\sqrt{(1-e^2)}}{r^2}\cdot\frac{d\varepsilon}{dt} = 0$$

From which $\frac{d\varepsilon}{dt} = \left\{-3\frac{n^2a^2e}{\sqrt{(1-e^2)}}t\frac{\sin(\theta-\varpi)}{r} + 2na\right.$

$$\left. + \frac{na^2(1-e^2)}{e}\left(\frac{1}{\sqrt{(1-e^2)}} - 1\right)\frac{\cos(\theta-\varpi)}{r}\right\}\text{A}'$$

$$+\left\{3\,n^2a^3\sqrt{(1-e^2)}\,\frac{t}{r}+\frac{na}{e}\left(\frac{1}{\sqrt{(1-e^2)}}-1\right)r\sin(\theta-\varpi)\left\{2+e\cos(\theta-\varpi)\right\}\right\}B'$$

(23) There remains but one point which requires attention. The variations of ϖ and ε are deduced exclusively from a consideration of the expressions for the projections of the small areas traced out in minute portions of time by the radius vector; as will be seen on examining (14) and (15). The variations, therefore, of ϖ and ε thus obtained are such as suppose longitude to be measured upon the constantly varying plane of the orbit, or rather upon successive portions of the orbit, no part of which is a continuous plane. But, for the purposes of astronomy, it is convenient to have longitude measured upon the ecliptic to the node of the actual orbit, and then on the actual plane of the orbit to the place of the body. It is evident that every small change in the place of the orbit will cause the longitude of any point (suppose the intersection of the old and new orbit) thus measured to undergo a change, dependent on the change in the place of the node, simply in consequence of our adopting this mode of measuring. Thus the longitude θ is composed of two parts, namely, ν on the plane of the ecliptic, and $\theta-\nu$ on the plane of the orbit: now, if the longitude of the node be increased by $\delta\nu$, a moment's consideration of the spherical triangles will show that the part of the longitude which is measured on the plane of the orbit is diminished by $\cos i\times\delta\nu$, and the longitude is now $(\nu+\delta\nu)+(\theta-\nu-\cos i\times\delta\nu)$, or $\theta+\delta\nu\times 2\sin^2\frac{i}{2}$: that is, the longitude, as we measure it, is to be increased by $2\sin^2\frac{i}{2}$ multiplied by every increase in the longitude of the node. We must therefore add both to $\frac{d\varpi}{dt}$ and to $\frac{d\varepsilon}{dt}$,

$$2\sin^2\frac{i}{2}\times\frac{d\nu}{dt}\text{ or }\frac{an}{2\sqrt{(1-e^2)}\cos^2\frac{i}{2}}\,z\,C'.$$

(24) The element n (the mean motion) depends entirely upon a, whose variation has already been found: it may, however, be useful to give its variation separately. Since $n^2a^3=\mu$, we have

$$2\,na^3\frac{dn}{dt}+3\,n^2a^2\frac{da}{dt}=0,\text{ from which }\frac{dn}{dt}=-\frac{3}{2}\cdot\frac{n}{a}\cdot\frac{da}{dt}$$

$$=\frac{3\,n^2a^2e}{\sqrt{(1-e^2)}}\cdot\frac{\sin(\theta-\varpi)}{r}A'-3\,n^2a^3\sqrt{(1-e^2)}\,\frac{B'}{r}$$

(25) Collecting, for convenience of reference, all the expressions that we have obtained, we have

$$\frac{da}{dt}=-2\,\frac{na^3e}{\sqrt{(1-e^2)}}\cdot\frac{\sin(\theta-\varpi)}{r}A'+2\,na^4\sqrt{(1-e^2)}\,\frac{B'}{r}$$

$$\frac{dn}{dt}=3\,\frac{n^2a^2e}{\sqrt{(1-e^2)}}\cdot\frac{\sin(\theta-\varpi)}{r}A'-3\,n^2a^3\sqrt{(1-e^2)}\,\frac{B'}{r}$$

$$\frac{d\varepsilon}{dt} = \left\{ -3\,\frac{n^2a^2e}{\sqrt{(1-e^2)}}\,t\,\frac{\sin(\theta-\varpi)}{r} + 2\,na \right.$$

$$\left. + \frac{na^2(1-e^2)}{e}\left(\frac{1}{\sqrt{(1-e^2)}}-1\right)\frac{\cos(\theta-\varpi)}{r} \right\} \mathrm{A}'$$

$$+\left\{3\,n^2a^3\sqrt{(1-e^2)}\,\frac{t}{r} + \frac{na}{e}\left(\frac{1}{\sqrt{(1-e^2)}}-1\right) r\sin(\theta-\varpi)\,\{2+e\cos(\theta-\varpi)\}\right\}\mathrm{B}'$$

$$+\frac{na}{2\sqrt{(1-e^2)}\,\cos^2\frac{i}{2}}\,z\,\mathrm{C}'$$

$$\frac{d\varpi}{dt} = \frac{na^2\sqrt{(1-e^2)}}{e}\cdot\frac{\cos(\theta-\varpi)}{r}\,\mathrm{A}'$$

$$+\frac{na}{e\sqrt{(1-e^2)}}\,r\sin(\theta-\varpi)\,\{2+e\cos(\theta-\varpi)\}\,\mathrm{B}' + \frac{na}{2\sqrt{(1-e^2)}\,\cos^2\frac{i}{2}}\,z\,\mathrm{C}'$$

$$\frac{de}{dt} = -\,na^2\sqrt{(1-e^2)}\,\frac{\sin(\theta-\varpi)}{r}\,\mathrm{A}' + \frac{na\sqrt{(1-e^2)}}{e}\left(a^2(1-e^2)-r^2\right)\frac{\mathrm{B}'}{r}$$

$$\frac{d\nu}{dt} = \frac{na}{\sqrt{(1-e^2)}\,\sin^2 i}\,z\,\mathrm{C}'$$

$$\frac{di}{dt} = \frac{na}{\sqrt{(1-e^2)}}\,r\cos(\theta-\nu)\,\mathrm{C}'.$$

(26) Conceiving all these expressions integrated through the interval for which we require the variation of elements, and applying the integrals to the values of the elements at the beginning of the time, we shall have the values of the elements which are to be used for calculations at and near the end of the time. Thus, suppose the elements so corrected to be a', n', ε', ϖ', e', ν', i', we must calculate the place at the time T, just as if the planet had been moving, since $t=0$, undisturbed in an ellipse, and as if ε' were the mean longitude when $t=0$, n' the mean motion since that time, and consequently $n'\mathrm{T}+\varepsilon'$ the mean longitude at the time T; and we must use all the other corrected elements as for undisturbed motion. If the integrations could be effected, this process would be absolutely rigorous.

(27) The first difficulty that occurs is this: that all the expressions involve the varying elements a, e, n, &c., as well as the co-ordinates r and θ, none of which can be calculated without a knowledge of the quantities whose values it is the very object of the investigation to find. To this it is to be answered, that the elements vary so slowly that it is generally quite accurate enough to make the calculations with the values which they had at the beginning of the time; or, at any rate, after the variations found from the formulæ have been used for a part of the time, we may correct the elements, and use the new corrected elements for the calculations of the values in the next portion of time, &c. The next difficulty is, that, even taking advantage of the process that we have described, the expressions cannot be integrated. The method of obviating this difficulty will form the subject of the next section.

SECTION II.

METHOD OF QUADRATURES.

(27) In the preceding investigations, all angles and variations of angles are supposed to be expressed as circular arcs in parts of the radius. It is convenient for practical purposes to express them in seconds. For this purpose we have only to make $n = \text{N} \sin 1''$, $\frac{dn}{dt} = [\text{N}] \sin 1''$, $\frac{d\epsilon}{dt} = [\epsilon] \sin 1''$, $\frac{d\varpi}{dt} = [\varpi] \sin 1''$, $\frac{d\nu}{dt} = [\nu] \sin 1''$, $\frac{di}{dt} = [i] \sin 1''$; also $\frac{da}{dt} = [a]$, $\frac{de}{dt} = [e]$; and we obtain equations which give us [N], $[\epsilon]$, $[\varpi]$, $[\nu]$, $[i]$, $[a]$, and $[e]$: the five former being now expressed in seconds of arc, the sixth in terms of the unit of linear measure, and the seventh in terms of unity. These expressions, which are the differential coefficients with respect to the time, are, in fact, the actual variations (measured as we have just mentioned) which would take place in the unit of time, if the rates of variation remained uniform during one unit of time.

(28) The unit of linear measure and the unit of time in these expressions are absolutely arbitrary. For the unit of linear measure, it will be convenient to use that generally employed in astronomy, namely, the Earth's mean distance from the Sun. The choice of the unit of time will be thus determined: In the actual operation we must divide the whole time, for which the changes of the elements are to be investigated, into a number of equal parts, of perhaps ten days or twenty days each, which we shall call *intervals:* Now, it is convenient to take one interval for the unit of time. Then N is the number of seconds in the planet's mean motion during one interval; [N], $[\epsilon]$, $[\varpi]$, $[\nu]$, $[i]$, $[a]$, and $[e]$, are the variations of the respective elements during one interval (supposing the variations to go on uniformly during the whole interval) measured, the five first in seconds of arc, the sixth in parts of the Earth's mean distance from the Sun, and the seventh in parts of unity. The quantity t, which is the number measuring the quotient of the whole time up to any particular instant by the unit of time, must evidently be the ordinal number corresponding to that particular interval, or some number differing from that ordinal number by a quantity less than unity. The following considerations will show precisely what t must be.

(29) To obtain for the variation of each of the elements a quantity which *nearly* represents the true variation in one interval (leaving an error which is hereafter to be taken into account), it will evidently be best to make all our calculations with the co-ordinates, &c., calculated for the middle of each interval. We must then take for t (in the expression for $[\epsilon]$) the value which it has for the middle of each interval; that is, for the first interval we must make $t = \frac{1}{2}$; for the second interval we must make $t = \frac{3}{2}$; and generally for the p^{th} interval we must make $t = p - \frac{1}{2}$.

(30) We have now obtained expressions for the variations of the elements in each interval, which are adapted to use, and which are rigorously correct on the supposition that the rate of variation for the middle of each interval may be taken without error for the actual variation during that interval. We shall now show how the error of this supposition is to be taken into account.

(31) Take three successive numbers (the variation of a, for instance), and take their first-differences and second-difference, thus

$$\begin{array}{lcc} & \text{First-Diff.} & \text{Second-Diff.} \\ [a]_{p-1} & & \\ & \Delta^{(1)}_{-1} & \\ [a]_p & & \Delta^{(2)} \\ & \Delta^{(1)}_{+1} & \\ [a]_{p+1} & & \end{array}$$

Then assuming the three numbers $[a]_{p-1}$, $[a]_p$, $[a]_{p+1}$, to be represented by the formula

$$[a]_p + bx + cx^2,$$

where x is the time from the instant to which $[a]_p$ corresponds, we must make the formula represent $[a]_{p-1}$ on putting -1 for x (since the unit of time is the same as the interval,) and must make it represent $[a]_{p+1}$ on putting $+1$ for x. From this we easily find

$$b = \frac{1}{2}\left(\Delta^{(1)}_{-1} + \Delta^{(1)}_{+1}\right)$$

and
$$c = \frac{1}{2}\Delta^{(2)}.$$

The formula $[a]_p + bx + cx^2$ will now represent with great accuracy all values of $[a]$ for the time $p + x$ where x is not greater than 1. Now, this is the quantity which we ought to integrate from $x = -\frac{1}{2}$ to $x = +\frac{1}{2}$, in order to get the true variation of a through the interval, for whose middle the quantity $[a]_p$ is calculated. The integral between these limits is

$$[a]_p\left(\frac{1}{2} + \frac{1}{2}\right) + \frac{b}{2}\left(\frac{1}{4} - \frac{1}{4}\right) + \frac{c}{3}\left(\frac{1}{8} + \frac{1}{8}\right)$$
$$= [a]_p + \frac{1}{24}\Delta^{(2)}.$$

Thus we find, that for the true variation of a through one interval we ought to add $[a]_p$ to $\frac{1}{24}$ of the second-difference which stands opposite to it.

(32) When we calculate a long series of such quantities as $[a]_p$ and take their differences, it is evident that the second-differences, corresponding to the first and last terms, will be wanting: these, however, can be supplied by estimation (from observation of the others), and by means of these the first-differences preceding the first term and following the last term can be formed. Then the whole variation of a through the whole of the intervals is =

$$\text{the sum of all the calculated quantities } [a]$$
$$+ \frac{1}{24} \text{ the sum of all the corresponding second-differences}$$
$$= \text{the sum of all the calculated quantities } [a]$$
$$+ \frac{1}{24} \left\{ \text{first-difference following last term—first-difference preceding first term} \right\}.$$

It will easily be seen that, if we assume the third-differences to be sensible, their effects during one interval will destroy each other in the same manner as those of the first-differences: the only effects omitted are, therefore, those of the fourth-differences, which (except the intervals are extravagantly large) may be neglected. If, however, it is wished to take them into account, the expression is =

$$\text{the sum of all the calculated quantities } [a]$$
$$+ \frac{1}{24} \text{ the sum of all the corresponding second-differences}$$
$$- \frac{17}{5760} \text{ the sum of all the corresponding fourth-differences.}$$

Section III.

PRACTICAL RULES FOR CALCULATION.

(33) Divide the time through which the variations of the elements are to be calculated into equal intervals. Experience alone can teach the calculator what will be the most advantageous length of the intervals: it will depend greatly upon the positions of the disturbing planets, especially Jupiter; but it is probable that, when Jupiter is nearest, intervals of 10 days each would not be found too long, and that at other times intervals of 20 days each might be safely used. It is desirable to retain the intervals of the same length through the whole of the time, even though the calculations at some parts should be made independently for only each alternate interval, and the others should be filled up by interpolation.

(34) All the calculations which follow are to be made for the middle day of each interval. Thus, suppose the intervals were of 10 days each, and we wished to calculate the variations of elements in the 400 days between September 17, 1834, and October 22, 1835, the calculations must be made for 1834, September 22, October 2, October 12, &c. In the following rules we shall express the order of the calculation by the letter p: so that for September 22 (in this instance) $p = 1$; for October 2, $p = 2$; for October 12, $p = 3$, &c.

(35) It is supposed that we know the planet's mean longitude at the beginning of the time, ϵ; its mean distance from the Sun, a; the number of seconds in its mean sidereal motion during one interval, N; its longitude of perihelion, ϖ; its excentricity, e; the longitude of its ascending node, ν; and the inclination of its orbit to the ecliptic, i: all for the beginning of the time. The mean longitude and the longitude of perihelion are supposed to be measured from the first point of Aries on the plane of the ecliptic, to the node, and then upon the plane of the orbit. With these elements, the planet's true longitude θ (measured as the others are measured), its radius vector r, and its co-ordinates, x, y, z, (of which x is drawn from the centre of the Sun towards the first point of Aries, y towards the first point of Cancer, and z perpendicular to the plane of the ecliptic towards the north) must be calculated approximately for the middle day of every interval. It is probable that an accuracy of $1'$ in true longitude and $\frac{1}{5000}$ of the whole radius vector will be sufficient: but experience will be the best guide on this point.

(36) The proportions $\frac{m_1}{\mu}$, $\frac{m_2}{\mu}$, &c., of the masses of the various disturbing planets to the Sun's mass are supposed to be known. The radii vectores r_1, r_2, &c., the co-ordinates x_1, y_1, z_1, x_2, y_2, z_2, &c., of the various disturbing planets, and λ_1, λ_2, &c., their distances from the disturbed planet, must be calculated for every middle day.

(37) The next step will be to calculate for every middle day the following quantities:—

$$\mathrm{A}=\frac{m_1}{\mu}\left\{\frac{x-x_1}{\lambda_1^3}+\frac{x_1}{r_1^3}\right\}+\frac{m_2}{\mu}\left\{\frac{x-x_2}{\lambda_2^3}+\frac{x_2}{r_2^3}\right\}+\&\mathrm{c}.$$

$$\mathrm{B}=\frac{m_1}{\mu}\left\{\frac{y-y_1}{\lambda_1^3}+\frac{y_1}{r_1^3}\right\}+\frac{m_2}{\mu}\left\{\frac{y-y_2}{\lambda_2^3}+\frac{y_2}{r_2^3}\right\}+\&\mathrm{c}.$$

$$\mathrm{C}=\frac{m_1}{\mu}\left\{\frac{z-z_1}{\lambda_1^3}+\frac{z_1}{r_1^3}\right\}+\frac{m_2}{\mu}\left\{\frac{z-z_2}{\lambda_2^3}+\frac{z_2}{r_2^3}\right\}+\&\mathrm{c}.$$

(38) Find the angles ψ and χ, where $\tan\psi=\tan\nu\cos i$, and $\tan\chi=\cot\nu\cos i$ (ψ and χ will therefore be constants): and calculate for every middle day the following expressions:—

$$\mathrm{A}'=\mathrm{A}x+\mathrm{B}y+\mathrm{C}z$$

$$\mathrm{B}'=\mathrm{A}\frac{\cos\nu}{\cos\psi}\sin(\theta-\nu+\psi)+\mathrm{B}\frac{\sin\nu}{\cos\chi}\sin(\theta-\nu-\chi)-\mathrm{C}\sin i\cos(\theta-\nu)$$

$$\mathrm{C}'=-\mathrm{A}\sin\nu\sin i+\mathrm{B}\cos\nu\sin i-\mathrm{C}\cos i.$$

(39) Find the angle ϕ, such that $\sin\phi=e$ (ϕ is therefore constant), and calculate for each middle day the following expressions (where p, as before mentioned, is the ordinal number of the interval)

$$[a] = -\left(2\,\mathrm{N}\sin 1''\,a^3\tan\phi\right)\frac{\sin(\theta-\varpi)}{r}\mathrm{A}' + \left(2\,\mathrm{N}\sin 1''\,a^4\cos\phi\right)\frac{\mathrm{B}'}{r}$$

$$[\mathrm{N}] = \left(3\,\mathrm{N}^2\sin 1''\,a^2\tan\phi\right)\frac{\sin(\theta-\varpi)}{r}\mathrm{A}' - \left(3\,\mathrm{N}^2\sin 1''\,a^3\cos\phi\right)\frac{\mathrm{B}'}{r}$$

$$[\epsilon] = \left\{-\left(3\,\mathrm{N}^2\sin 1''\,a^2\tan\phi\right)\left(p-\frac{1}{2}\right)\frac{\sin(\theta-\varpi)}{r} + \left(2\,\mathrm{N}a\right) + \left(\mathrm{N}\,a^2\cos\phi\tan\frac{\phi}{2}\right)\frac{\cos(\theta-\varpi)}{r}\right\}\mathrm{A}'$$

$$+\left\{\left(3\,\mathrm{N}^2\sin 1''\,a^3\cos\phi\right)\frac{p-\frac{1}{2}}{r} + \left(2\,\mathrm{N}a\frac{\tan\frac{\phi}{2}}{\cos\phi}\right)r\sin(\theta-\varpi) + \left(\mathrm{N}a\tan\phi\tan\frac{\phi}{2}\right)r\sin(\theta-\varpi)\cos(\theta-\varpi)\right\}\mathrm{B}'$$

$$+\left(\frac{\mathrm{N}a}{2\cos\phi\cos^2\frac{i}{2}}\right)z\,\mathrm{C}'$$

$$[\varpi] = \left(\mathrm{N}a^2\cot\phi\right)\frac{\cos(\theta-\varpi)}{r}\mathrm{A}' + \left(\frac{2\,\mathrm{N}a}{\sin\phi\cos\phi}\right)r\sin(\theta-\varpi)\,\mathrm{B}'$$
$$+\left(\frac{\mathrm{N}a}{\cos\phi}\right)r\sin(\theta-\varpi)\cos(\theta-\varpi)\,\mathrm{B}' + \left(\frac{\mathrm{N}a}{2\cos\phi\cos^2\frac{i}{2}}\right)z\,\mathrm{C}'$$

$$[e] = -\left(\mathrm{N}\sin 1''\,a^2\cos\phi\right)\frac{\sin(\theta-\varpi)}{r}\mathrm{A}' + \left(\mathrm{N}\sin 1''\,a^3\frac{\cos^3\phi}{\sin\phi}\right)\frac{\mathrm{B}'}{r} - \left(\mathrm{N}\sin 1''\,a\cot\phi\right)r\,\mathrm{B}'$$

$$[\nu] = \left(\frac{\mathrm{N}a}{\cos\phi\sin^2 i}\right)z\,\mathrm{C}'$$

$$[i] = \left(\frac{\mathrm{N}a}{\cos\phi}\right)r\cos(\theta-\nu)\,\mathrm{C}'.$$

The quantities within the large parentheses are constant.

(40) Collect the whole series of calculated quantities $[a]$, take the first-differences and second-differences, and supply by estimation a second-difference preceding the first and one following the last; and with these form a first-difference preceding the first $[a]$ and one following the last $[a]$. Then the sum of all the quantities $[a]$ with $\frac{1}{24}$ of the excess of the last first-difference over the first first-difference will be the whole variation of a, in parts of the earth's mean distance from the Sun. Similar operations performed with respect to all the quantities $[\mathrm{N}]$, $[\epsilon]$, $[\varpi]$, $[\nu]$, and $[i]$ will give the whole variations of N, ϵ, ϖ, ν, and i, in seconds of space; and similar

operations performed on all the quantities $[e]$ will give the whole variation of e in parts of unity. These whole variations we shall denote by the prefix δ.

(41) Now, the planet's place at and near the end of the time is to be calculated as if it were moving, undisturbed, in an elliptic orbit, whose mean distance $= a + \delta a$, excentricity $= e + \delta e$, longitude of perihelion $= \varpi + \delta\varpi$, longitude of node $= \nu + \delta\nu$, and inclination $= i + \delta i$: and its mean longitude in this orbit is to be calculated as if, *at the beginning of the time*, its mean longitude had been $\epsilon + \delta\epsilon$, and as if, *from the beginning of the time*, its mean sidereal motion had been $\mathrm{N} + \delta\mathrm{N}$ in every interval.

(42) If the planet's place is to be calculated for a considerable time before and after the day to which we have corrected the elements, it will not, perhaps, be sufficiently accurate to use one set of elements (though this is sufficient for the ordinary ephemeris for the opposition of a small planet). In that case it will only be necessary to terminate the summation of the quantities $[a]$, &c., at two or three different days, and to use the elements, thus corrected to two or three different days, for the calculation of places for times near to those days.

(43) If the change in the elements through the whole period appears to be great, the only method of making the calculation accurate will be, to sum the variations for a short time (as perhaps one half or one third of the whole period) and, correcting the elements, to use these corrected elements for the calculation of the co-ordinates and other quantities which are to be used in the calculation of the variations for the next part of the period. It is probable that this process will seldom be found necessary, except when the planet near its aphelion is acted on by powerful disturbing forces; a circumstance which occurs sometimes in the perturbations of Encke's Comet.

(44) It is only necessary to add, that the formulæ above suppose the longitudes to be measured from an invariable line on an invariable ecliptic. To take account of the alteration in the ecliptic and the first point of Aries, the longitude of the node must be increased by

$$\text{general precession during the interval} - \text{diminution of obliquity during the interval} \times \sin\nu \times \cot i;$$

the longitudes of the planet and its perihelion from the node must be increased by

$$\text{diminution of obliquity} \times \sin\nu \times \operatorname{cosec} i:$$

the complete mean longitude therefore of the planet, and longitude of its perihelion, must be increased by

$$\text{general precession} + \text{diminution of obliquity} \times \tan\frac{i}{2} \times \sin\nu.$$

And the inclination must be increased by

$$\text{diminution of obliquity} \times \cos\nu.$$

The true longitude, calculated with the elements thus further corrected and the mean sidereal motion, for any considerable interval before and after the day for which the elements are computed, must also be affected with precession proportional to that interval. If, however, in the process of calculating true longitudes, the motion of precession be added to the mean sidereal motion, and if the same motion of precession be applied to the longitudes of the node and perihelion (neglecting for short times the effect of change of obliquity) it will not be necessary to take account of precession afterwards.

G. B. AIRY.

OBSERVATORY, CAMBRIDGE,
Dec. 3, 1834.

ON THE

DETERMINATION OF THE LONGITUDE

FROM AN

OBSERVED SOLAR ECLIPSE OR OCCULTATION.

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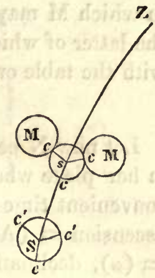
An accurate observation of a Solar Eclipse, or Occultation of a Star by the Moon, furnishes a favourable opportunity for the calculation of the longitude. This calculation may be effected by various methods, most of which are well known to astronomers: amongst the most simple and practically useful may be noticed the method of the late Dr. Young, and the improvements on the same by Mr. Thomas Henderson, now Astronomer Royal of Scotland, (see *Nautical Almanacs* from 1827 to 1833, inclusive); also two methods by Mr. Edward Riddle, and another by Mr. Thomas Maclear, published in the *Memoirs of the Royal Astronomical Society*, vol. iv. pages 305 and 531. To obviate the difficulties in the way of the calculation, these methods, as, indeed, all others that have come within my observation, suppose in the first instance that the estimated Greenwich time will suffice to take out the Moon's declination accurately from the ephemeris; or that the motion of the Moon in her orbit is uniform throughout a wide interval of time; consequently, when a good result is to be obtained, and the error of the estimated longitude is considerable, the computer is generally obliged to repeat the calculation with more accurate data, deduced for the Greenwich time according to the calculated, instead of the assumed, longitude. A method well adapted to computation, and, in all cases, free from inaccuracy or roughness of approximation has long been wanted. The following brief discussion of the problem is submitted by way of continuation of my paper on Eclipses, which forms the Appendix to the Nautical Almanac for 1836, and has for its object an easy, practical, and, at the same time, a correct solution. It is proposed, also, to supersede the necessity of having recourse, in these calculations, to the elements usually printed in Occultation lists, the use of which materially augments the chances of inaccuracy; and furthermore, to reduce the processes of calculation, for an Occultation, to plain and simple rules for the use of those who may be unaccustomed to analytical formulæ.

In the case of an Eclipse of the Sun, the apparent time of observation being converted into arc, at the rate of 15° for an hour, will show the true hour-angle of the Sun's centre at that instant; and as the declination of the Sun is never subject to a very rapid daily variation, it may be taken out from the Ephemeris with tolerable accuracy by the approximate Greenwich time, deduced from an estimated longitude or a rough longitude by account.



In the annexed figure, let Z represent the position of the zenith, S the Sun and $c'c'c'$ his limb. To illustrate the principle and simplify the reasoning that enters into the present investigation, it will be convenient to imagine, merely by way of convention, the limb $c'c'c'$ of the Sun to be an apparent one as affected by a parallax equal to the relative parallax of the two bodies: on this supposition, let s be the true place and ccc the true appearance of the limb as it would be seen from the centre of the Earth. Then, by the theory of the effects of parallax, the true semidiameter Sc' of the Sun will represent the fictitious semidiameter sc as augmented by the parallax; and if any point c be taken in the fictitious limb ccc it will be transferred to a corresponding point c' on the true limb of the Sun; consequently, the true limb of the Moon M being brought in contact with this disc, the parallax will exactly reduce her apparent limb to a contact with the Sun's true limb $c'c'c'$. Moreover, as the hour-angle and declination of the Sun S are known at the time of observation, and as this position is now viewed as an apparent one, the effects of the parallax or, in other words, the calculation of the relative right ascension and declination of s and the diminished semidiameter sc follows directly from the equations (2) of my paper on Eclipses, page 103. The problem is thus reduced to the determination of the corresponding Greenwich time when the true disc of the Moon comes in contact with the given disc ccc placed at a given relative right ascension and declination; and every consideration relating to parallax is hence eliminated from the inquiry.

Assume,

$$\left.\begin{array}{ll} \alpha & \text{the right ascension} \\ h & \text{the hour-angle} \\ \delta & \text{the declination} \\ \sigma & \text{the semidiameter} \end{array}\right\} \text{of the true Sun S}$$

$$\left.\begin{array}{ll} \alpha_0 & \text{the right ascension} \\ \delta_0 & \text{the declination} \\ \sigma_0 & \text{the semidiameter} \end{array}\right\} \text{of the fictitious Sun } s$$

and D, the declination of the Moon.

Then,

$$\left.\begin{aligned} \Delta\alpha &= \rho \mathrm{P} \frac{\cos l}{\cos \delta_0} \sin h \\ \Delta\delta &= \rho \mathrm{P} \{\sin l \cos \delta - \cos l \sin \delta \cos (h - \tfrac{1}{2} \Delta\alpha)\} \\ \alpha_0 &= \alpha + \Delta\alpha \\ \delta_0 &= \delta + \Delta\delta \end{aligned}\right\} \text{ - - - } (a)$$

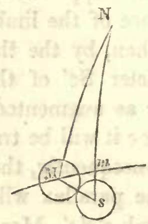
Or, following the method of resolution employed at page 104,

$$\left.\begin{array}{c} \Delta\alpha = \rho \mathrm{P} \dfrac{\cos l}{\cos \delta_0} \sin h \\ (h) = h - \tfrac{1}{2} \Delta\alpha \\ \tan\theta = \cos (h) \cot l \qquad \tan \mathrm{M} = \dfrac{\sin \theta}{\cos (\theta + \delta)} \tan (h) \\ \tan \epsilon = \tan (\theta + \delta) \cos \mathrm{M} \\ \Delta\delta = \rho \mathrm{P} \cos \mathrm{M} \cos \epsilon \\ \alpha_0 = \alpha + \Delta\alpha \\ \delta_0 = \delta + \Delta\delta \end{array}\right\} \text{ - - - } (b)$$

in which M may be regarded as the parallactic angle, and ϵ the altitude of the Sun, the latter of which will be wanted to take out the diminution of the Sun's semidiameter with the table on page 175, to get

$$\sigma_0 = \sigma - \text{diminution.}$$

Let now N be the North pole, M the place of the Moon at the time of contact, and m her place when in conjunction with s in right ascension. At a convenient time (t) near to this conjunction let the Moon's right ascension $=$ (A), declination $=$ (D); the Sun's right ascension $= (\alpha)$, declination $= (\delta)$, &c.; the relative hourly motion in right ascension $= A_1$ and the relative hourly motion in declination $= D_1$. Then for the time t_0 of conjunction with s, and the relative declination D_0 at that time, or the declination of m, we shall have



$$(\alpha_0) = (\alpha) + \Delta\alpha \qquad t_0 = (t) + \frac{(\alpha_0) - (A)}{A_1}$$

$$(\delta_0) = (\delta) + \Delta\delta \qquad D_0 = (D) + \frac{(\alpha_0) - (A)}{A_1} D_1$$

Assume $ms = D_0 - (\delta_0) = k$, $Ms = \Delta$, $\angle Mms = 90° + \eta$ and $\angle sMm = 90° + \psi$; and

$$\tan\eta = -\frac{D_1}{A_1 \cos (D)}$$

Also, by the small triangle mMs, considered as plane,

$$\cos\psi = \frac{k\cos\eta}{\Delta}$$

and, by the spherical triangle NMs,

$$\sin\angle MNs = -\sin\Delta\,\frac{\sin(\eta+\psi)}{\cos(D)}$$

or, as the small arcs may be assumed proportional to their sines,

$$\angle MNs = -\Delta\,\frac{\sin(\eta+\psi)}{\cos(D)}$$

The time of the Moon's passing over this angle, or the time elapsed in passing from M to m is therefore $-\dfrac{\Delta\sin(\eta+\psi)}{A_1\cos(D)}$, which deducted from t_0, there results, for the instant of contact, or observation,

The corresponding Greenwich time $=$

$$(t) + \frac{(\alpha_0) - (A)}{A_1} + \frac{\Delta\sin(\eta+\psi)}{A_1\cos(D)}$$

The longitude from Greenwich is hence determined by taking the difference between the Greenwich time and that of the observation, previously making them both apparent or both mean by the application of the equation of time if necessary; and it will be

$$\left.\begin{matrix}\text{West}\\ \text{East}\end{matrix}\right\}\ \text{when the Greenwich time is the}\ \left\{\begin{matrix}\text{greater}\\ \text{less}\end{matrix}\right.$$

For an Occultation of a Star by the Moon the calculation will, in some respects, be slightly abridged. The characters A_1, D_1, must then represent the absolute motions of the Moon in right ascension and declination; the semidiameter σ, and consequently its diminution, will disappear; hence, as the altitude ϵ may be dispensed with, the equations (a) will perhaps be preferable to the equations (b) for the calculation of

the parallactic quantities $\Delta\alpha$, $\Delta\delta$; or the equations (b) by eliminating M and ε, may be modified into the following convenient expressions:

$$\left.\begin{array}{ll} \Delta\alpha = \rho P \dfrac{\cos l}{\cos\delta_0} \sin h & (h) = h - \tfrac{1}{2}\Delta\alpha \\ \tan\theta = \cos(h)\cot l & \Delta\delta = \rho P \sin l \dfrac{\cos(\theta+\delta)}{\cos\theta} \end{array}\right\} \quad (c)$$

It will be useful here to recapitulate the expressions in a form suited to the facilities of arithmetical calculation, and separately arranged for an Eclipse of the Sun and an Occultation of a Star by the Moon, to preserve distinctness.

I.—ECLIPSE OF THE SUN.

1. With the longitude by account find the corresponding Greenwich time and thence from the Ephemeris take out the Sun's right ascension α, declination δ, and semidiameter σ; the horizontal parallaxes P, π; also, take out the Moon's declination D roughly to the minute.

Reduce the latitude by the table on page 57, and with ρ from the table on page 58 find

$$P' = \rho(P - \pi)$$

h = apparent time of observation reduced into *arc*.

2.
$$p = P'\cos l \sin h \qquad \Delta h \text{ in minutes} = [7{\cdot}92082]\,\frac{p}{\cos D} \qquad (h) = h - \Delta h$$

$$\tan\theta = \cos(h)\cot l \qquad\qquad G = \cos(h)\cos l$$

$$\tan M = \frac{\sin\theta}{\cos(\theta+\delta)}\tan(h) \qquad \tan\varepsilon = \tan(\theta+\delta)\cos M$$

$$B = \cos M \cos\varepsilon$$

$$\text{check} \;-\;- \quad \frac{\sin\theta}{\cos(\theta+\delta)} = \frac{G}{B}$$

$$\Delta\delta = B \,.\, P' \qquad\qquad \delta_0 = \delta + \Delta\delta$$

$$\Delta\alpha \text{ in time} = [8{\cdot}82391]\,\frac{p}{\cos\delta_0} \qquad\qquad \alpha_0 = \alpha + \Delta\alpha$$

M to be in the same semicircle with h.

3. With ε find the corresponding factor f in the annexed table; then, using P and σ each in minutes,

$$\text{diminution of } \sigma \text{ in seconds} = \left(\frac{P'}{10}\right)\left(\frac{\sigma}{10}\right)\cdot f$$

and thence

$$\sigma_0 = \sigma - \text{diminution}$$

$$s = [9{\cdot}43537]\,P$$

$$\text{For}\begin{cases}\text{partial}\\ \text{total or annular}\end{cases}\text{phase, } \Delta = \begin{cases} s + \sigma_0 \\ s - \sigma_0\end{cases}$$

ε	Factor f for diminution of ☉'s Semid.	
0°	0·01	+ ·30
10	0·31	·30
20	0·61	·28
30	0·89	·26
40	1·15	·22
50	1·37	·17
60	1·54	·13
70	1·67	·08
80	1·75	+ ·02
90	1·77	

4. In the hourly Ephemeris of the Moon fix on a convenient time (t) at which the Moon's right ascension is near to α_0, and for this time take out the right ascension (A) in time, the declination (D) and their hourly variations; also the Sun's right ascension (α), declination (δ), and their hourly variations. Then,

$$A_1 = \text{hourly var. (A)} - \text{hourly var. } (\alpha) \text{ in time}$$

$$D_1 = \text{hourly var. (D)} - \text{hourly var. } (\delta) \text{ in arc}$$

$$(\alpha_0) = (\alpha) + \Delta\alpha$$

$$(\delta_0) = (\delta) + \Delta\delta$$

5. $$m = \frac{(\alpha_0) - (A)}{A_1} \qquad t_0 = (t) + m[3{\cdot}55630]$$

$$D_0 = (D) + m \, . \, D_1 \qquad k = D_0 - (\delta_0)$$

$$n = [1{\cdot}17609] \, A_1 \cos (D)$$

$$\tan \eta = -\frac{D_1}{n} \qquad \cos \psi = \frac{k \cos \eta}{\Delta}$$

$$\text{Corresponding Greenwich mean time} = t_0 + [3{\cdot}55630] \frac{\Delta}{n} \sin (\eta \mp \psi)$$

η to have a different sign from D_1

$$\left.\begin{matrix}\text{upper}\\ \text{under}\end{matrix}\right\} \text{ sign when an } \left\{\begin{matrix}\text{immersion}\\ \text{emersion}\end{matrix}\right\} \text{ is observed.}$$

II.—OCCULTATION OF A STAR BY THE MOON.

6. With the estimated longitude find the corresponding Greenwich time, and thence take out the Moon's horizontal parallax P, and her declination D roughly to the minute; also

Sid. time = Apparent time + ☉'s right ascension

or Sid. time = Mean time + Sid. time Mean Noon from p. III. of Ephemeris + accel. on Greenwich mean time

$$h = \text{Sid. time} - \alpha, \text{ in arc}$$

$$P' = \rho P$$

α being the Star's right ascension.

7. $$p = P' \cos l \sin h \qquad \Delta h \text{ in minutes} = [7{\cdot}92082] \frac{p}{\cos D} \qquad (h) = h - \Delta h$$

$$\kappa = P' \sin l \cos \delta \qquad \kappa' = P' \cos l \sin \delta \cos (h) \qquad \delta_0 = \delta + \kappa - \kappa'$$

$$\Delta\alpha \text{ in time} = [8{\cdot}82391] \frac{p}{\cos \delta_0} \qquad \alpha_0 = \alpha + \Delta\alpha$$

8. In the hourly Ephemeris of the Moon fix on a convenient time (t) at which the Moon's right ascension is near to α_0, and for this time take out the right ascension (A),

the declination (D), and their hourly variations A_1, D_1. Then,

$$m = \frac{\alpha_0 - (A)}{A_1} \qquad t_0 = (t) + [3{\cdot}55630]\, m$$

$$D_0 = (D) + m \,.\, D_1 \qquad k = D_0 - \delta_0$$

$$n = [1{\cdot}17609]\, A_1 \cos(D)$$

$$\tan\eta = -\frac{D_1}{n} \qquad \cos\psi = [0{\cdot}56463]\,\frac{k\cos\eta}{P}$$

$$\text{Corresponding Greenwich mean time} = t_0 + [2{\cdot}99167]\,\frac{P}{n}\sin(\eta \mp \psi)$$

PRACTICAL RULES FOR CALCULATING THE LONGITUDE FROM AN OBSERVED OCCULTATION.

With the estimated longitude find the corresponding Greenwich time roughly to the minute, and for this time take out from the Ephemeris the Moon's declination roughly to the minute, her horizontal parallax to the tenth of a second, and the Sun's right ascension in time to the nearest second. To the Sun's right ascension add the apparent time of the observation which will give the right ascension of the meridian. The difference between this right ascension and that of the star will give the hour-angle of the star in time, which must be reduced into arc in the usual manner; it will be,

$$\left.\begin{matrix}\text{W.}\\ \text{E.}\end{matrix}\right\}\text{ when R.A. of meridian is }\left\{\begin{matrix}\text{greater}\\ \text{less}\end{matrix}\right\}\text{ than R.A. of } *.$$

Reduce the latitude of the place by subtracting the correction found in the table on page 57, (Appendix to Nautical Almanac, 1836), for which the nearest correction found in the table will be sufficient.

To the proportional logarithm of the Moon's horizontal parallax, add the correction answering to the latitude in the following series:

Lat. - -	0°	11°	19°	24°	29°	34°	38°	42°	46°	50°	54°	59°	64°	69°	77°	90°
Corr. - -	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	

To the proportional logarithm of the horizontal parallax, so corrected, add the log. secant of the reduced latitude and the log. cosecant of the hour-angle. To the sum (S_1) add the log. cosine of the Moon's declination and the constant log. 0·3010. The result will be the prop. log. of an arc, which subtracted from the hour-angle will give the hour-angle corrected.

To the corrected prop. log. of the horizontal parallax, add the log. secant of the $*$'s declination and the log. cosecant of the reduced latitude. To the same log. add the log. cosecant of the $*$'s declination the log. secant of the reduced latitude and the log. secant of the hour-angle corrected. These sums will be the prop. logs. of two arcs.

The former arc to have the same name as the latitude.

The latter to have

$\left.\begin{array}{l}\text{a different name from}\\ \text{the same name as}\end{array}\right\}$ the declination when the hour-angle is $\left\{\begin{array}{l}\text{less}\\ \text{greater}\end{array}\right\}$ than 90°.

The sum of these two arcs, having regard to their names, will give the correction to be applied to the ✻'s declination to get the declination corrected.

To the sum (S_1) add the constant log. 1·1761 and the log. cosine of the ✻'s declination corrected; the sum will be the prop. log. of an arc in time, to be

$\left.\begin{array}{l}\text{added to}\\ \text{subtracted from}\end{array}\right\}$ the ✻'s right ascension, when it is $\left\{\begin{array}{l}\text{West}\\ \text{East}\end{array}\right\}$ of the meridian,

to get the ✻'s right ascension corrected.

In the hourly ephemeris of the Moon, fix on a convenient time at which her right ascension is near to that of the star corrected; and, for this time, take out the right ascension, the declination, and their hourly variations.

Subtract the common log. of the difference between the corrected right ascension of the star and the right ascension of the Moon from the common log. of the hourly motion in right ascension; to the remainder add the constant log. 0·4771; to the same remainder add the prop. log. of the hourly motion in declination. The former sum will be the prop. log. of a time to be

$\left.\begin{array}{l}\text{added to}\\ \text{subtracted from}\end{array}\right\}$ the assumed time when ✻'s R.A. is $\left\{\begin{array}{l}\text{greater}\\ \text{less}\end{array}\right\}$ than ☽'s R.A.

to get the time corrected;

The latter will be the prop. log. of a correction of the ☽'s declination, to be applied with

$\left.\begin{array}{l}\text{the same name as}\\ \text{a different name from}\end{array}\right\}$ hourly var. when ✻'s R.A. is $\left\{\begin{array}{l}\text{greater}\\ \text{less}\end{array}\right\}$ than ☽'s R.A.

To the common log. of the hourly motion in right ascension, add the log. cosine of the ☽'s corrected declination; and to the sum (S_2) add the prop. log. of the hourly motion in declination and the constant log. 7·1427. The result will be the log cotangent of the 1st orbital inclination * and must take

$\left.\begin{array}{l}\text{the same name as}\\ \text{a different name from}\end{array}\right\}$ hourly motion in dec. when ✻ is $\left\{\begin{array}{l}\text{North}\\ \text{South}\end{array}\right\}$ of ☽.

To the prop. log. of the difference between the star's declination corrected and the Moon's declination corrected add the constant log. 9·4354 and the log. secant of the preceding orbital inclination; and from the sum deduct the prop. log. of the horizontal parallax. The remainder will be the log. secant of the 2nd orbital inclination† which must have the name

$\left.\begin{array}{l}\text{S.}\\ \text{N.}\end{array}\right\}$ when the observation is an $\left\{\begin{array}{l}\text{immersion}\\ \text{emersion}\end{array}\right.$

Add together the two orbital inclinations, having proper regard to their names; and to the log. cosecant of this sum add the preceding sum (S_2), the prop. log. of the horizontal parallax and the constant log. 8·1844. The sum will be the prop. log. of a correction to be applied to the time corrected to get the mean time at Greenwich: it must be

$\left.\begin{array}{l}\text{added}\\ \text{subtracted}\end{array}\right\}$ when the sum of the orbital inclinations is $\left\{\begin{array}{l}\text{N.}\\ \text{S.}\end{array}\right.$

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* With the parallel of declination. † With the Moon's limb.

By applying the equation of time from p. II. of the ephemeris there will result the Greenwich apparent time, and the difference between it and the apparent time of observation will show the longitude of the place from Greenwich; it will be

$\left.\begin{matrix}\text{W.}\\ \text{E.}\end{matrix}\right\}$ when the Greenwich time is $\left\{\begin{matrix}\text{greater}\\ \text{less}\end{matrix}\right\}$ than the observed.

EXAMPLES.

I.—SOLAR ECLIPSE.

For a Solar Eclipse take the example directly calculated in the Appendix to 1836, page 139:—

Suppose the beginning of the Solar Eclipse on May 15, 1836, to be observed to take place at $1^h\ 36^m\ 35^s{\cdot}6$ P.M., apparent time, in latitude 55° 57′ 20″ N., and longitude about 12^m W.

Here we have

	h m	
Observed apparent time -	1 36·6	
Longitude- - - - - - - -	12·0	$h = + 1^h\ 36^m\ 35^s{\cdot}6$
Greenwich apparent time	1 48·6	$= + \quad 24°\ \ 8'{\cdot}9$
Equation of time - - - -	3·9	
Greenwich mean time - -	1 44·7	

We hence take from the Ephemeris, $\alpha = 3^h\ 29^m\ 19^s$, $\delta = +\ 18°\ 57'{\cdot}6$, $\sigma = 15'\ 49''{\cdot}9$, D = + 19° 19′, P = 54′ 24″·4, $\pi = 8''{\cdot}5$, $P - \pi = 54'\ 15''{\cdot}9$.

Latitude	+ 55° 57′ 20″
Reduction	10 28
l - - -	+ 55 46 52 - - - $\rho = 9{\cdot}99902$

	$P-\pi$ 3·51267		cos (h) +9·96060	- - +9·96060
	ρ - - 9·99902		cot l - +9·83256	cos l +9·75001
	P′ - - 3·51169	θ +31° 50′·7	tan θ - +9·79316	G - +9·71061
	cos l - 9·75001	δ +18 57·6	sin θ - +9·72231	
	sin h +9·61183	$\theta+\delta$ +50 48·3	cos - +9·80069	B - +9·78899
	p - +2·87353 (1)		+9·92162	check +9·92162
	cos D 9·97484		tan (h) +9·64936	
	+2·89869		tan M +9·57098	
h +24° 8′·9	const. 7·92082		cos M +9·97180	- - +9·97180
Δh + 6·6	- - +0·81951		tan $(\theta+\delta)$ +0·08861	cos ε +9·81719
(h) +24 2·3		ε +48° 58′·3	tan ε - +0·06041	B - +9·78899
δ +18 57·6				P′ - +3·51169
+ 33·3	- - - - - - - - - - - - - - - - - -		$\Delta\delta$ - +33′ 18″·4	- - +3·30068
δ_0 +19 30·9	cos - 9·97430 (2)		σ - - - 15′ 49″·9	
	+2·89923 (1)—(2)		dim. - 11·6	P - - 3·51380
	const. 8·82391		σ_0 - - 15 38·3	const. 9·43537
	log. - - +1·72314		s - - - 14 49·6	- - - 2·94917
	$\Delta\alpha$ +0^h 0^m $52^s{\cdot}86$		Δ - - - 30 27·9	
	α 3 29 19			
	α_0 3 30 12			

By inspecting the hourly ephemeris of the Moon's right ascension on May 15th with $\alpha_0 = 3^h\ 30^m\ 12^s$, the most eligible time to assume is evidently $(t) = 3^h\ 0^m\ 0^s$; at this time we have $(A) = 3^h\ 30^m\ 42^s{\cdot}84$, $(A_1) = 2^m\ 0^s{\cdot}68$, $(D) = +\,19°\ 31'\ 34''{\cdot}0$, $(D_1) = +\,9'\ 55''{\cdot}2$, $(\alpha) = 3^h\ 29^m\ 31^s{\cdot}57$, $(\alpha_1) = +\,9^s{\cdot}89$, $(\delta) = +\,18°\ 58'\ 21''{\cdot}4$, $(\delta_1) = +\,34''{\cdot}8$: with these we proceed as follows:

	m s		′ ″	
(A_1) - -	2 0·68	(D_1) - -	+ 9 55·2	
(α_1) - -	9·89	(δ_1) - -	+ 34·8	
A_1 - -	1 50·79	D_1 - -	+ 9 20·4	
	h m s		° ′ ″	
(α) - -	3 29 31·57	(δ)	+ 18 58 21·4	
$\Delta\alpha$	+ 0 52·86	$\Delta\delta$	+ 33 18·4	
(α_0) - -	3 30 24·43	(δ_0)	+ 19 31 39·8	
(A) - -	3 30 42·84			
(α_0) — (A) -	— 0 18·41			
log. - - - - - -	— 1·26505			
A_1 - -	2·04450	D_1 - -	+ 2·74850	(1)
m - -	— 9·22055	- - - - - - - - -	— 9·22055	
const. -	3·55630	log - -	— 1·96905	
log - -	— 2·77685		— 0° 1′ 33″·1	
	— $0^h\ 9^m\ 58^s$·2	(D)	+ 19 31 34·0	
(t)	3 0 0	D_0	+ 19 30 0·9	
t_0	+ 2 50 1·8	(δ_0)	+ 19 31 39·8	
		k	— 1 38·9	
		cos (D) - -	9·97428	
		A_1 - -	2·04450	
		const. - -	1·17609	
		n - -	3·19487	(2)
η - -	— 19° 41′·2	tan η - -	— 9·55363	(1)—(2)
		cos η - -	+ 9·97384	
		k - -	— 1·99520	
			— 1·96904	
		Δ - -	3·26196	
ψ - -	+ 92 55·2	cos ψ - -	— 8·70708	
$\eta - \psi$ - -	—112 36·4 - - -	sin - -	— 9·96528	
		Δ - -	3·26196	
		const. - -	3·55630	
			— 6·78354	(3)
corr.	— $1^h\ 4^m\ 38^s$·5	- - - - - - - - -	3·58867	(3)—(2)
t_0 + corr.	+ 1 45 23·3	Greenwich mean time		
	3 56·0	Equation of time		
	1 49 19·3	Greenwich apparent time		
	1 36 35·6	Observed		
Longitude - - -	12 43·7	W. of Greenwich.		

II.—OCCULTATION OF A STAR.

Suppose, at Bedford, on January 7, 1836, in latitude 52° 8′ 28″ N., the Immersion of *i* Leonis to be observed at 10^h 39^m 22^s·4 P. M., apparent time, and the estimated longitude to be about 0^h 1^m W. Required the longitude?

	h m		° ′ ″
Apparent time (observation)	10 39	Latitude	N. 52 8 28
Longitude	0 1 W.	Reduc.	10 57
Apparent time (Greenwich)	10 40		N. 51 57 31
Equation of time	7	Reduced or geocentric latitude.	
Mean time (Greenwich)	10 47		

For Jan. 7, at 10^h 47^m, we find, from the Ephemeris, ☉'s R. A. = 19^h 12^m 40^s, ☽'s dec. = N. 15° 50′, and ☽'s equ. hor. par. = 56′ 1″·9.

	h m s		
☉'s R. A.	19 12 40	P. L. ☽'s hor. par.	0·5068
App. time	10 39 22	corr. for lat.	9
R. A. meridian	5 52 2	P. L. corr^d. hor. par.	0·5077
—— ✳	10 23 26	sec. red. lat.	0·2103
✳'s hour angle E. { in time	4 31 24	cosec. hour angle	0·0333
✳'s hour angle E. { in arc	67° 51′	sum (S_1)	0·7513
		cos. ☽'s dec.	9·9832
		const. log.	0·3010
corr^n.	17	P. L. corr^n.	1·0355
✳'s hour angle E. corr^d.	67 34		

P. L. corr^d hor. par.	0·5077		0·5077
sec. ✳'s dec.	0·0150	cosec.	0·5876
cosec. red. lat.	0·1037	sec.	0·2103
N. 0° 42′ 33″·0 P.L.	0·6264	sec. corr^d. hour angle	0·4184
S. 0 3 23·9		P. L.	1·7240
corr^n. N. 0 39 9·1		sum (S_1)	0·7513
✳'s dec. N. 14 58 38·8		const. log.	1·1761
✳'s dec. corr^d. N. 15 37 47·9		cos.	9·9836
corr^n.	0^h 2^m 12^s·56	P.L. corr^n.	1·9110
✳'s R. A.	10 23 26·39		
✳'s R. A. corr^d.	10 21 13·83		

On referring with the ✳'s corrected R.A. to the hourly ephemeris of the Moon, it will evidently be most convenient to take out the data at 11^h: for this time we have ☽'s R. A. = 10^h 20^m 58^s·47, hourly motion ☽'s R. A. = 2^m 2^s·9, ☽'s dec. = N. 15° 47′ 11″·0, hourly motion ☽'s dec. = S. 11′ 41″·5.

	h m s		
*'s corr^d. R. A. -	10 21 13 ·83		
☽'s R. A. - - - - -	10 20 58 ·47		
diff. - - - - - - -	0 15 ·36		
common log. - - - - -	1 ·1864		
com. log. h. m. ☽'s R. A. -	2 ·0896		
Remainder - -	0 ·9032	- - - - - - - - - - - - -	0 ·9032
const. log. - - - - - - -	0 ·4771	P. L. h. m. ☽'s dec. - - - -	1 ·1874
corr^n. - - - 0^h 7^m 29^s ·9 P.L.	1 ·3803	corr^n. - - S. 0° 1′ 27″ ·7 P.L.	2 ·0906
Time assumed 11 0 0		☽'s dec. - - N. 15 47 11 ·0	
Time corr^d. - 11 7 29 ·9		☽'s dec. corr^d. N. 15 45 43 ·3	
com. log. h m. ☽'s R. A. - - - -	2 ·0896	*'s corr^d. dec. - - - N.	15° 37′ 47″ ·9
cos. ☽'s corr^d. dec. - - - - - - -	9 ·9834	☽'s ——— - - N.	15 45 43 ·3
sum (S_2) - - - - - - - - - - -	2 ·0730	diff. (* S. of ☽) - -	7 55 ·4
P. L. h. m. ☽'s dec. - - - - - - -	1 ·1874	P. L. - - - - - - - -	1 ·3563
const. log. - - - - - - - - - - - -	7 ·1427	const. log. - - - - - -	9 ·4354
1st Orb. incl. - - N. 21° 34′ cot.	0 ·4031	- - sec. - - - - - - - - - -	0 ·0315
			0 ·8232
		P. L. ☽'s hor. par. - -	0 ·5068
			0 ·3164
2nd Orb. incl. - - S. 61 9	- - - - - - - -	sec. - - - - - - - - - -	0 ·3164
sum - - - - - S. 39 35	- - - - - - - -	cosec. - - - - - - - -	0 ·1957
		sum (S_2) - - - - - - -	2 ·0730
		P. L. ☽'s hor. par. - -	0 ·5068
		const. log. - - - - - - -	8 ·1844
corr^n.	0^h 19^m 44^s ·5	P. L. - - - - - - - - -	0 ·9599
Time corr^d.	11 7 29 ·9		
Greenw^h mean time - -	10 47 45 ·4		
Equation of time - - -	6 31 ·0		
Greenw^h app. time - -	10 41 14 ·4		
Observed ——— - -	10 39 22 ·4		
Longitude - - - - - -	1 52 ·0 W.		

P.S.—The principle of reversing the effect of the relative horizontal parallax on the position of the Sun, instead of using the actual effect on the position of the Moon, may be advantageously employed in the direct calculation of an Eclipse for a particular place. It will only be necessary to use the parallaxes for the Sun viewed as an apparent position, and to diminish the semidiameter by the amount derived from the table on page 175. Thus, it appears, at the beginning of the Eclipse, for instance, that the contact may be mathematically tested in two ways. First, we may apply the actual effects of the parallax to the true position of the Moon, then augment her semidiameter, and thus establish a contact of the limbs. But, if we reverse the operation, and consider the Sun to be an apparent body under the influence of the relative parallax, then clearing it from this supposed influence by reversing the parallax, and

diminishing the semidiameter, a contact will similarly be established with the true limb of the Moon; and this principle, in its application to solar eclipses, possesses an advantage similar to that derived in the case of an occultation, by considering the Star as an apparent place. (See *Appendix to Nautical Almanac* for 1836, page 123*.)

The formulæ, Nos. 2, 3, 4, and 5, pages 130 and 131 may, according to this method, be supplied by the following:

2.

$$P' = \rho\,(P - \pi) \qquad m = P' \cos l$$

$$Q_1 = [9\cdot4180] \qquad Q_2 = [9\cdot4180]\; m \sin\delta$$

$$s = [9\cdot43537]\; P$$

3.

$$k = \frac{m}{\cos D}$$

$$\Delta h \text{ in minutes} = [7\cdot92082]\; k \sin h$$

$$(h) = h - \Delta h$$

$$\tan\theta = \cos(h)\cot l \qquad G = \cos(h)\cos l$$

$$\tan M = \frac{\sin\theta}{\cos(\theta+\delta)}\tan(h) \qquad \tan\varepsilon = \tan(\theta+\delta)\cos M$$

$$B = \cos M \cos\varepsilon$$

$$\text{check - - - } \frac{\sin\theta}{\cos(\theta+\delta)} = \frac{G}{B}$$

$$\Delta\delta = B.\,P'$$

$$\sigma_0 = \sigma - \text{diminution for } \varepsilon$$

$$\text{For} \left\{ \begin{array}{l} \text{partial} \\ \text{total or annular} \end{array} \right\} \text{phase, } \Delta' = \left\{ \begin{array}{l} s + \sigma_0 \\ s - \sigma_0 \end{array} \right.$$

4.

$$k_0 = \frac{m}{\cos\delta_0} \qquad \Delta\alpha = k_0 \sin h$$

$$\Delta\alpha_1 = Q_1 k_0 \cos h \qquad \Delta\delta_1 = Q_2 \sin(h)$$

5.

$$\delta_0 = \delta + \Delta\delta \qquad \alpha' = \alpha - \Delta\alpha$$

$$y = (\alpha - \Delta\alpha)\cos D \qquad y_1 = (\alpha_1 - \Delta\alpha_1)\cos D$$

$$x = (D + \alpha' \text{ corr.}) - \delta_0 \qquad x_1 = D_1 - \Delta\delta_1$$

* This was inadvertently ascribed to Carlini; Professor Henderson, by whom a paper has appeared upon this very point, in the Quarterly Journal for 1828, page 411, informs me that the method has been long in practice, and that it was employed at an early period by Dr. Maskelyne.

ON THE ELEMENTS OF THE ORBIT

OF

HALLEY'S COMET,

AT ITS APPEARANCE IN THE YEARS 1835 AND 1836.

BY LIEUT. W. S. STRATFORD, R.N.,

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The object of the present paper is to afford the most accurate means of determining the Elements of the Orbit of Halley's Comet, at the instant of its Perihelion Passage in 1835, from *all* the Observations of that Body; and to explain in detail the various operations which have been performed at the Nautical Almanac Office for its accomplishment.

It was originally intended to trace the Comet's history from the period of its return in 1759, but this has been rendered unnecessary by the masterly address of Mr. Airy, the Astronomer Royal, to the Fellows of the Royal Astronomical Society, at their annual general meeting in 1837, on the occasion of presenting the gold medal of the society to Professor Rosenberger, "for his elaborate calculations relating to the return of Halley's Comet." It is impossible to mention Mr. Airy without, at the same time, acknowledging the cordial co-operation which the Author of this paper has experienced from that gentleman, not only in the particular instance of the cometary discussions, but at all times, and on all occasions, in which matters connected with the perfection of the Nautical Almanac and the interests of science have been concerned.

In the Supplement to the Nautical Almanac for the year 1833, with the view of attracting the early attention of astronomers to the subject, there was first given an Ephemeris of the Comet, from Aug. 3, 1835, to Feb. 11, 1836, founded upon the following elements of its orbit, given by M. de Pontécoulant in the *Conn. des Tems pour l'An* 1833, page 112.

Passage of the perihelion, 1835, Nov. 7·2, Paris mean astronomical time.

	°	′	″
Place of the perihelion on the orbit - - - - -	304	31	43
Longitude of the ascending node - - - - - - -	55	30	0
Inclination of the orbit - - - - - - - - - -	17	44	24
Ratio of the excentricity to the semi-axis major -	0·9675212		
Semi-axis major - - - - - - - - - -	17·98705		

Motion retrograde.

This Ephemeris was reprinted in the Nautical Almanac for 1835; and as the principal difference of M. de Pontécoulant's elements from those of M. Damoiseau related to the time of passage of the perihelion, a double Ephemeris was added, for the purpose of affording means of an early rectification of this element. The double

Ephemeris contained, for each 8th day, from Aug. 7, 1835, to Feb. 7, 1836, the Right Ascension and Declination, each to minutes, of the Comet, on two suppositions of the arrival at the perihelion, viz., Nov. 3·2 and Nov. 11·2, 1835.

In the same work were also given the co-ordinates of the Sun and Comet, together with a plan of the Heavens, showing, from three different sets of elements, the paths of the Comet amongst the fixed stars, and the relative position of the Comet in each on certain days, so as to indicate the direction in which the Comet should be sought for, with the greatest probability of its rediscovery.

It appears that the Comet was first seen at Rome by M. Dumouchel, Director of the Observatory of the Roman College, at $0^h\ 20^m$, sidereal time at the place, on August 5, 1835, in Right Ascension $5^h\ 26^m$, and Declination $+22°\ 27'$. (*Ast. Nach.*, No. 288.) It was observed generally in Europe after the 20th of August.

From a comparison of observations made at the latter end of August with the double Ephemeris, it was estimated that the Comet would arrive at its perihelion about 8·5 days later than the time stated by M. de Pontécoulant.

With a view to a nearer approximation to this element, another double Ephemeris was published on September 30, 1835, containing, for the month of October, 1835, the places of the Comet, on the supposition of the perihelion passage occurring respectively on Nov. 15·1935 and Nov. 16·1935, astronomical mean time at Greenwich.

Additional observations indicated that Nov. 16·1935 might be adopted for the time of passage, without much liability to error. With this time, and the other elements of Pontécoulant unchanged, an Ephemeris was computed from Aug. 20·5 to to Sept. 30·5, 1835, which, united with the October Ephemeris computed previously from the same elements, embraced the period between Aug. 20·5 and Oct. 31·5, 1835. With this and six other Ephemerides computed in a similar manner from elements in which a small variation was given to each in succession, whilst the other five remained constant, a general Ephemeris was formed for the same period, in which the Right Ascension and Declination consisted each of one known and six unknown quantities.

Having collected 56 Right Ascensions and 56 Declinations from roughly-reduced observations, made between Aug. 20 and Oct. 19, 1835, with these and the corresponding Right Ascensions and Declinations interpolated from the last-mentioned Ephemeris, there were formed 112 equations of condition, from which were deduced corrections for the assumed elements.

From these approximate elements an Ephemeris was immediately published for the month of November; but there being no doubt of some error having crept into the calculations, a revision of the whole was entered upon, and the following results ultimately obtained.

Perihelion passage, 1835, Nov. 15·93546, Mean Astronomical Time at Greenwich.

Semi-axis major - - - - - - - - - - -	18·0779386	
Ratio of the excentricity to the semi-axis major	0·9675509	
	° ′ ″	
Inclination of the orbit - - - - - - - - -	17 45 56·7	
Longitude of the ascending node - - - - -	55 8 21·2	From Mean Equinox
Longitude of the perihelion on the orbit - -	304 32 9·2	of Nov. 15, 1835.

With these results an Ephemeris for the month of December was prepared, and published on December 1, 1835.

It now remained to ascertain, by a rough comparison with observations, whether these elements were sufficiently approximate for the calculation of perturbations and their own final rectification, and for this purpose the following comparisons were made.

Date.	Right Ascension.			Declination.		
1835.	Observed.	Computed.	O—C.	Observed.	Computed.	O—C.
	h m s	h m s	s	° ′	° ′	′
Aug. 20·5091	5 40 52	5 40 54	— 2	+23 30·0	+23 29·4	+0·6
Sept. 2·6526	5 52 10	5 52 12	— 2	25 10·4	25 10·1	+0·3
Oct. 8·3122	8 36 49	8 36 49	0	+57 53·7	+57 54·6	—0·9
Nov. 8·2233	17 15 0	17 14 59	+ 1	—12 51·7	—12 51·2	—0·5

The observation of August 20 was made at Dorpat by Professor Struve; that of September 2, at Hayes, by the Rev. T. J. Hussey; and those of October 8 and November 8, with the meridian instruments at Cambridge, by Professor Airy.

The results of these comparisons showed that the elements represented the orbit with sufficient accuracy for the purposes in view, and the calculations were immediately commenced. The *first* part of the Series containing the Apparent Right Ascension and Declination and the Logarithm of the *true* distance of the Comet from the Earth, between August 1·5, 1835 and March 31·5, 1836, was published on December 30, 1835, with the view of affording early facilities for the reduction of the observations of the Comet throughout the whole period of its probable visibility. It is here reprinted in a different form, in Table X, the Right Ascensions being expressed in *arc* instead of *time*.

The various calculations relating to the Ephemeris of the Comet were performed agreeably to the method described by Mr. Woolhouse, in the Appendix to the Nautical Almanac for 1835, and those relating to the Perturbations by Professor Airy's method, in the Appendix to the Nautical Almanac for 1837, using in all cases the data from the Nautical Almanac.

In order to prevent confusion, it has been deemed expedient to alter the notation occasionally; and, for facility of reference, it is here collected and arranged in order of the letters of the small italic, the small roman, the large roman, and the greek alphabets.

NOTATION.

a The semi-axis major of the Comet's orbit, at mean noon of July 30, 1835.

$[a]$ The variation of a during one of the equal intervals into which the whole period, through which the variations of the elements are calculated, is divided.

δa	The whole variation of a in any given number of intervals, from July 30·0, 1835.
$c = \dfrac{e}{\sin 1''}$	
c_1, c_2, &c.	Constants used in the calculation of the Variations.
e	The ratio of the excentricity to the semi-axis major of the Comet's orbit.
$[e]$	The variation of e during one interval.
δe	The whole variation of e.
$\sin f = \dfrac{\cos \nu}{\sin F}$	
$\sin g = \dfrac{\sin \nu \cos \omega}{\sin G}$	
$\sin h = \dfrac{\sin \nu \sin \omega}{\sin H}$	
i	The inclination of the Comet's orbit to the ecliptic.
$[i]$	The variation of i during one interval.
δi	The whole variation of i.
$k_1 = \sqrt{a(1-e)}$	
$k_2 = \sqrt{a(1+e)}$	
m_1, m_2, m_3, &c.	The masses of the disturbing planets in the order of their distances from the Sun, the mass of the Sun being supposed 1.
n	The mean daily sidereal motion of the Comet.
p	The number of the interval.
r, r_1, r_2, &c.	The radii vectores of the Comet, and the disturbing Planets in the order of their distances from the Sun.
r', r'_1, r'_2, &c.	The projections of r, r_1, r_2, &c. on the ecliptic.
t	The number of days from the passage of the perihelion.
u	The Comet's excentric anomaly from the perihelion.
v	The Comet's true anomaly from the perihelion.
x, y, z	The Comet's heliocentric co-ordinates: x, being measured on a line passing through the mean vernal equinox of January 1, 1835; y, perpendicular to x in the plane of the ecliptic (supposed invariable), and z, perpendicular to the plane of the ecliptic, towards the North.
x_1, x_2, &c. y_1, y_2, &c. z_1, z_2, &c.	The heliocentric co-ordinates of the disturbing planets, in the order of their distances from the Sun, and measured as x, y, z are measured.

$\left.\begin{matrix} x \\ y \\ z \end{matrix}\right\}$ The heliocentric co-ordinates of the Comet, measured in directions parallel to those of X, Y, Z respectively.

$\odot$ The Sun's *true* longitude from the *true* equinox.

$\odot'$ ——————————— *mean* equinox of Jan. 1, 1835.

A The united effects of the disturbing Planets upon the Comet in the direction of the co-ordinate x.

$A' = Ax + By + Cz.$

B The united effects of the disturbing Planets upon the Comet in the direction of the co-ordinate y.

$B' = Ac_1 \sin(\mu + \psi) + Bc_2 \sin(\mu - \chi) + Cc_3 \cos\mu.$

C The united effects of the disturbing Planets upon the Comet in the direction of the co-ordinate z.

$C' = Ac_4 + Bc_5 + Cc_6.$

$E = (\alpha' - \alpha)\cos\delta.$

$E' = \delta' - \delta.$

$$F = \tan^{-1}\left(-\frac{\cot\nu}{\cos i}\right)$$

$$F' = F + (\varpi - \nu)$$

$$G = \tan^{-1}\left(\frac{\sin\nu\sin\sigma\cos\omega}{\sin i\cos(\sigma+\omega)}\right)$$

$$G' = G + (\varpi - \nu)$$

$$H = \tan^{-1}\left(\frac{\sin\nu\sin\sigma\sin\omega}{\sin i\sin(\sigma+\omega)}\right)$$

$$H' = H + (\varpi - \nu)$$

$$L = \frac{\sin\zeta}{r}NA' - c_7\frac{NB'}{r}$$

$$M = c_7\frac{\cos\zeta}{r}NA' + c_8 r\sin\zeta NB' + r\sin\zeta\cos\zeta NB'$$

N The number of seconds in the mean sidereal motion of the Comet during one interval.

[N] The variation of N during one interval.

δN The whole variation of N.

$\left.\begin{matrix} P \\ Q \\ R \\ S \\ U \\ V \end{matrix}\right\}$ Numbers to be determined from the equations of condition, and by which the assumed variation of each of the elements of the orbit, T, a, e, ϖ, ν, and i, is to be respectively multiplied to obtain the true variation.

T The time of passage of the perihelion.

$\left\{\begin{matrix} X \\ Y \\ Z \end{matrix}\right.$ The *true* geocentric co-ordinates of the Sun; X, being measured on a line passing through the true vernal equinox of the date; Y, perpendicular to the direction of X, and in the plane of the equator; and Z, perpendicular to the plane of the equator, towards the North.

α	The *apparent* geocentric right ascension of the Comet, deduced from the fundamental elements.
α'	The *apparent* geocentric right ascension of the Comet, deduced from observation.
α_1, α_2, &c.	The *apparent* right ascension of the Comet, deduced by varying each of the fundamental elements in succession, in the order T, a, e, ϖ, ν, and i.
$\Delta\alpha$	The variation of α in one day.
β, β_1, β_2	The heliocentric North latitude of the Comet and the disturbing Planets.
δ	The *apparent* geocentric North declination of the Comet, deduced from the fundamental elements.
δ'	The *apparent* geocentric North declination of the Comet, deduced from observation.
δ_1, δ_2, &c.	The *apparent* geocentric North declination of the Comet, deduced as α_1, α_2, &c., are deduced.
$\Delta\delta$	The variation of δ in one day.
Δ_1, Δ_2, &c.	The coefficients of P, Q, &c., in the equations of condition dependent upon right ascension.
Δ'_1, Δ'_2, &c.	The coefficients of P, Q, &c., in the equations of condition dependent upon declination.
ε	The Comet's mean longitude.
$[\varepsilon]$	The variation of ε during one interval.
$\delta\varepsilon$	The whole variation of ε.

$\zeta = \mu + \nu - \varpi =$ the angular distance of the Comet from the perihelion.

θ, θ_1, θ_2, &c.	The *true* heliocentric longitude of the Comet and the disturbing Planets on the ecliptic.
λ_1, λ_2, &c.	The distance of each disturbing planet from the Comet.
μ	The argument of latitude.
μ'	The projection of μ on the ecliptic.
ν	The longitude of the Comet's ascending node.
$[\nu]$	The variation of ν during one interval.
$\delta\nu$	The whole variation of ν.
ϖ	The longitude of the perihelion, measured from the first point of Aries, on the ecliptic, to the node, and thence on the orbit.
$[\varpi]$	The variation of ϖ during one interval.
$\delta\varpi$	The whole variation of ϖ.

$$\sigma = \tan^{-1}\frac{\tan i}{\cos \nu}$$

$$\phi = \sin^{-1} e.$$

$$\chi = \text{cotan}^{-1} (\text{cotan } \nu \ \cos i.)$$

$$\psi = \tan^{-1} (\tan \nu \ \cos i.)$$

ω The obliquity of the ecliptic.

CALCULATION OF THE EPHEMERIS OF THE COMET FROM THE APPROXIMATE ELEMENTS OF ITS ORBIT, CONSIDERED AS INVARIABLE, BETWEEN AUGUST 1, 1835, AND MARCH 31, 1836.

At the rediscovery of the Comet on August 5, 1835, about 102 days from the perihelion, it was only just perceptible in the best telescopes. The extension of the ephemeris therefore to March 31, 1836, or 136 days after the perihelion passage, would it was presumed embrace the utmost possible limits of visibility. Sir J. Herschel and Mr. Maclear, however, saw it at the Cape so late as May 5, 1836. It will therefore be necessary, hereafter, to extend the whole of the calculations to this period, to render the work complete.

The longitudes of the ascending node, and of the perihelion, having been reduced to the true equinox of Aug. 7, and Nov. 15, 1835, and Feb. 23, 1836, by the application of precession and nutation, three independent values of F, G, H, and sin f, sin g, sin h, were accurately computed for those dates from the formulæ,

$$\tan \mathrm{F} = -\frac{\cot \nu}{\cos i} \qquad \sin f = \frac{\cos \nu}{\sin \mathrm{F}}$$

$$\tan \mathrm{G} = \frac{\sin \nu \sin \sigma \cos \omega}{\sin i \cos(\sigma+\omega)} \qquad \sin g = \frac{\sin \nu \cos \omega}{\sin \mathrm{G}}$$

$$\tan \mathrm{H} = \frac{\sin \nu \sin \sigma \sin \omega}{\sin i \sin(\sigma+\omega)} \qquad \sin h = \frac{\sin \nu \sin \omega}{\sin \mathrm{H}}$$

And the values for each tenth day obtained by interpolation with differences to the second order, and thence for each day by simple proportion.

The excentric anomaly (u), true anomaly (v), and radius vector (r), were then computed for every mean midnight, commencing with Aug. 1·5, 1835, and terminating with March 31·5, 1836, from the formulæ:—

$$n = \frac{3548''\cdot 19269}{a^{\frac{3}{2}}} \qquad c = \frac{e}{\sin 1''}$$

$$k_1 = \sqrt{a(1-e)} \qquad k_2 = \sqrt{a(1+e)}$$

$$u - c \sin u = nt$$

$$\tan \tfrac{1}{2} v = \frac{k_2 \sin \frac{1}{2} u}{k_1 \cos \frac{1}{2} u}$$

$$\sqrt{r} = \frac{k_1 \cos \frac{1}{2} u}{\cos \frac{1}{2} v}$$

The true heliocentric co-ordinates of the Comet were obtained for each mean midnight, from

$$\mathrm{x} = r \sin f \sin(\mathrm{F}'+v)$$
$$\mathrm{y} = r \sin g \sin(\mathrm{G}'+v)$$
$$\mathrm{z} = r \sin h \sin(\mathrm{H}'+v)$$

The excentric anomaly, the logarithm of the radius vector, and the heliocentric co-ordinates of the Comet are inserted in Table I.

The true geocentric co-ordinates of the Sun were obtained for each mean midnight from the formulæ,

$$X = r_3 \cos \odot$$
$$Y = r_3 \sin \odot \cos \omega$$
$$Z = r_3 \sin \odot \sin \omega = Y \tan \omega$$

and are inserted in Table II.

Combining the true heliocentric co-ordinates of the Comet with the true geocentric co-ordinates of the Sun we obtain for the true geocentric co-ordinates of the Comet $X+x$, $Y+y$, and $Z+z$, and thence

$$\tan \alpha = \frac{Y+y}{X+x}$$
$$\tan \delta = \frac{Z+z}{X+x} \cos \alpha$$
$$\lambda_3 = \frac{Z+z}{\sin \delta}$$

The results being *true* Right Ascensions and Declinations were then reduced to *apparent* by applying aberration, viz.,

$$\text{Aberration in} \begin{cases} \text{Right Asc.} = -\cdot 0057382\, \lambda_3 . \Delta \alpha \\ \text{Declination} = -\cdot 0057382\, \lambda_3 . \Delta \delta \end{cases}$$

The apparent right ascension and declination, and the logarithm of the true distance of the Comet from the Earth, are given in Table X.

Between October 1 and 28 the daily variation of the place of the Comet appeared too irregular to admit of easy or accurate interpolation. The intervals were therefore reduced to six and three hours. The α and δ and λ_3, for intervals of six hours, were obtained from interpolated values of the geocentric co-ordinates of the Comet, and those for intervals of three hours, by interpolating the resulting values for six hours.

CALCULATION OF THE PERTURBATIONS.

Having decided on ascertaining the effects produced by each of the planets, Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Georgian, between Aug. 1, 1835, and March 31, 1836, the whole period was divided, in the first instance, into intervals of eight days each, and for the middle day of each interval, the first being Aug. 5, the heliocentric co-ordinates of the Comet and the disturbing Planets were computed.

The longitudes of the ascending node and perihelion were first reduced to the mean equinox of Jan. 1, 1835, by the application of precession; and the true anomalies, for mean noon of each middle day, interpolated from those computed for mean midnight for the Ephemeris. The heliocentric co-ordinates of the Comet for mean noon of each eighth day were then determined as follows:

$$\mu = (\varpi - \nu) + v = 249^\circ\ 23'\cdot 8 + v$$
$$\tan \mu' = \tan \mu \cos i \qquad \theta = \mu' + \nu$$
$$\sin \beta = \sin \mu \sin i \qquad r' = r \cos \beta$$

$$x = r' \cos \theta$$
$$y = r' \sin \theta$$
$$z = r \sin \beta.$$

The results are inserted in Table III.

The heliocentric longitudes of the disturbing planets were reduced from the apparent equinox to the same mean equinox as the Comet, viz., Jan. 1, 1835, by the application of precession and nutation.

With the heliocentric longitude from the mean equinox, and the heliocentric latitude, the co-ordinates of the planets were obtained from formulæ similar to those used for the Comet, viz.:

$$r_1' = r_1 \cos \beta_1$$
$$x_1 = r_1' \cos \theta_1$$
$$y_1 = r_1' \sin \theta_1$$
$$z_1 = r_1 \sin \beta_1$$

and those of the Earth from

$$x_3 = - r_3 \cos \odot'$$
$$y_3 = - r_3 \sin \odot'$$
$$z_3 = 0$$

The heliocentric co-ordinates of Mercury, Venus, the Earth, and Mars, were computed on each eighth day in succession, and those of Jupiter, Saturn, and the Georgian, on each sixteenth day, commencing at mean noon of August 5, 1835, and are contained in Table III.

FORCES A, B, C.

The forces A, B, C, in the directions of the Comet's heliocentric co-ordinates, x, y, z, respectively, were obtained for every eighth day, commencing with Aug. 5, 1835; those of Mercury, Venus, the Earth, and Mars, by direct calculations, and those of Jupiter, Saturn, and the Georgian by interpolation of 16 day intervals.

The masses used were the following:

For Mercury the log of	—	$m_1 = 3 \cdot 69340$
Venus	—	$m_2 = 4 \cdot 39663$
Earth	—	$m_3 = 4 \cdot 44985$
Mars	—	$m_4 = 3 \cdot 57181$
Jupiter	—	$m_5 = 6 \cdot 97935$
Saturn	—	$m_6 = 6 \cdot 45445$
Georgian	—	$m_7 = 5 \cdot 74671$

Putting $\lambda_1 = \left\{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2\right\}^{\frac{1}{2}}$

$$A = m_1 \left\{\frac{x - x_1}{\lambda_1^3} + \frac{x_1}{r_1^3}\right\} + m_2 \left\{\frac{x - x_2}{\lambda_2^3} + \frac{x_2}{r_2^3}\right\} + \&c.$$

$$B = m_1 \left\{\frac{y - y_1}{\lambda_1^3} + \frac{y_1}{r_1^3}\right\} + m_2 \left\{\frac{y - y_2}{\lambda_2^3} + \frac{y_2}{r_2^3}\right\} + \&c.$$

$$C = m_1 \left\{\frac{z - z_1}{\lambda_1^3} + \frac{z_1}{r_1^3}\right\} + m_2 \left\{\frac{z - z_2}{\lambda_2^3} + \frac{z_2}{r_2^3}\right\} + \&c.$$

The irregularity of the values of A, B, C thus obtained for intervals of eight days, suggested the expediency of shortening the interval. An interval of four days was then adopted; and, taking Aug. 1 as the middle of the first interval, the forces A, B, C were now obtained for every eighth day, commencing with Aug. 1, thus affording a series for each fourth day.

For the additional values of A, B, C, the co-ordinates of the Comet and Mercury were obtained independently, those of Venus, the Earth, and Mars, by interpolating the values previously computed. The A, B, C, for Jupiter, Saturn, and the Georgian, were interpolated immediately from the values already obtained for intervals of eight days.

The values of A for each Planet, separately and collectively, are inserted in Table IV.; those of B in Table V.; and those of C in Table VI.

The powerful influence exercised on the Comet by the Earth in the month of October, 1835, on account of the proximity of those bodies at that time; and by Venus about the latter end of November, for a similar cause, appeared to render it necessary to shorten the interval.

Adopting an interval of 1 day, the variations $[a]$, $[\epsilon]$, &c., dependent on the Earth alone, were then computed for the period commencing with October 4, and ending with October 24, 1835, and thence the total variation δa, $\delta \epsilon$, &c., between October 6, and October 22, 1835.

The variations dependent upon Venus alone were computed for an interval of 1 day for the period commencing with November 17, and ending with December 11, 1835, and thence the total variation between November 19, and December 9, 1835. The results were then compared with the total variation of the elements for the same periods by means of Table IX. and found to be almost insensible.

CALCULATION OF THE VARIATION OF ELEMENTS.

These were computed for every fourth day, from the following formulæ:

$$\tan \psi = \tan \nu \cos i$$

$$\tan \chi = \operatorname{cotan} \nu \cos i$$

$$c_1 = \frac{\cos \nu}{\cos \psi}$$

$$c_2 = \frac{\sin \nu}{\cos \chi}$$

$$c_3 = -\sin i$$

$$c_4 = -\sin i \sin \nu$$

$$c_5 = +\sin i \cos \nu$$

$$c_6 = -\cos i$$

$$A' = Ax + By + Cz$$

$$B' = Ac_1 \sin(\mu + \psi) + Bc_2 \sin(\mu - \chi) + Cc_3 \cos \mu$$

$$C' = Ac_4 + Bc_5 + Cc_6$$

$$c_7 = + a \operatorname{cotan} \phi \cos \phi$$

$$c = + \frac{2}{\sin \phi}$$

$$c_9 = -\ 2a^3 \sin 1'' \tan \phi$$
$$c_{10} = +\ 3a^2 \sin 1'' \tan \phi$$
$$c_{11} = +\ 2a$$
$$c_{12} = +\ a \tan \phi \tan \tfrac{1}{2}\phi$$
$$c_{13} = +\ \frac{a}{2 \cos \phi \cos^2 \frac{1}{2} i}$$
$$c_{14} = +\ \frac{a}{\cos \phi}$$
$$c_{15} = -\ a^2 \sin 1'' \cos \phi$$
$$c_{16} = -\ a \sin 1'' \text{cotan}\ \phi$$
$$c_{17} = +\ \frac{a}{\cos \phi \sin^2 i}$$
$$\zeta = \mu + \nu - \varpi$$

$$\mathrm{L} = \frac{\sin \zeta}{r} \mathrm{NA}' - c_7 \frac{\mathrm{NB}'}{r}$$

$$\mathrm{M} = c_7 \frac{\cos \zeta}{r} \mathrm{NA}' + c_8 r \sin \zeta \mathrm{NB}' + r \sin \zeta \cos \zeta \mathrm{NB}'$$

$$[a] = c_9\, \mathrm{L}$$
$$[\epsilon] = -(p - \tfrac{1}{2})\, c_{10} \mathrm{NL} + c_{11} \mathrm{NA}' + c_{12} \mathrm{M} + c_{13}\, z \mathrm{NC}'$$
$$[\varpi] = c_{13}\, z \mathrm{NC}' + c_{14} \mathrm{M}$$
$$[e] = c_{15}\, \mathrm{L} + c_{16}\, r \mathrm{NB}'$$
$$[\nu] = c_{17}\, z \mathrm{NC}'$$
$$[i] = c_{14}\, r \cos \mu \mathrm{NC}'$$

The variations $[a]$, $[\epsilon]$, &c. of the elements of the orbit in each interval of four days are inserted in Table VIII.

As August 1·0 is the middle of the first interval of four days, the commencement of that interval will be July 30·0: it is from this moment, therefore, that our departure has been taken, and the approximate elements were assumed to represent the actual orbit of the Comet on July 30, at mean noon at Greenwich.

The variations $[a]$, $[\epsilon]$, &c. were then each differenced to the 2nd order, and the total variations δa, $\delta \epsilon$, &c. obtained for each successive fourth day from July 30·0, by adding each variation $[a]$ in succession to the sum of all the variations which preceded it, and increasing each result by $\frac{1}{24}$ of the second difference standing opposite.

The corrections of $\delta\epsilon$, $\delta\varpi$, δi, and $\delta\nu$, for diminution of obliquity, were found from the formulæ,

$$\text{correction of} \left\{ \begin{matrix} \delta\epsilon \\ \delta\varpi \end{matrix} \right\} = \text{dimin. of obliq.} \times \sin \nu \tan \tfrac{1}{2} i$$
$$\text{———} \qquad \delta i = \text{dimin. of obliq.} \times \cos \nu$$
$$\text{———} \qquad \delta \nu = \text{dimin. of obliq.} \times \sin \nu \ \text{cotan}\ i$$

the amount of diminution being reckoned from July 30, assuming $0''{\cdot}457$ as the annual diminution.

The results are inserted in Table IX. They represent for any date in the Table the

total amount of alteration which each element of the orbit of July 30·0 has undergone by the action of all the disturbing Planets.

Having obtained these values for every fourth day, commencing with July 30, 1835, the elements of the perturbed orbits were obtained for each fourth day, by the successive addition of each $\delta\alpha$, $\delta\varepsilon$, &c., to the original elements: the mean longitude on July 30 having been taken

$$\varepsilon = \varpi + nt$$

t being the interval in days between July 30·0 and the assumed time of passing the perihelion, viz., Nov. 15·93546.

It now only remained to compute the Right Ascension and Declination of the Comet from each set of perturbed elements, and by a comparison of the results with those in the original Ephemeris derived from the imperturbed elements of July 30·0, to ascertain the alterations produced by the disturbing Planets on the Right Ascension and Declination of the Comet.

The value of ε for the orbit of each date was assumed to be the mean longitude in that orbit on July 30, and was in each case reduced to the date with the mean motion belonging to the orbit, as determined from its semi-axis major.

With the longitudes ν and ϖ, reduced to the true equinox of each date, by applying precession and nutation, the apparent Right Ascension and Declination of the Comet were determined from each set of perturbed elements for every fourth day, from July 30, 1835. These calculations were conducted in a manner precisely similar to those for the original Ephemeris.

Subtracting the α and δ of the original Ephemeris from the α and δ derived from the disturbed elements, the effect of perturbation upon the Right Ascension and Declination for each fourth day, from July 30, was obtained, and thence, by interpolation, the daily effect.

These perturbations are inserted in Table X. They are to be applied with the proper sign to the Right Ascension and Declination of the Comet in the same Table, to furnish the apparent Right Ascension and Declination, such as should be exhibited by observation, on the presumption that the elements of July 30, 1835, are the true elements of the Comet's orbit at that period.

CORRECTION OF THE ASSUMED ELEMENTS.

Let it be now supposed that the true elements of the orbit on July 30·0, 1835, were

$$T + 0^{d}\cdot 02\ P$$
$$a + 0\cdot 01\ Q$$
$$e + 0\cdot 0001\ R$$
$$\varpi + 1'\ S$$
$$\nu + 1'\ U$$
$$i + 1'\ V$$

T, a, &c., being the numerical values of the assumed elements on July 30, (page 186), and 0·02 P, 0·01 Q, &c., the corrections due to those elements.

If with the elements $T+0{\cdot}02$, a, e, ϖ, ν, i, a right ascension α_1 and declination δ_1 be computed for the time t in a way similar in every respect to that described in pages 191 and 192, it is plain that $\alpha-\alpha_1$ and $\delta-\delta_1$ are the variations of right ascension and declination produced by the given variation (0·02) of T alone. Now if the elements have been obtained sufficiently near in the first instance to justify the presumption that the variations of the right ascension and declination will be proportional simply to the variation of the element which produces them, then for the true variation of T, viz. 0·02 P, the variation of right ascension will be $(\alpha-\alpha_1)$ P, or reduced to the arc of a great circle, $(\alpha-\alpha_1)\cos\delta\,.\,P$, and of declination $(\delta-\delta_1)$ P.

In the same manner the elements T, $a+0{\cdot}01$, e, ϖ, ν, i will furnish a right ascension α_2 and declination δ_2; and the variations $\alpha-\alpha_2$, $\delta-\delta_2$ will be dependent upon the given variation of a alone. The variations produced by the true variation of a, viz. 0·01 Q, will therefore be $(\alpha-\alpha_2)\cos\delta\,.\,Q$ in right ascension, and $(\delta-\delta_2)\,.\,Q$ in declination.

By thus varying each of the other elements in succession by a given minute quantity, a knowledge is obtained of its separate influence in altering the right ascension and declination, and hence the influence of the assumed unknown total variations, viz.

$$\text{For variation of } e = {\cdot}0001\,\text{R, the variation in} \begin{cases} \text{R.A.} = (\alpha-\alpha_3)\cos\delta\,.\,\text{R} \\ \text{Dec.} = (\delta-\delta_3)\,.\,\text{R} \end{cases}$$

$$\text{———} \quad \varpi = 1'\,\text{S}' \quad \text{———} \begin{cases} \text{R.A.} = (\alpha-\alpha_4)\cos\delta\,.\,\text{S} \\ \text{Dec.} = (\delta-\delta_4)\,.\,\text{S} \end{cases}$$

$$\text{———} \quad \nu = 1'\,\text{U} \quad \text{———} \begin{cases} \text{R.A.} = (\alpha-\alpha_5)\cos\delta\,.\,\text{U} \\ \text{Dec.} = (\delta-\delta_5)\,.\,\text{U} \end{cases}$$

$$\text{———} \quad i = 1'\,\text{V} \quad \text{———} \begin{cases} \text{R.A.} = (\alpha-\alpha_6)\cos\delta\,.\,\text{V} \\ \text{Dec.} = (\delta-\delta_6)\,.\,\text{V} \end{cases}$$

Having the variation of α and δ consequent upon a variation of each element singly, now suppose all the elements to vary together, the total variation of right ascension and declination of the Comet will be

$$+\Delta_1\,\text{P}+\Delta_2\,\text{Q}+\Delta_3\,\text{R}+\Delta_4\,\text{S}+\Delta_5\,\text{U}+\Delta_6\,\text{V} = (\alpha'-\alpha)\cos\delta = \text{E}$$
$$+\Delta'_1\,\text{P}+\Delta'_2\,\text{Q}+\Delta'_3\,\text{R}+\Delta'_4\,\text{S}+\Delta'_5\,\text{U}+\Delta'_6\,\text{V} = (\delta'-\delta) = \text{E}'$$

where P, Q, R, S, U, V are the unknown quantities to be determined.

$\left.\begin{array}{l}\Delta_1, \Delta_2, \&\text{c.}\\ \Delta'_1, \Delta'_2, \&\text{c.}\end{array}\right\}$ The variations of right ascension and declination determined from given minute variation of elements, as before explained.

α', δ' An observed right ascension and declination at the time for which α and δ have been computed.

On these principles have the equations of condition in Table XI. been formed. Six different Ephemerides have been computed from six different sets of elements; and subtracting the resulting Right Ascension and Declination of each from the Right Ascension and Declination in Table X, derived from the original elements, the differences (those of R. A. being first multiplied by cos δ,) form the coefficients of P, Q, R, &c.; indicating, for any given time within the limits of the table, the numerical amount of variation caused by a minute variation of each element.

The mode of using the table is as follows:—Having a reduced observation of Right

Ascension and Declination at a given mean time, find, by interpolating Table XI, the coefficients of P, Q, R, &c., for that time:—find also, by interpolating Table X, the Right Ascension α and Declination δ, including the perturbations, for the same instant: subtract the interpolated from the observed Right Ascension, and multiply the remainder by cos δ, the product is to be substituted for E, the right hand term in the equations of condition dependent upon Right Ascension.

Subtract the interpolated from the observed Declination (North Declination being +, and South —), and the remainder is to be substituted for E′, the right hand term in the equations of condition dependent upon Declination.

The unknown quantities being 6, require absolutely only 6 equations for their complete determination, but from the uncertainty attached to observations of Comets, it is desirable to procure as many as possible, and form similar equations of condition, the resolution of any number of which may be effected by the *method of least squares.*

Each observation of Right Ascension and Declination furnishes, conjointly with the Ephemeris, a value of E and E′; and all the equations combined as before mentioned, and resolved, will furnish the value of the unknown quantities P, Q, R, &c., and hence the corrections to be applied to the assumed orbit of July 30, to obtain the true orbit of the Comet on that day.

When from a complete discussion of all the observations deserving of confidence, the assumed orbit of July 30, with the position of the perihelion therein, and the time of perihelion passage, shall have been corrected, the total variation of each of the elements for each fourth day, from July 30, given in Table IX., furnishes a ready means of reducing the orbit and the position of the perihelion to the instant of the Comet's passage by that point.

W. S. Stratford.

TABLES.

TABLE I.

Containing, for each Mean Midnight at Greenwich (Astronomical time) from August 1, 1835, to March 31, 1836, the Comet's Excentric Anomaly (u), Logarithm of the Radius Vector (r), and True Heliocentric co-ordinates (x, y, z,); x, being measured on a line passing through the true Vernal Equinoctial point of the date; y, on a line in a plane parallel to that of the Equator, and perpendicular to the direction of x; and z, perpendicular to the plane of the Equator, towards the North.

Date.	u	Log. of r	x	y	z
1835.	° ′ ″				
Aug. 1 ·5	23 13 56 ·58	0 ·3021016	+0 ·9934592	+1 ·6196067	+0 ·6400825
2 ·5	23 6 58 ·85	0 ·2990714	0 ·9942075	1 ·6032837	0 ·6364889
3 ·5	22 59 58 ·31	0 ·2960138	0 ·9949189	1 ·5869050	0 ·6328720
4 ·5	22 52 54 ·63	0 ·2929266	0 ·9955925	1 ·5704610	0 ·6292300
5 ·5	22 45 47 ·96	0 ·2898106	0 ·9962267	1 ·5539570	0 ·6255636
6 ·5	22 38 38 ·33	0 ·2866660	0 ·9968209	1 ·5373970	0 ·6218734
7 ·5	22 31 25 ·49	0 ·2834906	0 ·9973750	1 ·5207700	0 ·6181573
8 ·5	22 24 9 ·45	0 ·2802842	0 ·9978869	1 ·5040790	0 ·6144149
9 ·5	22 16 50 ·08	0 ·2770462	0 ·9983565	1 ·4873210	0 ·6106458
10 ·5	22 9 27 ·50	0 ·2737772	0 ·9987832	1 ·4705020	0 ·6068508
11 ·5	22 2 1 ·60	0 ·2704760	0 ·9991612	1 ·4536180	0 ·6030291
12 ·5	21 54 32 ·11	0 ·2671406	0 ·9995002	1 ·4366610	0 ·5991783
13 ·5	21 46 59 ·20	0 ·2637726	0 ·9997897	1 ·4196410	0 ·5953005
14 ·5	21 39 22 ·77	0 ·2603706	1 ·0000309	1 ·4025540	0 ·5913941
15 ·5	21 31 42 ·81	0 ·2569340	1 ·0002227	1 ·3853990	0 ·5874591
16 ·5	21 23 59 ·20	0 ·2534626	1 ·0003650	1 ·3681770	0 ·5834953
17 ·5	21 16 11 ·61	0 ·2499532	1 ·0004558	1 ·3508750	0 ·5794993
18 ·5	21 8 20 ·35	0 ·2464082	1 ·0004936	1 ·3335080	0 ·5754738
19 ·5	21 0 25 ·36	0 ·2428272	1 ·0004786	1 ·3160740	0 ·5714190
20 ·5	20 52 26 ·12	0 ·2392058	1 ·0004070	1 ·2985570	0 ·5673298
21 ·5	20 44 23 ·12	0 ·2355478	1 ·0002793	1 ·2809760	0 ·5632107
22 ·5	20 36 15 ·98	0 ·2318502	1 ·0000940	1 ·2633190	0 ·5590588
23 ·5	20 28 4 ·45	0 ·2281108	0 ·9998470	1 ·2455800	0 ·5548718
24 ·5	20 19 48 ·87	0 ·2243320	0 ·9995412	1 ·2277720	0 ·5506526
25 ·5	20 11 28 ·62	0 ·2205090	0 ·9991710	1 ·2098760	0 ·5463960
26 ·5	20 3 4 ·17	0 ·2166456	0 ·9987370	1 ·1919110	0 ·5421066
27 ·5	19 54 35 ·12	0 ·2127380	0 ·9982370	1 ·1738640	0 ·5377804
28 ·5	19 46 1 ·47	0 ·2087864	0 ·9976685	1 ·1557380	0 ·5334177
29 ·5	19 37 23 ·12	0 ·2047902	0 ·9970305	1 ·1375330	0 ·5290182
30 ·5	19 28 39 ·96	0 ·2007480	0 ·9963210	1 ·1192460	0 ·5245807
31 ·5	19 19 51 ·84	0 ·1966586	+0 ·9955375	+1 ·1008750	+0 ·5201040

TABLE I.—*continued.*

Date.	u	Log. of r	x	y	z
1835.	° ′ ″				
Aug.31 ·5	19 19 51 ·84	0 ·1966586	+0 ·9955375	+1 ·1008750	+0 ·5201040
Sep. 1 ·5	19 10 58 ·73	0 ·1925220	0 ·9946785	1 ·0824220	0 ·5155884
2 ·5	19 2 0 ·47	0 ·1883362	0 ·9937410	1 ·0638820	0 ·5110315
3 ·5	18 52 56 ·97	0 ·1841010	0 ·9927245	1 ·0452580	0 ·5064341
4 ·5	18 43 47 ·97	0 ·1798138	0 ·9916247	1 ·0265400	0 ·5017930
5 ·5	18 34 33 ·77	0 ·1754778	0 ·9904415	1 ·0077480	0 ·4971120
6 ·5	18 25 13 ·86	0 ·1710878	0 ·9891705	0 ·9888602	0 ·4923859
7 ·5	18 15 48 ·02	0 ·1666426	0 ·9878092	0 ·9698784	0 ·4876133
8 ·5	18 6 16 ·51	0 ·1621440	0 ·9863554	0 ·9508074	0 ·4827964
9 ·5	17 56 39 ·11	0 ·1575906	0 ·9848072	0 ·9316510	0 ·4779343
10 ·5	17 46 55 ·37	0 ·1529786	0 ·9831605	0 ·9123946	0 ·4730224
11 ·5	17 37 5 ·46	0 ·1483094	0 ·9814125	0 ·8930468	0 ·4680629
12 ·5	17 27 9 ·13	0 ·1435812	0 ·9795602	0 ·8736058	0 ·4630536
13 ·5	17 17 6 ·38	0 ·1387936	0 ·9776010	0 ·8540718	0 ·4579945
14 ·5	17 6 56 ·59	0 ·1339424	0 ·9755304	0 ·8344322	0 ·4528812
15 ·5	16 56 40 ·13	0 ·1290300	0 ·9733450	0 ·8147010	0 ·4477163
16 ·5	16 46 16 ·54	0 ·1240534	0 ·9710422	0 ·7948690	0 ·4424965
17 ·5	16 35 45 ·79	0 ·1190126	0 ·9686180	0 ·7749395	0 ·4372221
18 ·5	16 25 7 ·51	0 ·1139044	0 ·9660667	0 ·7549054	0 ·4318895
19 ·5	16 14 21 ·67	0 ·1087290	0 ·9633849	0 ·7347707	0 ·4264990
20 ·5	16 3 28 ·09	0 ·1034856	0 ·9605694	0 ·7145355	0 ·4210495
21 ·5	15 52 26 ·47	0 ·0981718	0 ·9576147	0 ·6941950	0 ·4155386
22 ·5	15 41 16 ·77	0 ·0927882	0 ·9545167	0 ·6737546	0 ·4099664
23 ·5	15 29 58 ·38	0 ·0873298	0 ·9512687	0 ·6531994	0 ·4043276
24 ·5	15 18 31 ·63	0 ·0818006	0 ·9478680	0 ·6325483	0 ·3986265
25 ·5	15 6 55 ·91	0 ·0761956	0 ·9443070	0 ·6117866	0 ·3928567
26 ·5	14 55 11 ·00	0 ·0705142	0 ·9405810	0 ·5909151	0 ·3870176
27 ·5	14 43 17 ·03	0 ·0647582	0 ·9366846	0 ·5699451	0 ·3811104
28 ·5	14 31 13 ·11	0 ·0589216	0 ·9326099	0 ·5488564	0 ·3751282
29 ·5	14 18 59 ·48	0 ·0530076	0 ·9283524	0 ·5276640	0 ·3690735
30 ·5	14 6 35 ·82	0 ·0470144	0 ·9239040	0 ·5063664	0 ·3629438
Oct. 1 ·5	13 54 1 ·63	0 ·0409398	0 ·9192580	0 ·4849564	0 ·3567352
2 ·5	13 41 16 ·79	0 ·0347846	0 ·9144076	0 ·4634403	0 ·3504477
3 ·5	13 28 21 ·03	0 ·0285478	0 ·9093442	0 ·4418178	0 ·3440792
4 ·5	13 15 13 ·82	0 ·0222278	0 ·9040590	0 ·4200841	0 ·3376257
5 ·5	13 1 55 ·17	0 ·0158266	0 ·8985456	0 ·3982486	0 ·3310880
6 ·5	12 48 24 ·48	0 ·0093420	0 ·8927934	0 ·3763035	0 ·3244616
7 ·5	12 34 41 ·59	0 ·0027756	0 ·8867936	0 ·3542575	0 ·3177458
8 ·5	12 20 46 ·04	9 ·9961268	0 ·8805362	0 ·3321065	0 ·3109374
9 ·5	12 6 37 ·48	9 ·9893966	0 ·8740114	0 ·3098547	0 ·3040343
10 ·5	11 52 15 ·64	9 ·9825862	+0 ·8672078	+0 ·2875047	+0 ·2970345

TABLE I.—*continued.*

Date.	u	Log. of r	x	y	z
1835.	° ′ ″				
Oct. 10·5	11 52 15·64	9·9825862	+0·8672078	+0·2875047	+0·2970345
11·5	11 37 40·05	9·9756970	0·8601150	0·2650579	0·2899355
12·5	11 22 50·50	9·9687316	0·8527220	0·2425212	0·2827358
13·5	11 7 46·26	9·9616898	0·8450138	0·2198898	0·2754305
14·5	10 52 27·30	9·9545774	0·8369808	0·1971773	0·2680199
15·5	10 36 53·09	9·9473970	0·8286088	0·1743849	0·2605010
16·5	10 21 3·22	9·9401526	0·8198842	0·1515173	0·2528708
17·5	10 4 57·41	9·9328506	0·8107940	0·1285853	0·2451286
18·5	9 48 35·00	9·9254944	0·8013202	0·1055881	0·2372695
19·5	9 31 55·85	9·9180934	0·7914509	0·0825416	0·2292937
20·5	9 14 59·54	9·9106550	0·7811695	0·0594537	0·2211993
21·5	8 57 45·46	9·9031872	0·7704582	0·0363288	0·2129819
22·5	8 40 13·51	9·8957022	0·7593024	+0·0131855	0·2046422
23·5	8 22 23·23	9·8882118	0·7476850	−0·0099666	0·1961779
24·5	8 4 14·56	9·8807306	0·7355907	0·0331082	0·1875894
25·5	7 45 46·69	9·8732712	0·7229972	0·0562329	0·1788723
26·5	7 26 59·92	9·8658540	0·7098927	0·0793132	0·1700299
27·5	7 7 53·64	9·8584958	0·6962557	0·1023373	0·1610593
28·5	6 48 27·88	9·8512188	0·6820712	0·1252805	0·1519619
29·5	6 28 42·66	9·8440474	0·6673242	0·1481181	0·1427398
30·5	6 8 37·86	9·8370056	0·6519966	0·1708276	0·1333933
31·5	5 48 13·56	9·8301216	0·6360744	0·1933811	0·1239252
Nov. 1·5	5 27 30·00	9·8234252	0·6195440	0·2157490	0·1143388
2·5	5 6 27·36	9·8169476	0·6023926	0·2379015	0·1046375
3·5	4 45 6·08	9·8107226	0·5846093	0·2598051	0·0948263
4·5	4 23 26·85	9·8047858	0·5661888	0·2814225	0·0849124
5·5	4 1 30·13	9·7991722	0·5471216	0·3027204	0·0749012
6·5	3 39 16·85	9·7939190	0·5274071	0·3236594	0·0648014
7·5	3 16 47·94	9·7890630	0·5070447	0·3442008	0·0546219
8·5	2 54 4·57	9·7846406	0·4860391	0·3643039	0·0443735
9·5	2 31 7·96	9·7806866	0·4643965	0·3839309	0·0340668
10·5	2 7 59·57	9·7772344	0·4421295	0·4030404	0·0237145
11·5	1 44 40·93	9·7743138	0·4192531	0·4215950	0·0133293
12·5	1 21 13·77	9·7719514	0·3957876	0·4395563	+0·0029255
13·5	0 57 39·86	9·7701694	0·3717559	0·4568896	−0·0074825
14·5	0 34 1·10	9·7689852	0·3471868	0·4735619	0·0178797
15·5	0 10 19·56	9·7684100	0·3221121	0·4895424	0·0282503
16·5	0 13 22·99	9·7684498	0·2965645	0·5048074	0·0385803
17·5	0 37 4·34	9·7691038	0·2705833	0·5193319	0·0488533
18·5	1 0 42·58	9·7703662	0·2442074	0·5331004	0·0590554
19·5	1 24 15·70	9·7722240	+0·2174783	−0·5460981	−0·0691720

TABLE I.—*continued.*

Date.	u	Log. of r	x	y	z
1835.	° ′ ″				
Nov. 19·5	1 24 15·70	9·7722240	+0·2174783	−0·5460981	−0·0691720
20·5	1 47 41·88	9·7746596	0·1904396	0·5583170	0·0791901
21·5	2 10 59·30	9·7776506	0·1631328	0·5697534	0·0890973
22·5	2 34 6·25	9·7811692	0·1356024	0·5804060	0·0988819
23·5	2 57 1·23	9·7851854	0·1078894	0·5902805	0·1085341
24·5	3 19 42·85	9·7896660	0·0800351	0·5993855	0·1180451
25·5	3 42 9·78	9·7945750	0·0520802	0·6077318	0·1274063
26·5	4 4 20·93	9·7998768	+0·0240623	0·6153356	0·1366118
27·5	4 26 15·50	9·8055344	−0·0039870	0·6222141	0·1456570
28·5	4 47 52·40	9·8115104	0·0320279	0·6283870	0·1545361
29·5	5 9 11·21	9·8177696	0·0600360	0·6338764	0·1632477
30·5	5 30 11·45	9·8242774	0·0879834	0·6387059	0·1717897
Dec. 1·5	5 50 52·58	9·8309996	0·1158424	0·6428983	0·1801603
2·5	6 11 14·39	9·8379058	0·1435917	0·6464802	0·1883598
3·5	6 31 16·68	9·8449658	0·1712102	0·6494756	0·1963884
4·5	6 50 59·34	9·8521522	0·1986796	0·6519104	0·2042471
5·5	7 10 22·67	9·8594414	0·2259913	0·6538097	0·2119395
6·5	7 29 26·30	9·8668076	0·2531207	0·6551979	0·2194648
7·5	7 48 10·63	9·8742314	0·2800633	0·6560999	0·2268274
8·5	8 6 35·94	9·8816940	0·3068094	0·6565386	0·2340303
9·5	8 24 42·38	9·8891782	0·3333506	0·6565376	0·2410765
10·5	8 42 30·21	9·8966684	0·3596787	0·6561188	0·2479692
11·5	8 59 59·82	9·9041516	0·3857907	0·6553030	0·2547123
12·5	9 17 11·55	9·9116158	0·4116820	0·6541116	0·2613094
13·5	9 34 5·77	9·9190510	0·4373501	0·6525640	0·2677644
14·5	9 50 42·67	9·9264462	0·4627882	0·6506786	0·2740799
15·5	10 7 2·88	9·9337952	0·4880003	0·6484727	0·2802610
16·5	10 23 6·65	9·9410906	0·5129837	0·6459639	0·2863110
17·5	10 38 54·54	9·9483274	0·5377420	0·6431681	0·2922343
18·5	10 54 26·71	9·9554990	0·5622698	0·6401000	0·2980335
19·5	11 9 43·79	9·9626026	0·5865737	0·6367742	0·3037131
20·5	11 24 46·01	9·9696340	0·6106513	0·6332049	0·3092764
21·5	11 39 33·75	9·9765896	0·6345044	0·6294039	0·3147259
22·5	11 54 7·53	9·9834690	0·6581395	0·6253846	0·3200668
23·5	12 8 27·70	9·9902692	0·6815563	0·6211567	0·3253015
24·5	12 22 34·49	9·9969886	0·7047555	0·6167336	0·3304329
25·5	12 36 28·40	0·0036268	0·7277430	0·6121230	0·3354646
26·5	12 50 9·76	0·0101834	0·7505212	0·6073370	0·3403998
27·5	13 3 38·95	0·0166578	0·7730940	0·6023821	0·3452415
28·5	13 16 56·11	0·0230484	0·7954590	0·5972692	0·3499912
29·5	13 30 1·79	0·0293576	−0·8176264	−0·5920045	−0·3546530

TABLE I.—*continued.*

Date.	u ° ′ ″	Log. of r	x	y	z
1835.					
Dec. 29·5	13 30 1·79	0·0293576	−0·8176264	−0·5920045	−0·3546530
30·5	13 42 56·14	0·0355838	0·8395930	0·5865984	0·3592290
31·5	13 55 39·57	0·0417286	0·8613670	0·5810555	0·3637218
1836.					
Jan. 1·5	14 8 12·27	0·0477918	0·8829480	0·5753848	0·3681338
2·5	14 20 34·76	0·0537754	0·9043446	0·5695897	0·3724678
3·5	14 32 47·08	0·0596792	0·9255554	0·5636797	0·3767258
4·5	14 44 49·48	0·0655036	0·9465822	0·5576599	0·3809095
5·5	14 56 42·60	0·0712524	0·9674383	0·5515331	0·3850229
6·5	15 8 26·05	0·0769220	0·9881102	0·5453091	0·3890650
7·5	15 20 0·71	0·0825180	1·0086172	0·5389883	0·3930407
8·5	15 31 26·37	0·0880382	1·0289507	0·5325780	0·3969498
9·5	15 42 43·65	0·0934870	1·0491251	0·5260810	0·4007962
10·5	15 53 52·37	0·0988622	1·0691320	0·5195026	0·4045793
11·5	16 4 52·97	0·1041670	1·0889800	0·5128475	0·4083024
12·5	16 15 45·44	0·1094006	1·1086656	0·5061187	0·4119655
13·5	16 26 30·28	0·1145670	1·1282024	0·4993195	0·4155723
14·5	16 37 7·59	0·1196668	1·1475900	0·4924528	0·4191239
15·5	16 47 37·41	0·1246992	1·1668241	0·4855227	0·4226202
16·5	16 58 0·05	0·1296674	1·1859141	0·4785318	0·4260641
17·5	17 8 15·70	0·1345720	1·2048622	0·4714824	0·4294566
18·5	17 18 24·46	0·1394142	1·2236687	0·4643785	0·4327990
19·5	17 28 26·49	0·1441950	1·2423366	0·4572223	0·4360926
20·5	17 38 21·85	0·1489146	1·2608652	0·4500181	0·4393375
21·5	17 48 10·98	0·1535766	1·2792653	0·4427634	0·4425371
22·5	17 57 53·91	0·1581812	1·2975366	0·4354630	0·4456916
23·5	18 7 30·59	0·1627276	1·3156733	0·4281209	0·4488008
24·5	18 17 1·38	0·1672194	1·3336871	0·4207380	0·4518678
25·5	18 26 26·37	0·1716568	1·3515778	0·4133141	0·4548926
26·5	18 35 45·55	0·1760400	1·3693435	0·4058548	0·4578756
27·5	18 44 59·12	0·1803700	1·3869864	0·3983598	0·4608183
28·5	18 54 7·29	0·1846496	1·4045150	0·3908298	0·4637230
29·5	19 3 10·13	0·1888784	1·4219270	0·3832684	0·4665887
30·5	19 12 7·73	0·1930580	1·4392260	0·3756758	0·4694171
31·5	19 21 0·19	0·1971884	1·4564090	0·3680542	0·4722084
Feb. 1·5	19 29 47·70	0·2012720	1·4734870	0·3604040	0·4749648
2·5	19 38 30·30	0·2053086	1·4904530	0·3527285	0·4776857
3·5	19 47 8·07	0·2092994	1·5073140	0·3450256	0·4803725
4·5	19 55 41·13	0·2132450	1·5240680	0·3373003	0·4830257
5·5	20 4 9·53	0·2171468	1·5407210	0·3295531	0·4856463
6·5	20 12 33·37	0·2210042	1·5572660	0·3217829	0·4882339
7·5	20 20 52·99	0·2248214	−1·5737210	−0·3139910	−0·4907921

TABLE I.—*continued.*

Date.	u	Log. of r	x	y	z
1836.	° ′ ″				
Feb. 7 ·5	20 20 52 ·99	0 ·2248214	—1 ·5737210	—0 ·3139910	—0 ·4907921
8 ·5	20 29 7 ·99	0 ·2285944	1 ·5900680	0 ·3061834	0 ·4933176
9 ·5	20 37 18 ·83	0 ·2323276	1 ·6063210	0 ·2983541	0 ·4958136
10 ·5	20 45 25 ·58	0 ·2360212	1 ·6224810	0 ·2905048	0 ·4982807
11 ·5	20 53 28 ·15	0 ·2396750	1 ·6385450	0 ·2826437	0 ·5007183
12 ·5	21 1 26 ·81	0 ·2432908	1 ·6545200	0 ·2747628	0 ·5031283
13 ·5	21 9 21 ·45	0 ·2468684	1 ·6704020	0 ·2668677	0 ·5055107
14 ·5	21 17 12 ·17	0 ·2504082	1 ·6861920	0 ·2589599	0 ·5078655
15 ·5	21 24 59 ·18	0 ·2539120	1 ·7018960	0 ·2510363	0 ·5101936
16 ·5	21 32 42 ·41	0 ·2573796	1 ·7175110	0 ·2431004	0 ·5124952
17 ·5	21 40 21 ·83	0 ·2608110	1 ·7330360	0 ·2351553	0 ·5147712
18 ·5	21 47 57 ·92	0 ·2642096	1 ·7484860	0 ·2271943	0 ·5170232
19 ·5	21 55 30 ·28	0 ·2675730	1 ·7638460	0 ·2192253	0 ·5192495
20 ·5	22 2 59 ·18	0 ·2709030	1 ·7791240	0 ·2112468	0 ·5214518
21 ·5	22 10 24 ·74	0 ·2742006	1 ·7943220	0 ·2032570	0 ·5236307
22 ·5	22 17 46 ·98	0 ·2774660	1 ·8094430	0 ·1952574	0 ·5257862
23 ·5	22 25 5 ·86	0 ·2806996	1 ·8244830	0 ·1872502	0 ·5279182
24 ·5	22 32 21 ·41	0 ·2839010	1 ·8394410	0 ·1792374	0 ·5300280
25 ·5	22 39 33 ·93	0 ·2870734	1 ·8543290	0 ·1712136	0 ·5321165
26 ·5	22 46 43 ·20	0 ·2902144	1 ·8691360	0 ·1631855	0 ·5341824
27 ·5	22 53 49 ·38	0 ·2933258	1 ·8838690	0 ·1551516	0 ·5362271
28 ·5	23 0 52 ·75	0 ·2964100	1 ·8985370	0 ·1471069	0 ·5382521
29 ·5	23 7 52 ·89	0 ·2994636	1 ·9131230	0 ·1390603	0 ·5402550
Mar. 1 ·5	23 14 50 ·21	0 ·3024902	1 ·9276420	0 ·1310079	0 ·5422388
2 ·5	23 21 44 ·56	0 ·3054882	1 ·9420870	0 ·1229515	0 ·5442020
3 ·5	23 28 36 ·25	0 ·3084602	1 ·9564670	0 ·1148859	0 ·5461461
4 ·5	23 35 25 ·06	0 ·3114052	1 ·9707780	0 ·1068224	0 ·5480715
5 ·5	23 42 11 ·02	0 ·3143228	1 ·9850160	0 ·0987545	0 ·5499772
6 ·5	23 48 54 ·47	0 ·3172162	1 ·9991950	0 ·0906788	0 ·5518650
7 ·5	23 55 35 ·07	0 ·3200826	2 ·0133000	0 ·0826053	0 ·5537340
8 ·5	24 2 13 ·04	0 ·3229238	2 ·0273360	0 ·0745275	0 ·5555845
9 ·5	24 8 48 ·56	0 ·3257418	2 ·0413220	0 ·0664464	0 ·5574191
10 ·5	24 15 21 ·51	0 ·3285348	2 ·0552370	0 ·0583611	0 ·5592349
11 ·5	24 21 51 ·81	0 ·3313032	2 ·0690850	0 ·0502796	0 ·5610338
12 ·5	24 28 19 ·74	0 ·3340486	2 ·0828730	0 ·0421943	0 ·5628160
13 ·5	24 34 45 ·27	0 ·3367712	2 ·0966030	0 ·0341058	0 ·5645820
14 ·5	24 41 8 ·35	0 ·3394706	2 ·1102700	0 ·0260183	0 ·5663312
15 ·5	24 47 29 ·21	0 ·3421488	2 ·1238840	0 ·0179263	0 ·5680652
16 ·5	24 53 47 ·56	0 ·3448034	2 ·1374320	0 ·0098370	0 ·5697827
17 ·5	25 0 3 ·63	0 ·3474364	2 ·1509200	—0 ·0017496	0 ·5714849
18 ·5	25 6 17 ·54	0 ·3500484	—2 ·1643540	+0 ·0063418	—0 ·5731712

TABLE I.—*continued.*

Date.	u	Log. of r	x	y	z
1836.	° ′ ″				
Mar. 18·5	25 6 17·54	0·3500484	—2·1643540	+0·0063418	—0·5731712
19·5	25 12 29·15	0·3526392	2·1777300	0·0144311	0·5748430
20·5	25 18 38·55	0·3552088	2·1910480	0·0225203	0·5764996
21·5	25 24 45·69	0·3577576	2·2043100	0·0306062	0·5781412
22·5	25 30 50·83	0·3602870	2·2175190	0·0386944	0·5797692
23·5	25 36 53·88	0·3627968	2·2306750	0·0467840	0·5813829
24·5	25 42 54·71	0·3652858	2·2437730	0·0548675	0·5829816
25·5	25 48 53·50	0·3677556	2·2568170	0·0629496	0·5845670
26·5	25 54 50·43	0·3702076	2·2698160	0·0710352	0·5861395
27·5	26 0 45·23	0·3726398	2·2827560	0·0791168	0·5876977
28·5	26 6 38·03	0·3750534	2·2956460	0·0871938	0·5892438
29·5	26 12 28·86	0·3774486	2·3084840	0·0952698	0·5907748
30·5	26 18 18·01	0·3798274	2·3212790	0·1033490	0·5922947
31·5	26 24 4·99	0·3821934	—2·3340520	+0·1114423	—0·5938052

TABLE II.

Containing, for each Mean Midnight at Greenwich (Astronomical time) from August 1, 1835, to March 31, 1836, the Sun's True Geocentric co-ordinates (X, Y, Z,); X, being measured on a line passing through the True Vernal Equinoctial point of the date; Y, on a line in the plane of the Equator, and perpendicular to the direction of X; and Z, perpendicular to the plane of the Equator, towards the North.

Date.	X	Y	Z
1835.			
Aug. 1·5	−0·6377144	+0·7239212	+0·3141959
2·5	0·6507172	0·7139433	0·3098651
3·5	0·6635348	0·7037629	0·3054467
4·5	0·6761640	0·6933840	0·3009422
5·5	0·6886010	0·6828106	0·2963532
6·5	0·7008426	0·6720445	0·2916807
7·5	0·7128858	0·6610891	0·2869258
8·5	0·7247270	0·6499469	0·2820903
9·5	0·7363624	0·6386212	0·2771750
10·5	0·7477904	0·6271152	0·2721814
11·5	0·7590069	0·6154303	0·2671100
12·5	0·7700086	0·6035713	0·2619630
13·5	0·7807926	0·5915402	0·2567409
14·5	0·7913554	0·5793399	0·2514459
15·5	0·8016942	0·5669735	0·2460787
16·5	0·8118056	0·5544446	0·2406408
17·5	0·8216860	0·5417562	0·2351336
18·5	0·8313320	0·5289113	0·2295588
19·5	0·8407414	0·5159127	0·2239171
20·5	0·8499097	0·5027664	0·2182115
21·5	0·8588338	0·4894753	0·2124431
22·5	0·8675108	0·4760423	0·2066131
23·5	0·8759393	0·4624726	0·2007236
24·5	0·8841147	0·4487694	0·1947763
25·5	0·8920348	0·4349377	0·1887731
26·5	0·8996971	0·4209812	0·1827156
27·5	0·9070998	0·4069035	0·1766055
28·5	0·9142407	0·3927093	0·1704449
29·5	0·9211173	0·3784028	0·1642355
30·5	0·9277274	0·3639887	0·1579793
31·5	−0·9340702	+0·3494706	+0·1516782

TABLE II.—*continued.*

Date.	X	Y	Z
1835.			
Aug. 31 ·5	−0 ·9340702	+0 ·3494706	+0 ·1516782
Sep. 1 ·5	0 ·9401430	0 ·3348529	0 ·1453338
2 ·5	0 ·9459447	0 ·3201410	0 ·1389485
3 ·5	0 ·9514735	0 ·3053374	0 ·1325236
4 ·5	0 ·9567283	0 ·2904466	0 ·1260608
5 ·5	0 ·9617067	0 ·2754731	0 ·1195620
6 ·5	0 ·9664088	0 ·2604215	0 ·1130293
7 ·5	0 ·9708324	0 ·2452950	0 ·1064639
8 ·5	0 ·9749755	0 ·2300975	0 ·0998680
9 ·5	0 ·9788376	0 ·2148339	0 ·0932431
10 ·5	0 ·9824170	0 ·1995069	0 ·0865909
11 ·5	0 ·9857127	0 ·1841211	0 ·0799130
12 ·5	0 ·9887233	0 ·1686809	0 ·0732117
13 ·5	0 ·9914467	0 ·1531899	0 ·0664882
14 ·5	0 ·9938820	0 ·1376537	0 ·0597451
15 ·5	0 ·9960284	0 ·1220750	0 ·0529836
16 ·5	0 ·9978838	0 ·1064592	0 ·0462060
17 ·5	0 ·9994476	0 ·0908113	0 ·0394144
18 ·5	1 ·0007185	0 ·0751365	0 ·0326112
19 ·5	1 ·0016956	0 ·0594383	0 ·0257979
20 ·5	1 ·0023781	0 ·0437224	0 ·0189768
21 ·5	1 ·0027662	0 ·0279936	0 ·0121501
22 ·5	1 ·0028587	+0 ·0122569	+0 ·0053200
23 ·5	1 ·0026553	−0 ·0034839	−0 ·0015119
24 ·5	1 ·0021567	0 ·0192232	0 ·0083432
25 ·5	1 ·0013618	0 ·0349567	0 ·0151719
26 ·5	1 ·0002713	0 ·0506792	0 ·0219959
27 ·5	0 ·9988861	0 ·0663847	0 ·0288126
28 ·5	0 ·9972056	0 ·0820709	0 ·0356207
29 ·5	0 ·9952314	0 ·0977301	0 ·0424172
30 ·5	0 ·9929635	0 ·1133602	0 ·0492011
Oct. 1 ·5	0 ·9904031	0 ·1289545	0 ·0559695
2 ·5	0 ·9875505	0 ·1445099	0 ·0627209
3 ·5	0 ·9844073	0 ·1600214	0 ·0694533
4 ·5	0 ·9809734	0 ·1754841	0 ·0761646
5 ·5	0 ·9772506	0 ·1908938	0 ·0828530
6 ·5	0 ·9732391	0 ·2062490	0 ·0895174
7 ·5	0 ·9689400	0 ·2215423	0 ·0961551
8 ·5	0 ·9643546	0 ·2367707	0 ·1027644
9 ·5	0 ·9594834	0 ·2519284	0 ·1093433
10 ·5	−0 ·9543277	−0 ·2670131	−0 ·1158903

TABLE II.—*continued.*

Date.	X	Y	Z
1835.			
Oct. 10·5	—0·9543277	—0·2670131	—0·1158903
11·5	0·9488880	0·2820198	0·1224035
12·5	0·9431659	0·2969437	0·1288809
13·5	0·9371623	0·3117803	0·1353204
14·5	0·9308786	0·3265255	0·1417202
15·5	0·9243160	0·3411742	0·1480782
16·5	0·9174755	0·3557214	0·1543921
17·5	0·9103593	0·3701632	0·1606602
18·5	0·9029685	0·3844938	0·1668801
19·5	0·8953061	0·3987093	0·1730500
20·5	0·8873732	0·4128050	0·1791678
21·5	0·8791725	0·4267767	0·1852318
22·5	0·8707056	0·4406199	0·1912399
23·5	0·8619757	0·4543289	0·1971897
24·5	0·8529850	0·4679004	0·2030800
25·5	0·8437366	0·4813287	0·2089085
26·5	0·8342334	0·4946128	0·2146737
27·5	0·8244780	0·5077449	0·2203735
28·5	0·8144746	0·5207219	0·2260058
29·5	0·8042254	0·5335404	0·2315694
30·5	0·7937343	0·5461961	0·2370624
31·5	0·7830035	0·5586856	0·2424831
Nov. 1·5	0·7720366	0·5710053	0·2478301
2·5	0·7608375	0·5831521	0·2531021
3·5	0·7494084	0·5951215	0·2582969
4·5	0·7377541	0·6069100	0·2634133
5·5	0·7258759	0·6185154	0·2684502
6·5	0·7137773	0·6299339	0·2734059
7·5	0·7014629	0·6411615	0·2782789
8·5	0·6889342	0·6521953	0·2830678
9·5	0·6761963	0·6630313	0·2877708
10·5	0·6632517	0·6736663	0·2923865
11·5	0·6501038	0·6840969	0·2969137
12·5	0·6367569	0·6943192	0·3013504
13·5	0·6232138	0·7043302	0·3056957
14·5	0·6094795	0·7141259	0·3099473
15·5	0·5955570	0·7237032	0·3141040
16·5	0·5814503	0·7330599	0·3181650
17·5	0·5671633	0·7421914	0·3221281
18·5	0·5527009	0·7510947	0·3259921
19·5	—0·5380680	—0·7597662	—0·3297556

TABLE II.—*continued.*

Date.	X	Y	Z
1835.			
Nov. 19 ·5	−0 ·5380680	−0 ·7597662	−0 ·3297556
20 ·5	0 ·5232679	0 ·7682038	0 ·3334175
21 ·5	0 ·5083063	0 ·7764042	0 ·3369766
22 ·5	0 ·4931886	0 ·7843640	0 ·3404311
23 ·5	0 ·4779192	0 ·7920813	0 ·3437806
24 ·5	0 ·4625022	0 ·7995539	0 ·3470239
25 ·5	0 ·4469443	0 ·8067783	0 ·3501597
26 ·5	0 ·4312499	0 ·8137524	0 ·3531868
27 ·5	0 ·4154234	0 ·8204754	0 ·3561047
28 ·5	0 ·3994705	0 ·8269446	0 ·3589124
29 ·5	0 ·3833962	0 ·8331580	0 ·3616093
30 ·5	0 ·3672041	0 ·8391143	0 ·3641943
Dec. 1 ·5	0 ·3509002	0 ·8448118	0 ·3666671
2 ·5	0 ·3344888	0 ·8502491	0 ·3690266
3 ·5	0 ·3179752	0 ·8554240	0 ·3712725
4 ·5	0 ·3013635	0 ·8603357	0 ·3734042
5 ·5	0 ·2846594	0 ·8649822	0 ·3754208
6 ·5	0 ·2678671	0 ·8693622	0 ·3773217
7 ·5	0 ·2509927	0 ·8734741	0 ·3791064
8 ·5	0 ·2340384	0 ·8773168	0 ·3807744
9 ·5	0 ·2170117	0 ·8808888	0 ·3823249
10 ·5	0 ·1999168	0 ·8841886	0 ·3837572
11 ·5	0 ·1827584	0 ·8872146	0 ·3850708
12 ·5	0 ·1655416	0 ·8899654	0 ·3862649
13 ·5	0 ·1482738	0 ·8924400	0 ·3873390
14 ·5	0 ·1309573	0 ·8946376	0 ·3882928
15 ·5	0 ·1135992	0 ·8965570	0 ·3891256
16 ·5	0 ·0962042	0 ·8981975	0 ·3898372
17 ·5	0 ·0787781	0 ·8995575	0 ·3904274
18 ·5	0 ·0613261	0 ·9006370	0 ·3908958
19 ·5	0 ·0438550	0 ·9014350	0 ·3912420
20 ·5	0 ·0263694	0 ·9019513	0 ·3914662
21 ·5	−0 ·0088756	0 ·9021856	0 ·3915681
22 ·5	+0 ·0086187	0 ·9021381	0 ·3915477
23 ·5	0 ·0261103	0 ·9018086	0 ·3914051
24 ·5	0 ·0435920	0 ·9011976	0 ·3911402
25 ·5	0 ·0610586	0 ·9003058	0 ·3907531
26 ·5	0 ·0785044	0 ·8991338	0 ·3902444
27 ·5	0 ·0959239	0 ·8976816	0 ·3896141
28 ·5	0 ·1133131	0 ·8959504	0 ·3888627
29 ·5	+0 ·1306663	−0 ·8939408	−0 ·3879904

TABLE II.—*continued.*

Date.	X	Y	Z
1835.			
Dec. 29 ·5	+0 ·1306663	−0 ·8939408	−0 ·3879904
30 ·5	0 ·1479764	0 ·8916536	0 ·3869975
31 ·5	0 ·1652420	0 ·8890896	0 ·3858847
1836.			
Jan. 1 ·5	0 ·1824538	0 ·8862483	0 ·3846513
2 ·5	0 ·1996086	0 ·8831344	0 ·3832999
3 ·5	0 ·2167012	0 ·8797475	0 ·3818298
4 ·5	0 ·2337263	0 ·8760878	0 ·3802417
5 ·5	0 ·2506797	0 ·8721565	0 ·3785354
6 ·5	0 ·2675558	0 ·8679550	0 ·3767122
7 ·5	0 ·2843501	0 ·8634839	0 ·3747721
8 ·5	0 ·3010571	0 ·8587450	0 ·3727154
9 ·5	0 ·3176716	0 ·8537391	0 ·3705428
10 ·5	0 ·3341886	0 ·8484678	0 ·3682549
11 ·5	0 ·3506041	0 ·8429315	0 ·3658521
12 ·5	0 ·3669118	0 ·8371324	0 ·3633350
13 ·5	0 ·3831065	0 ·8310725	0 ·3607048
14 ·5	0 ·3991837	0 ·8247514	0 ·3579612
15 ·5	0 ·4151367	0 ·8181733	0 ·3551061
16 ·5	0 ·4309614	0 ·8113392	0 ·3521401
17 ·5	0 ·4466519	0 ·8042515	0 ·3490642
18 ·5	0 ·4622034	0 ·7969121	0 ·3458791
19 ·5	0 ·4776097	0 ·7893248	0 ·3425862
20 ·5	0 ·4928658	0 ·7814911	0 ·3391866
21 ·5	0 ·5079681	0 ·7734146	0 ·3356814
22 ·5	0 ·5229109	0 ·7650978	0 ·3320718
23 ·5	0 ·5376908	0 ·7565431	0 ·3283588
24 ·5	0 ·5523017	0 ·7477544	0 ·3245443
25 ·5	0 ·5667400	0 ·7387340	0 ·3206292
26 ·5	0 ·5810008	0 ·7294867	0 ·3166155
27 ·5	0 ·5950812	0 ·7200125	0 ·3125034
28 ·5	0 ·6089770	0 ·7103177	0 ·3082956
29 ·5	0 ·6226830	0 ·7004046	0 ·3039931
30 ·5	0 ·6361972	0 ·6902761	0 ·2995972
31 ·5	0 ·6495148	0 ·6799352	0 ·2951092
Feb. 1 ·5	0 ·6626323	0 ·6693855	0 ·2905308
2 ·5	0 ·6755458	0 ·6586305	0 ·2858629
3 ·5	0 ·6882509	0 ·6476734	0 ·2811075
4 ·5	0 ·7007469	0 ·6365163	0 ·2762656
5 ·5	0 ·7130268	0 ·6251638	0 ·2713381
6 ·5	0 ·7250890	0 ·6136183	0 ·2663271
7 ·5	+0 ·7369287	−0 ·6018841	−0 ·2612341

TABLE II.—*continued.*

Date.	X	Y	Z
1836.			
Feb. 7·5	+0·7369287	−0·6018841	−0·2612341
8·5	0·7485434	0·5899637	0·2560599
9·5	0·7599287	0·5778589	0·2508063
10·5	0·7710818	0·5655759	0·2454750
11·5	0·7819977	0·5531174	0·2400677
12·5	0·7926742	0·5404874	0·2345861
13·5	0·8031068	0·5276899	0·2290318
14·5	0·8132926	0·5147293	0·2234068
15·5	0·8232278	0·5016092	0·2177124
16·5	0·8329091	0·4883346	0·2119510
17·5	0·8423340	0·4749097	0·2061244
18·5	0·8514993	0·4613391	0·2002344
19·5	0·8604026	0·4476267	0·1942829
20·5	0·8690409	0·4337777	0·1882717
21·5	0·8774127	0·4197970	0·1822038
22·5	0·8855150	0·4056887	0·1760802
23·5	0·8933456	0·3914576	0·1699033
24·5	0·9009036	0·3771077	0·1636751
25·5	0·9081862	0·3626450	0·1573979
26·5	0·9151928	0·3480724	0·1510729
27·5	0·9219204	0·3333960	0·1447036
28·5	0·9283680	0·3186195	0·1382896
29·5	0·9345352	0·3037472	0·1318348
Mar. 1·5	0·9404191	0·2887843	0·1253406
2·5	0·9460190	0·2737349	0·1188088
3·5	0·9513338	0·2586029	0·1122411
4·5	0·9563625	0·2433919	0·1056393
5·5	0·9611028	0·2281080	0·0990056
6·5	0·9655541	0·2127539	0·0923411
7·5	0·9697146	0·1973345	0·0856490
8·5	0·9735837	0·1818541	0·0789302
9·5	0·9771600	0·1663172	0·0721868
10·5	0·9804420	0·1507290	0·0654212
11·5	0·9834286	0·1350940	0·0586352
12·5	0·9861185	0·1194168	0·0518310
13·5	0·9885118	0·1037026	0·0450106
14·5	0·9906069	0·0879563	0·0381762
15·5	0·9924031	0·0721828	0·0313301
16·5	0·9939002	0·0563861	0·0244738
17·5	0·9950982	0·0405740	0·0176109
18·5	+0·9959961	−0·0247498	−0·0107423

TABLE II.—*continued.*

Date.	X	Y	Z
1836.			
Mar. 18 ·5	+0 ·9959961	−0 ·0247498	−0 ·0107423
19 ·5	0 ·9965947	−0 ·0089170	−0 ·0038708
20 ·5	0 ·9968940	+0 ·0069177	+0 ·0030020
21 ·5	0 ·9968942	0 ·0227492	0 ·0098734
22 ·5	0 ·9965958	0 ·0385728	0 ·0167414
23 ·5	0 ·9959996	0 ·0543835	0 ·0236038
24 ·5	0 ·9951064	0 ·0701763	0 ·0304584
25 ·5	0 ·9939175	0 ·0859477	0 ·0373036
26 ·5	0 ·9924335	0 ·1016914	0 ·0441370
27 ·5	0 ·9906551	0 ·1174037	0 ·0509567
28 ·5	0 ·9885840	0 ·1330796	0 ·0577607
29 ·5	0 ·9862208	0 ·1487153	0 ·0645472
30 ·5	0 ·9835663	0 ·1643064	0 ·0713143
31 ·5	+0 ·9806227	+0 ·1798484	+0 ·0780601

TABLE III.

Containing, for Mean Noon at Greenwich, the Heliocentric co-ordinates (x, x_1, &c., y, y_1, &c., z, z_1, &c.) of the Comet and the disturbing Planets; x, x_1, &c., being measured on a line passing through the Mean Vernal Equinox of January 1, 1835; y, y_1, &c., perpendicular to x, x_1, &c., in the plane of the Ecliptic (supposed invariable); and z, z_1, &c., perpendicular to the plane of the Ecliptic, towards the North.

Date.	Heliocentric Co-ordinates of					
	The Comet.			Mercury.		
1835.	x	y	z	x_1	y_1	z_1
Aug. 1	+0 ·99318	+1 ·74856	−0 ·05926	+0 ·35692	−0 ·06352	−0 ·03711
5	0 ·99605	1 ·68279	0 ·04645	0 ·33668	+0 ·05450	0 ·02532
9	0 ·99827	1 ·61585	0 ·03360	0 ·27709	0 ·16571	−0 ·01059
13	0 ·99977	1 ·54778	0 ·02073	0 ·17940	0 ·25364	+0 ·00555
17	1 ·00052	1 ·47856	−0 ·00786	+0 ·05418	0 ·30220	0 ·02080
21	1 ·00046	1 ·40815	+0 ·00503	−0 ·07972	0 ·30270	0 ·03275
25	0 ·99949	1 ·33638	0 ·01792	0 ·20210	0 ·25798	0 ·03984
29	0 ·99752	1 ·26322	0 ·03079	0 ·29857	0 ·17949	0 ·04176
Sep. 2	0 ·99433	1 ·18872	0 ·04361	0 ·36284	+0 ·08114	0 ·03913
6	0 ·98992	1 ·11270	0 ·05638	0 ·39447	−0 ·02482	0 ·03297
10	0 ·98408	1 ·03500	0 ·06908	0 ·39625	0 ·12912	0 ·02429
14	0 ·97670	0 ·95572	0 ·08165	0 ·37227	0 ·22519	0 ·01400
18	0 ·96745	0 ·87462	0 ·09409	0 ·32692	0 ·30845	+0 ·00292
22	0 ·95618	0 ·79166	0 ·10632	0 ·26451	0 ·37570	−0 ·00834
26	0 ·94252	0 ·70681	0 ·11828	0 ·18919	0 ·42466	0 ·01919
30	0 ·92621	0 ·61993	0 ·12991	0 ·10497	0 ·45370	0 ·02914
Oct. 4	0 ·90680	0 ·53095	0 ·14109	−0 ·01587	0 ·46164	0 ·03773
8	0 ·88377	0 ·43990	0 ·15172	+0 ·07396	0 ·44764	0 ·04453
12	0 ·85647	0 ·34673	0 ·16162	0 ·16005	0 ·41133	0 ·04910
16	0 ·82431	0 ·25157	0 ·17060	0 ·23743	0 ·35283	0 ·05103
20	0 ·78639	0 ·15474	0 ·17837	0 ·30045	0 ·27322	0 ·04987
24	0 ·74170	+0 ·05655	0 ·18461	0 ·34263	0 ·17469	0 ·04529
28	0 ·68924	−0 ·04218	0 ·18890	0 ·35691	−0 ·06262	0 ·03704
Nov. 1	0 ·62786	0 ·14032	0 ·19075	0 ·33638	+0 ·05541	0 02520
5	0 ·55672	0 ·23622	0 ·18961	0 ·27648	0 ·16651	−0 ·01046
9	0 ·47529	0 ·32763	0 ·18496	0 ·17853	0 ·25419	+0 ·00567
13	0 ·38379	0 ·41220	0 ·17640	+0 ·05316	0 ·30238	0 ·02091
17	0 ·28356	0 ·48727	0 ·16380	−0 ·08074	0 ·30252	0 ·03283
21	0 ·17673	0 ·55104	0 ·14740	0 ·20296	0 ·25748	0 ·03987
25	+0 ·06600	0 ·60262	0 ·12774	0 ·29920	0 ·17877	0 ·04176
29	−0 ·04613	−0 ·64230	+0 ·10554	−0 ·36320	+0 ·08033	+0 ·03910

TABLE III.—*continued.*

Date.	Heliocentric Co-ordinates of					
	The Comet.			Mercury.		
1835.	x	y	z	x_1	y_1	z_1
Nov. 29	—0 ·04613	—0 ·64230	+0 ·10554	—0 ·36320	+0 ·08033	+0 ·03910
Dec. 3	0 ·15751	0 ·67104	0 ·08152	0 ·39459	—0 ·02564	0 ·03290
7	0 ·26672	0 ·69032	0 ·05635	0 ·39616	0 ·12991	0 ·02421
11	0 ·37289	0 ·70155	0 ·03050	0 ·37199	0 ·22590	0 ·01392
15	0 ·47555	0 ·70617	+0 ·00437	0 ·32650	0 ·30902	+0 ·00282
19	0 ·57458	0 ·70539	—0 ·02181	0 ·26397	0 ·37615	—0 ·00842
23	0 ·66999	0 ·70013	0 ·04786	0 ·18856	0 ·42497	0 ·01927
27	0 ·76196	0 ·69123	0 ·07366	0 ·10429	0 ·45385	0 ·02920
31	0 ·85065	0 ·67934	0 ·09915	—0 ·01516	0 ·46162	0 ·03778
1836.						
Jan. 4	0 ·93620	0 ·66500	0 ·12427	+0 ·07465	0 ·44744	0 ·04457
8	1 ·01892	0 ·64859	0 ·14902	0 ·16070	0 ·41096	0 ·04913
12	1 ·09896	0 ·63046	0 ·17338	0 ·23799	0 ·35231	0 ·05102
16	1 ·17652	0 ·61089	0 ·19735	0 ·30087	0 ·27253	0 ·04986
20	1 ·25176	0 ·59015	0 ·22092	0 ·34285	0 ·17408	0 ·04524
24	1 ·32483	0 ·56837	0 ·24412	0 ·35689	—0 ·06171	0 ·03696
28	1 ·39589	0 ·54573	0 ·26696	0 ·33606	+0 ·05632	0 ·02510
Feb. 1	1 ·46508	0 ·52241	0 ·28942	0 ·27585	0 ·16731	—0 ·01034
5	1 ·53250	0 ·49838	0 ·31154	0 ·17764	0 ·25472	+0 ·00580
9	1 ·59834	0 ·47385	0 ·33334	+0 ·05213	—0 ·30257	0 ·02102
13	1 ·66253	0 ·44885	0 ·35480	—0 ·08175	0 ·30232	0 ·03290
17	1 ·72536	0 ·42347	0 ·37597	0 ·20380	0 ·25699	0 ·03990
21	1 ·78682	0 ·39776	0 ·39683	0 ·29982	0 ·17807	0 ·04176
25	1 ·84697	0 ·37174	0 ·41740	0 ·36356	+0 ·07954	0 ·03906
29	1 ·90594	0 ·34557	0 ·43770	0 ·39471	—0 ·02646	0 ·03285
Mar. 4	1 ·96372	0 ·31911	0 ·45774	0 ·39606	0 ·13068	0 ·02413
8	2 ·02041	0 ·29241	0 ·47753	0 ·37173	0 ·22658	0 ·01383
12	2 ·07601	0 ·26564	0 ·49705	0 ·32608	0 ·30960	+0 ·00275
16	2 ·13074	0 ·23875	0 ·51636	0 ·26344	0 ·37658	—0 ·00850
20	2 ·18443	0 ·21175	0 ·53544	0 ·18794	0 ·42526	0 ·01934
24	2 ·23723	0 ·18459	0 ·55428	0 ·10363	0 ·45398	0 ·02927
28	2 ·28929	0 ·15747	0 ·57293	—0 ·01448	0 ·46159	0 ·03785
Apr. 1	—2 ·34045	—0 ·13017	—0 ·59137	+0 ·07533	—0 ·44726	—0 ·04462

TABLE III.—*continued.*

Date.	Heliocentric Co-ordinates of Venus.			The Earth.		
1835.	x_2	y_2	z_2	x_3	y_3	z_3
Aug. 1	+0·04533	+0·71820	+0·00829	+0·63109	−0·79458	- -
5	−0·03582	0·71834	0·01293	0·68234	0·75020	- -
9	0·11652	0·70932	0·01742	0·73053	0·70243	- -
13	0·19574	0·69129	0·02168	0·77537	0·65149	- -
17	0·27247	0·66447	0·02567	0·81672	0·59758	- -
21	0·34572	0·62917	0·02934	0·85436	0·54094	- -
25	0·41456	0·58584	0·03263	0·88806	0·48178	- -
29	0·47812	0·53504	0·03551	0·91768	0·42041	- -
Sep. 2	0·53559	0·47742	0·03793	0·94303	0·35712	- -
6	0·58622	0·41371	0·03986	0·96405	0·29220	- -
10	0·62938	0·34472	0·04128	0·98064	0·22593	- -
14	0·66452	0·27134	0·04218	0·99270	0·15862	- -
18	0·69120	0·19451	0·04255	1·00010	0·09056	- -
22	0·70910	0·11521	0·04239	1·00283	−0·02206	- -
26	0·71802	+0·03446	0·04167	1·00082	+0·04657	- -
30	0·71783	−0·04673	0·04043	0·99412	0·11497	- -
Oct. 4	0·70856	0·12734	0·03868	0·98272	0·18278	- -
8	0·69034	0·20634	0·03644	0·96670	0·24971	- -
12	0·66342	0·28275	0·03373	0·94609	0·31548	- -
16	0·62817	0·35560	0·03061	0·92098	0·37976	- -
20	0·58505	0·42398	0·02710	0·89140	0·44223	- -
24	0·53461	0·48705	0·02325	0·85755	0·50259	- -
28	0·47750	0·54405	0·01912	0·81956	0·56050	- -
Nov. 1	0·41443	0·59428	0·01476	0·77764	0·61569	- -
5	0·34621	0·63714	0·01020	0·73194	0·66787	- -
9	0·27370	0·67208	0·00551	0·68270	0·71682	- -
13	0·19779	0·69869	+0·00075	0·63010	0·76229	- -
17	0·11944	0·71667	−0·00402	0·57443	0·80406	- -
21	−0·03962	0·72583	0·00873	0·51591	0·84186	- -
25	+0·04068	0·72606	0·01333	0·45486	0·87553	- -
29	0·12048	0·71735	0·01777	0·39158	0·90483	- -
Dec. 3	0·19880	0·69983	0·02199	0·32638	0·92967	- -
7	0·27469	0·67373	0·02594	- -	- -	- -
11	0·34722	0·63937	0·02958	0·19150	0·96554	- -
15	0·41549	0·59718	0·03285	- -	- -	- -
19	+0·47867	−0·54768	−0·03572	+0·05274	+0·98229	- -

TABLE III.—*continued.*

Date.	Heliocentric Co-ordinates of					
	Venus.			The Earth.		
1835.	x_2	y_2	z_2	x_3	y_3	z_3
Dec. 19	+0 ·47867	—0 ·54768	—0 ·03572	+0 ·05274	+0 ·98229	- -
23	0 ·53598	0 ·49147	0 ·03814	- -	- -	- -
27	0 ·58672	0 ·42923	0 ·04011	—0 ·08707	0 ·97942	- -
31	0 ·63027	0 ·36173	0 ·04159	- -	- -	- -
1836.						
Jan. 4	0 ·66610	0 ·28980	0 ·04257	0 ·22505	0 ·95711	- -
8	0 ·69376	0 ·21432	0 ·04300	- -	- -	- -
12	0 ·71290	0 ·13621	0 ·04291	0 ·35860	0 ·91584	- -
16	0 ·72326	—0 ·05641	0 ·04229	- -	- -	- -
20	0 ·72470	+0 ·02408	0 ·04115	0 ·48511	0 ·85631	- -
28	0 ·70084	0 ·18316	0 ·03739	0 ·60191	0 ·77976	- -
Feb. 5	0 ·64230	0 ·33320	0 ·03176	0 ·70677	0 ·68788	- -
13	0 ·55190	0 ·46670	0 ·02457	0 ·79781	0 ·58241	- -
21	0 ·43399	0 ·57690	0 ·01613	0 ·87315	0 ·46544	- -
29	0 ·29435	0 ·65822	—0 ·00690	0 ·93141	0 ·33944	- -
Mar. 8	+0 ·13990	0 ·70643	+0 ·00268	0 ·97165	0 ·20691	- -
16	—0 ·02161	0 ·71899	0 ·01214	0 ·99316	+0 ·07032	- -
24	0 ·18203	0 ·69510	0 ·02097	0 ·99561	—0 ·06767	- -
Apr. 1	—0 ·33317	+0 ·63592	+0 ·02873	—0 ·97911	—0 ·20428	- -

Date.	Mars.			Jupiter.		
1835.	x_4	y_4	z_4	x_5	y_5	z_5
Aug. 5	—1 ·54181	—0 ·48934	+0 ·02664	+0 ·15810	+5 ·13050	—0 ·02150
13	1 ·49982	0 ·58556	0 ·02356	- -	- -	- -
21	1 ·45104	0 ·67914	0 ·02037	+0 ·03617	5 ·13854	0 ·01868
29	1 ·39553	0 ·76959	0 ·01706	- -	- -	- -
Sep. 6	1 ·33355	0 ·85641	0 ·01369	—0 ·08574	5 ·14375	0 ·01601
14	1 ·26523	0 ·93918	0 ·01027	- -	- -	- -
22	1 ·19078	1 ·01744	0 ·00679	0 ·20774	5 ·14600	0 ·01318
30	1 ·11045	1 ·09064	+0 ·00331	- -	- -	- -
Oct. 8	1 ·02452	1 ·15838	—0 ·00022	0 ·32959	5 ·14553	0 ·01050
16	0 ·93338	1 ·22017	0 ·00375	- -	- -	- -
24	—0 ·83739	—1 ·27562	—0 ·00728	—0 ·45124	+5 ·14209	—0 ·00766

TABLE III.—*continued.*

Date.	Heliocentric Co-ordinates of					
	Mars.			Jupiter.		
1835.	x_4	y_4	z_4	x_5	y_5	z_5
Oct. 24	−0·83739	−1·27562	−0·00728	−0·45124	+5·14209	−0·00766
Nov. 1	0·73690	1·32422	0·01076	- -	- -	- -
5	0·68512	1·34586	0·01246	- -	- -	- -
9	0·63241	1·36565	0·01414	0·57261	5·13594	0·00496
13	0·57881	1·38354	0·01582	- -	- -	- -
17	0·52441	1·39949	0·01748	0·63317	5·13181	0·00353
21	0·46928	1·41346	0·01912	- -	- -	- -
25	0·41348	1·45241	0·02072	0·69365	5·12696	0·00211
29	0·35706	1·43526	0·02230	- -	- -	- -
Dec. 3	0·30011	1·44301	0·02384	0·75409	5·12135	−0·00075
7	0·24274	1·44867	0·02536	- -	- -	- -
11	0·18500	1·45218	0·02683	0·81442	5·11505	+0·00060
15	0·12697	1·45347	0·02824	- -	- -	- -
19	0·06875	1·45255	0·02961	- -	- -	- -
23	−0·01043	1·44941	0·03095	- -	- -	- -
27	+0·04790	1·44401	0·03224	0·93457	5·10047	0·00347
31	0·10618	1·43632	0·03348	- -	- -	- -
1836.						
Jan. 4	0·16428	1·42636	0·03467	- -	- -	- -
8	0·22210	1·41414	0·03580	- -	- -	- -
12	0·27955	1·39965	0·03688	1·05436	5·08323	0·00619
20	0·39305	1·36386	0·03886	- -	- -	- -
28	0·50394	1·31899	0·04055	1·17355	5·06302	0·00892
Feb. 5	0·61145	1·26532	0·04199	- -	- -	- -
13	0·71474	1·20298	0·04316	1·29205	5·04011	0·01181
21	0·81319	1·13235	0·04401	- -	- -	- -
29	0·90590	1·05385	0·04452	1·41000	5·01453	0·01455
Mar. 8	0·99222	0·96792	0·04477	- -	- -	- -
16	1·07152	0·87512	0·04469	1·52700	4·98632	0·01729
24	1·14319	0·77609	0·04429	- -	- -	- -
Apr. 1	+1·20670	−0·67151	−0·04356	−1·64327	+4·95541	+0·02005

TABLE III.—*continued.*

Date.	Heliocentric Co-ordinates of					
	Saturn.			The Georgian.		
1835.	x_6	y_6	z_6	x_7	y_7	z_7
Aug. 5	−8 ·89774	−3 ·88947	+0 ·42209	+17 ·0573	−10 ·4978	−0 ·26210
21	8 ·86645	3 ·97109	0 ·42219	17 ·0899	10 ·4472	0 ·26231
Sep. 6	8 ·83446	4 ·05257	0 ·42224	17 ·1214	10 ·3966	0 ·26251
22	8 ·80195	4 ·13352	0 ·42226	17 ·1538	10 ·3452	0 ·26272
Oct. 8	8 ·76859	4 ·21415	0 ·42227	17 ·1854	10 ·2941	0 ·26292
24	8 ·73474	4 ·29447	0 ·42222	17 ·2171	10 ·2431	0 ·26313
Nov. 1	8 ·71738	4 ·33460	0 ·42218	- -	- -	- -
9	8 ·69981	4 ·37462	0 ·42213	17 ·2484	10 ·1920	0 ·26334
17	8 ·68210	4 ·41440	0 ·42206	- -	- -	- -
25	8 ·66423	4 ·45410	0 ·42199	17 ·2795	10 ·1405	0 ·26354
Dec. 11	8 ·62800	4 ·53367	0 ·42180	17 ·3101	10 ·0890	0 ·26374
27	8 ·59112	4 ·61233	0 ·42167	17 ·3412	10 ·0370	0 ·26395
1836.						
Jan. 12	8 ·55342	4 ·69105	0 ·42142	17 ·3724	9 ·9860	0 ·26416
28	8 ·51530	4 ·76947	0 ·42115	17 ·4028	9 ·9344	0 ·26436
Feb. 13	8 ·47618	4 ·84719	0 ·42086	17 ·4333	9 ·8828	0 ·26457
29	8 ·43665	4 ·92436	0 ·42057	17 ·4639	9 ·8308	0 ·26478
Mar. 16	8 ·39615	5 ·00138	0 ·42018	17 ·4936	9 ·7787	0 ·26488
Apr. 1	−8 ·35526	−5 ·07797	+0 ·41981	+17 ·5235	− 9 ·7268	−0 ·26509

TABLE IV.

Containing, for Greenwich Mean Noon of each fourth day, from Aug. 1, 1835, to April 1, 1836, the united Effects (A) of the attractions of the disturbing Planets upon the Comet in the direction of the co-ordinate x, expressed in 10,000,000,000th parts of an unit, and distinguishing the separate Effect of each Planet.

Date.	Mercury.	Venus.	Earth.	Mars.	Jupiter.	Saturn.	Georgian.	A.
1835.								
Aug. 1	+36848	+ 11611	+ 17620	—1116	+196320	— 8558	+ 97	+252822
5	42145	6705	19032	1104	190031	8398	90	248501
9	41413	+ 1537	20367	1091	183562	8232	83	237639
13	30642	— 3781	21616	1077	176930	8061	76	216345
17	+10536	9111	22773	1061	170155	7883	69	185478
21	—11180	14318	23831	1042	163284	7700	62	152937
25	26320	19277	24780	1021	156341	7511	55	127047
29	32794	23886	25610	998	149332	7314	48	109998
Sep. 2	33032	28053	26301	972	142265	7109	41	99441
6	30023	31713	26831	943	135164	6894	33	92455
10	25728	34806	27158	911	128052	6671	25	87119
14	21124	37288	27207	876	120921	6437	17	82420
18	16592	39123	26818	837	113761	6190	9	77846
22	12213	40286	25627	795	106585	5930	+ 1	72989
26	7923	40762	22652	748	99401	5657	— 7	66956
30	— 3584	40530	+ 14783	696	92202	5367	15	56793
Oct. 4	+ 996	39585	— 10274	639	84976	5056	23	+ 30395
8	6069	37916	110218	575	77723	4723	30	— 69670
12	11984	35512	356598	503	70435	4368	37	314599
16	19294	32346	185594	421	63116	3986	43	—139980
20	29065	28372	41067	327	55773	3571	50	+ 11451
24	43494	23494	— 2076	219	48367	3118	58	62896
28	61638	17522	+ 9898	— 93	40863	2624	72	92088
Nov. 1	62110	10080	14024	+ 57	33326	2086	88	97263
5	49829	— 397	15260	239	25834	1500	101	89164
9	34105	+ 13233	15201	459	18374	866	108	80398
13	+12161	34958	14462	724	10939	— 188	102	72954
17	—10130	75992	13318	1026	+ 3631	+ 517	90	84264
21	25236	163607	11907	1329	— 3453	1239	75	149318
25	31375	+ 95881	10307	1526	10273	1962	61	+ 67967
29	31111	—379845	8563	1404	16801	2668	52	—415174
Dec. 3	27607	159455	6708	+ 721	23033	3354	44	199356
7	23508	64310	4761	— 487	28985	4011	36	108554
11	22283	24629	2742	1721	34681	4635	28	75965
15	27377	— 3809	+ 662	2466	40156	5217	19	67948
19	—29934	+ 9163	— 1466	—2652	— 45427	+ 5760	— 10	— 64566

TABLE IV.—*continued.*

Date.	Mercury.	Venus.	Earth.	Mars.	Jupiter.	Saturn.	Georgian.	A.
1835.								
Dec. 19	−29934	+ 9163	− 1466	−2652	− 45427	+ 5760	− 10	− 64566
23	23121	18202	3630	2486	50516	6266	− 1	55286
27	14508	24939	5822	2172	55457	6737	+ 8	46275
31	7112	30130	8033	1825	60285	7174	17	39934
1836.								
Jan. 4	− 583	34157	10251	1495	65003	7577	26	35572
8	+ 5756	37230	12468	1198	69614	7948	35	32311
12	12401	39462	14686	935	74141	8288	44	29567
16	19643	40921	16909	702	78607	8598	52	27004
20	27431	41656	19129	495	83013	8879	60	24611
24	35007	41697	21339	309	87360	9132	68	23104
28	40235	41074	23544	− 140	91660	9359	76	24600
Feb. 1	39220	39809	25748	+ 14	95926	9562	84	32985
5	28021	37932	27954	157	100160	9740	91	52173
9	+ 7558	35473	30160	290	104363	9894	99	81209
13	−14278	32469	32359	413	108544	10024	106	112169
17	29373	28959	34531	529	112709	10132	113	136880
21	35787	24988	36635	638	116860	10217	120	153319
25	35984	20606	38612	741	120999	10281	127	163840
29	32924	15869	40369	838	125129	10323	134	171258
Mar. 4	28534	10837	41812	930	129256	10345	141	177349
8	23802	5574	42839	1017	133382	10347	147	182938
12	19136	+ 149	43370	1098	137508	10229	153	188385
16	14651	− 5367	43386	1174	141637	10293	159	193415
20	10313	10901	42902	1245	145769	10239	165	198236
24	6007	16378	41990	1312	149910	10167	171	202635
28	− 1576	21724	40748	1374	154070	10078	177	206489
Apr. 1	+ 3162	− 26863	− 39257	+1430	−158248	+ 9972	+182	−209622

TABLE V.

Containing, for Greenwich Mean Noon of each fourth day, from August 1, 1835, to April 1, 1836, the united Effects (B) of the attractions of the disturbing Planets upon the Comet in the direction of the co-ordinate y, expressed in 10,000,000,000th parts of an unit, and distinguishing the separate Effect of each Planet.

Date.	Mercury.	Venus.	Earth.	Mars.	Jupiter.	Saturn.	Georgian.	B.
1835.								
Aug. 1	— 5216	+ 57349	— 17205	— 171	—406607	— 1181	+100	—372931
5	+ 8207	56610	15626	214	374253	1251	101	326426
9	25951	55158	13876	257	343563	1320	102	277805
13	43508	53098	11937	299	314472	1389	103	231388
17	52882	50506	9784	341	286878	1458	103	194970
21	49407	47440	7386	383	260718	1526	104	173062
25	36660	43938	4688	424	235887	1594	104	161891
29	21620	40039	— 1616	464	212329	1662	104	154308
Sep. 2	+ 8683	35776	+ 1945	504	189992	1729	104	145717
6	— 988	31188	6180	543	168775	1794	103	134629
10	7785	26320	11395	582	148579	1858	103	120986
14	12466	21226	18102	617	129376	1921	102	104950
18	15689	15966	27247	651	111145	1981	101	86152
22	17922	10602	40670	684	93796	2038	100	63068
26	19462	+ 5207	62251	714	77242	2094	99	— 31955
30	20468	— 150	101095	741	61494	2147	97	+ 16192
Oct. 4	20975	5395	179393	765	46574	2194	94	103584
8	20893	10450	322598	785	32398	2235	91	255928
12	19957	15237	+142675	799	18893	2269	87	+ 85607
16	17648	19669	—270096	804	— 6133	2294	82	—316562
20	13162	23651	169564	800	+ 5794	2308	77	203614
24	6516	27069	88291	784	16937	2306	72	107957
28	4848	29767	47178	747	27331	2283	70	57422
Nov. 1	— 6950	31491	24781	684	36851	2234	69	— 29220
5	+11045	31769	11301	581	45367	2154	66	+ 10673
9	32602	29539	— 2483	415	52783	2034	61	50975
13	44395	— 21919	+ 3658	— 154	59022	1868	47	83181
17	42131	+ 1103	8144	+ 256	64012	1652	29	114023
21	29800	87166	11528	886	67726	1386	+ 11	195731
25	14477	407564	14142	1803	70263	1074	— 6	507169
29	+ 395	+128198	16181	2975	71763	720	19	218773
Dec. 3	—11754	— 31315	17773	4115	72380	— 330	29	50840
7	23274	46031	18996	4662	72280	+ 90	38	26685
11	35285	45337	19911	4269	71616	531	47	15658
15	42472	42287	20547	3216	70524	986	56	10458
19	—36628	— 38598	+ 20932	+2034	+ 69034	+ 1454	— 64	+ 18164

TABLE V.—*continued.*

Date.	Mercury.	Venus.	Earth.	Mars.	Jupiter.	Saturn.	Georgian.	B.
1835.								
Dec. 19	−36628	− 38598	+ 20932	+2034	+ 69034	+ 1454	− 64	+ 18164
23	28757	34561	21077	1034	67152	1936	72	27809
27	25462	30235	20999	+ 288	65009	2426	79	32946
31	24555	25650	20706	− 244	62721	2919	86	35811
1836.								
Jan. 4	24285	20837	20205	619	60267	3416	93	38054
8	23845	15830	19502	884	57617	3917	99	40378
12	22682	10679	18605	1073	54823	4418	105	43307
16	20136	5431	17524	1208	51927	4919	110	47485
20	15257	− 140	16268	1305	48925	5419	115	53795
24	− 6764	+ 5132	14854	1372	45807	5918	120	63455
28	+ 6499	10328	13298	1418	42593	6416	124	77592
Feb. 1	24053	15388	11624	1447	39302	6911	128	95703
5	41413	20250	9862	1462	35930	7403	132	113264
9	50646	24855	8052	1466	32473	7891	136	122315
13	47144	29143	6249	1459	28938	8375	140	118250
17	34506	33058	4513	1444	25330	8854	144	104673
21	19663	36547	2916	1422	21650	9327	147	88534
25	+ 6965	39566	1520	1394	17896	9795	150	74198
29	− 2462	42071	+ 373	1358	14072	10256	153	62799
Mar. 4	9054	44027	− 506	1317	10180	10710	157	53883
8	13600	45405	1156	1271	6220	11156	160	46594
12	16773	46185	1643	1220	+ 2193	11595	163	40174
16	19047	46354	2078	1165	− 1904	12025	166	34019
20	20714	45911	2582	1106	6074	12446	169	27712
24	21939	44862	3215	1043	10307	12859	171	21046
28	22788	43225	4039	977	14604	13263	173	13907
Apr. 1	−23220	+ 41025	− 5116	− 908	− 18965	+13657	−175	+ 6298

TABLE VI.

Containing, for Greenwich Mean Noon of each fourth day, from August 1, 1835, to April 1, 1836, the united Effects (C) of the attractions of the disturbing Planets upon the Comet in the direction of the co-ordinate z, expressed in 10,000,000,000th parts of an unit, and distinguishing the separate Effect of each Planet.

Date.	Mercury.	Venus.	Earth.	Mars.	Jupiter.	Saturn.	Georgian.	C.
1835.								
Aug. 1	— 3800	— 57	— 98	+ 15	— 9965	+ 383	— 4	— 13526
5	3143	+ 342	89	16	6842	400	3	9319
9	— 1577	740	74	16	4037	417	2	— 4517
13	+ 877	1123	54	16	— 1535	435	— 1	+ 861
17	3484	1482	— 24	16	+ 679	453	0	6090
21	5149	1810	+ 19	16	2639	472	+ 1	10106
25	5441	2104	84	17	4378	491	2	12517
29	4761	2361	181	18	5906	509	3	13739
Sep. 2	3701	2578	332	18	7231	527	4	14391
6	2629	2758	571	19	8379	545	5	14906
10	1686	2899	966	20	9377	563	6	15517
14	896	3004	1649	21	10228	581	7	16386
18	+ 242	3073	2902	22	10933	599	8	17779
22	— 307	3109	5394	24	11512	616	9	20357
26	776	3113	10918	26	11982	633	10	25906
30	1182	3092	25176	28	12341	649	11	40115
Oct. 4	1525	3051	70609	32	12585	664	12	85428
8	1783	2998	251704	37	12726	677	13	266372
12	1871	2944	691560	43	12776	689	14	706155
16	1550	2907	373844	50	12726	699	15	388691
20	— 113	2913	113050	59	12569	704	16	129198
24	+ 4445	3006	42511	72	12306	704	16	63060
28	13336	3266	19803	89	11938	699	16	49147
Nov. 1	12114	3847	10738	113	11460	686	16	38974
5	5105	5084	6427	147	10869	663	16	28311
9	3783	7806	4127	192	10159	629	17	26713
13	5027	14496	2785	255	9329	579	16	32487
17	6143	34615	1945	342	8392	514	15	51966
21	6192	119356	1387	457	7364	431	13	135200
25	5417	518654	998	595	6264	333	11	532272
29	4348	285750	711	721	5111	221	9	296871
Dec. 3	3333	48633	489	763	3920	+ 97	7	57242
7	2471	10795	311	656	2706	— 38	4	16905
11	1591	+ 2015	158	426	1477	180	2	+ 6489
15	+ 254	— 841	+ 22	182	+ 240	328	+ 1	— 470
19	— 1140	— 2018	— 105	+ 4	— 1007	— 481	— 1	— 4748

TABLE VI.—*continued.*

Date.	Mercury.	Venus.	Earth.	Mars.	Jupiter.	Saturn.	Georgian.	C.
1835.								
Dec. 19	− 1140	− 2018	− 105	+ 4	− 1007	− 481	− 1	− 4748
23	1768	2594	229	− 101	2270	639	3	7604
27	2057	2913	354	154	3543	800	5	9826
31	2341	3100	485	176	4824	963	7	11896
1836.								
Jan. 4	2676	3208	626	183	6115	1127	9	13944
8	3051	3261	781	180	7422	1293	11	15999
12	3442	3270	955	175	8744	1460	12	18058
16	3803	3238	1153	168	10083	1628	13	20086
20	4039	3178	1381	162	11439	1796	15	22010
24	3966	3087	1645	156	12816	1964	16	23650
28	3295	2957	1951	150	14215	2132	18	24718
Feb. 1	− 1720	2804	2305	144	15637	2300	19	24929
5	+ 725	2625	2715	139	17083	2467	21	24325
9	3287	2423	3182	135	18556	2634	22	23665
13	4868	2201	3707	132	20057	2800	23	24052
17	5056	1961	4280	129	21588	2965	24	25891
21	4279	1709	4882	126	23151	3129	26	28744
25	3146	1448	5483	123	24747	3292	27	31974
29	2040	1183	6034	121	26377	3453	28	35156
Mar. 4	1105	918	6488	118	28043	3612	29	38103
8	+ 351	657	6792	116	29747	3770	30	40761
12	− 260	406	6907	114	31490	3926	31	43134
16	775	− 169	6825	112	33275	4079	32	45267
20	1233	+ 48	6558	110	35104	4230	33	47220
24	1661	242	6144	108	36977	4379	34	49061
28	2080	410	5634	106	38896	4525	34	50865
Apr. 1	− 2500	+ 549	− 5076	− 105	− 40863	− 4669	− 35	− 52699

TABLE VII.

Containing, for Greenwich Mean Noon of each fourth day, from August 1, 1835, to April 1, 1836, the Values of A′, B′, C′, expressed in 10,000,000,000th parts of an unit.

Date.	A′	B′	C′
1835.			
Aug. 1	−400192	+379032	+141231
5	301357	358155	128032
9	211518	329654	112257
13	141858	292643	93706
17	102748	252128	74645
21	90639	217317	58853
25	89146	193228	48127
29	84787	177804	41372
Sep. 2	73714	166523	36611
6	57437	156071	32437
10	38417	144762	28140
14	− 18463	131718	23336
18	+ 1635	116138	17587
22	22026	96485	+ 9889
26	43585	69390	− 2335
30	67851	+ 26861	26810
Oct. 4	94611	− 50570	91816
8	+ 91421	182278	315760
12	−125628	− 1588	766170
16	−128719	+356292	349965
20	+ 544	226084	84652
24	52187	119597	25473
28	75176	54565	13733
Nov. 1	72601	+ 8246	7670
5	52487	− 44330	6502
9	+ 26453	87584	14205
13	− 557	112055	27186
17	23152	137530	48286
21	61543	229148	125518
25	233150	−262744	578345
29	− 90035	+329951	424816
Dec. 3	+ 1952	181003	113288
7	+ 11485	+101222	− 47929

Date.	A′	B′	C′
1835.			
Dec. 7	+ 11485	+101222	− 47929
11	17539	69220	27928
15	24926	59426	18386
19	24389	59864	14810
23	17936	58438	11451
27	13211	55640	7974
31	10821	53820	4916
1836.			
Jan. 4	9730	53215	− 2265
8	9118	53611	+ 103
12	8320	55031	2240
16	6727	57842	4084
20	3923	62825	5414
24	+ 317	71329	5668
28	− 1407	84977	+ 3844
Feb. 1	+ 5544	104188	− 1213
5	31087	125261	9655
9	79727	139831	19132
13	141938	141288	25805
17	201580	131390	27870
21	250151	116780	26453
25	288374	102408	23510
29	320097	90076	20348
Mar. 4	348516	79732	17511
8	375450	70725	15107
12	401858	62406	13092
16	427372	54192	11245
20	452446	45735	9491
24	476649	36849	7676
28	499672	27466	5678
Apr. 1	+520954	+ 17609	− 3389

TABLE VIII.

Containing the Variations of the Elements of the Comet's Orbit for each interval of four days, between the Noon of July 30, 1835, and the Noon of April 3, 1836, Greenwich Mean Time; the tabular date being the middle of each interval.

*** The figure in a parenthesis, at the head of a column, indicates the number of cyphers to be prefixed to all the values in that column.

Date.	[a]	[e]	[ν]	[i]	[ϖ]	[ε]
1835.	0·(2)	0·(5)	″	″	″	″
Aug. 1	−16336	−26591	+0·11876	−0·37360	−1·60324	−0·91704
5	14500	23503	0·08438	0·32982	1·47658	0·86797
9	12507	20180	0·05353	0·28125	1·32495	0·80042
13	10549	16974	0·02757	0·22796	1·15008	0·70786
17	08993	14511	+0·00832	0·17603	0·97468	0·59770
21	08075	13152	−0·00420	0·13429	0·83170	0·49304
25	07645	12589	0·01224	0·10604	0·73334	0·41255
29	07377	12253	0·01807	0·08781	0·66679	0·35717
Sep. 2	07047	11770	0·02266	0·07467	0·61397	0·31813
6	06578	11022	0·02595	0·06337	0·56339	0·28726
10	05958	09999	0·02758	0·05248	0·51025	0·26016
14	05188	08707	0·02704	0·04139	0·45250	0·23475
18	04243	07101	0·02348	0·02952	0·38839	0·21037
22	03028	05018	−0·01492	−0·01563	0·31387	0·18683
26	−01341	−02101	+0·00392	+0·00345	0·21975	0·16348
30	+01292	+02486	0·04942	0·03677	−0·08391	0·13809
Oct. 4	05904	10548	0·18382	0·11572	+0·14815	−0·09948
8	+12478	+22003	0·67978	0·36132	0·53807	+0·01473
12	−05187	−09472	1·75710	0·78360	+0·13846	0·35552
16	25208	44683	0·84717	0·31333	−0·82022	0·23627
20	13279	23452	0·21425	0·06448	0·54292	+0·01356
24	05189	09083	0·06673	0·01583	0·32026	−0·10245
28	−00389	−00545	0·03681	0·00652	0·21084	0·19390
Nov. 1	+02334	+04282	0·02076	0·00247	0·13961	0·23658
5	05154	09287	0·01749	0·00107	−0·05269	0·26766
9	07611	13666	0·03728	+0·00005	+0·01254	0·30519
13	09168	16452	0·06805	−0·00426	0·04645	0·33634
17	11410	20483	0·11223	0·01516	0·04872	0·42751
21	19649	35284	0·26254	0·05820	0·02358	0·79611
25	+27384	+49289	1·04830	0·34892	0·29122	−0·85178
29	−21037	−37484	+0·63619	−0·31053	+0·69732	+1·57194

TABLE VIII.—*continued.*

Date.	[a]	[e]	[ν]	[i]	[ϖ]	[ε]
1835.	0 ·(2)	0 ·(5)	″	″	″	″
Nov. 29	−21037	−37484	+0 ·63619	−0 ·31053	+0 ·69732	+1 ·57194
Dec. 3	12683	22616	0 ·13105	0 ·09588	0 ·33595	0 ·86342
7	07103	12645	0 ·03832	0 ·04551	0 ·20249	0 ·49779
11	04997	08886	0 ·01209	0 ·02910	0 ·14724	0 ·35496
15	04475	07954	+0 ·00114	0 ·02068	0 ·13443	0 ·32039
19	04221	07474	−0 ·00458	0 ·01775	0 ·15017	0 ·32548
23	03644	06411	0 ·00778	0 ·01448	0 ·16087	0 ·31196
27	03121	05455	0 ·00833	0 ·01056	0 ·16190	0 ·29140
31	02779	04823	0 ·00692	0 ·00678	0 ·16363	0 ·27892
1836.						
Jan. 4	02567	04425	−0 ·00399	−0 ·00323	0 ·16760	0 ·27406
8	02430	04162	+0 ·00022	+0 ·00015	0 ·17396	0 ·27479
12	02341	03976	0 ·00551	0 ·00339	0 ·18317	0 ·28028
16	02286	03845	0 ·01144	0 ·00633	0 ·19665	0 ·29149
20	02277	03784	0 ·01697	0 ·00858	0 ·21716	0 ·31165
24	02366	03877	0 ·01963	0 ·00916	0 ·24971	0 ·34794
28	02661	04309	+0 ·01456	+0 ·00633	0 ·30101	0 ·41156
Feb. 1	03318	05359	−0 ·00498	−0 ·00203	0 ·37505	0 ·51292
5	04418	07193	0 ·04268	0 ·01639	0 ·46219	0 ·64562
9	05766	09518	0 ·09049	0 ·03292	0 ·53426	0 ·77337
13	06940	11615	0 ·12991	0 ·04496	0 ·56422	0 ·85163
17	07661	12974	0 ·14868	0 ·04911	0 ·55240	0 ·86922
21	07978	13636	0 ·14895	0 ·04711	0 ·51949	0 ·84892
25	08070	13894	0 ·13924	0 ·04227	0 ·48343	0 ·81621
29	08077	13986	0 ·12638	0 ·03691	0 ·45205	0 ·78468
Mar. 4	08065	14032	0 ·11374	0 ·03203	0 ·42626	0 ·75796
8	08053	14070	0 ·10236	0 ·02784	0 ·40421	0 ·73495
12	08044	14113	0 ·09234	0 ·02430	0 ·38386	0 ·71378
16	08018	14129	0 ·08239	0 ·02101	0 ·36283	0 ·69122
20	07978	14123	0 ·07211	0 ·01784	0 ·34001	0 ·66621
24	07914	14083	0 ·06037	0 ·01451	0 ·31449	0 ·63751
28	07822	14001	0 ·04616	0 ·01078	0 ·28596	0 ·60464
Apr. 1	−07698	−13869	−0 ·02844	−0 ·00647	+0 ·25435	+0 ·56728

TABLE IX.

Containing the total amount of Variation of each of the Elements of the Comet's Orbit, on every fourth day, commencing at the Noon of July 30, 1835, and ending at Noon of April 3, 1836, Greenwich Mean Time.

⁎ The figures in the parentheses indicate the number of cyphers between the decimal point and the first significant figure.

Date.	δa	δe	$\delta \nu$	δi	$\delta \varpi$	$\delta \varepsilon$
1835.			″	″	″	″
July 30	0 ·0000000	0 ·00000000	0 ·00	0 ·00	0 ·00	0 ·00
Aug. 3	−0 ·(2) 16329	−0 ·(5) 266	+0 ·13	−0 ·37	− 1 ·60	−0 ·92
7	30822	501	0 ·23	0 ·70	3 ·08	1 ·79
11	43330	703	0 ·30	0 ·98	4 ·40	2 ·58
15	53896	0 ·(5) 873	0 ·34	1 ·20	5 ·56	3 ·29
19	62915	0 ·(4)1018	0 ·36	1 ·37	6 ·53	3 ·89
23	71010	1150	0 ·37	1 ·51	7 ·37	4 ·39
27	78662	1276	0 ·37	1 ·61	8 ·10	4 ·80
31	86036	1398	0 ·36	1 ·70	8 ·77	5 ·16
Sep. 4	93077	1516	0 ·35	1 ·77	9 ·38	5 ·48
8	0 ·(2) 99649	1626	0 ·34	1 ·83	9 ·95	5 ·77
12	0 ·(1)105601	1726	0 ·32	1 ·88	10 ·46	6 ·03
16	110782	1813	0 ·31	1 ·92	10 ·91	6 ·26
20	115014	1884	0 ·30	1 ·94	11 ·30	6 ·47
24	118022	1934	0 ·30	1 ·95	11 ·61	6 ·66
28	119324	1954	0 ·32	1 ·95	11 ·83	6 ·82
Oct. 2	117949	1928	0 ·38	1 ·91	11 ·91	6 ·96
6	111963	1820	0 ·59	1 ·78	11 ·76	7 ·06
10	100495	1619	1 ·31	1 ·41	11 ·25	7 ·04
14	105780	1715	3 ·00	0 ·66	11 ·14	6 ·70
18	129657	2138	3 ·87	0 ·33	11 ·91	6 ·47
22	143096	2376	4 ·12	0 ·26	12 ·46	6 ·45
26	148422	2469	4 ·20	0 ·24	12 ·78	6 ·56
30	148898	2476	4 ·25	0 ·23	12 ·99	6 ·75
Nov. 3	146560	2433	4 ·29	0 ·22	13 ·13	6 ·98
7	141421	2340	4 ·32	0 ·22	13 ·19	7 ·25
11	133848	2204	4 ·37	0 ·22	13 ·18	7 ·56
15	124652	2039	4 ·45	0 ·22	13 ·13	7 ·90
19	0 ·(1)112992	1830	4 ·58	0 ·23	13 ·09	8 ·34
23	0 ·(2) 93364	1477	4 ·88	0 ·30	13 ·05	9 ·12
27	−0 ·(2) 68320	−0 ·(4)1027	+5 ·89	−0 ·63	−12 ·75	−9 ·87

TABLE IX—*continued.*

Date.	δa	δe	$\delta \nu$	δi	$\delta \varpi$	$\delta \varepsilon$
1835.			″	″	″	″
Nov. 27	−0 ·(2) 68320	−0 ·(4)1027	+5 ·89	−0 ·63	−12 ·75	−9 ·87
Dec. 1	86991	1359	6 ·54	0 ·93	12 ·09	8 ·43
5	0 ·(2) 99790	1587	6 ·70	1 ·03	11 ·75	7 ·55
9	0 ·(1)107038	1716	6 ·75	1 ·07	11 ·54	7 ·05
13	112101	1806	6 ·78	1 ·10	11 ·39	6 ·69
17	116587	1886	6 ·79	1 ·12	11 ·26	6 ·37
21	120795	1961	6 ·80	1 ·13	11 ·11	6 ·04
25	124441	2025	6 ·80	1 ·15	10 ·95	5 ·73
29	127570	2079	6 ·81	1 ·15	10 ·79	5 ·44
1836.						
Jan. 2	130354	2128	6 ·81	1 ·16	10 ·62	5 ·16
6	132924	2172	6 ·82	1 ·16	10 ·46	4 ·89
10	135356	2214	6 ·84	1 ·16	10 ·28	4 ·61
14	137698	2253	6 ·85	1 ·15	10 ·10	4 ·33
18	139986	2292	6 ·88	1 ·14	9 ·91	4 ·04
22	142267	2330	6 ·91	1 ·13	9 ·69	3 ·73
26	144642	2369	6 ·94	1 ·12	9 ·44	3 ·38
30	147318	2412	6 ·97	1 ·11	9 ·14	2 ·97
Feb. 3	150654	2466	6 ·98	1 ·11	8 ·76	2 ·46
7	155082	2538	6 ·94	1 ·12	8 ·30	1 ·81
11	160841	2633	6 ·87	1 ·15	7 ·77	1 ·04
15	167762	2749	6 ·75	1 ·19	7 ·21	−0 ·19
19	175406	2879	6 ·61	1 ·24	6 ·66	+0 ·67
23	183375	3015	6 ·48	1 ·28	6 ·14	1 ·52
27	191441	3154	6 ·35	1 ·32	5 ·65	2 ·34
Mar. 2	199517	3293	6 ·24	1 ·36	5 ·20	3 ·12
6	207582	3434	6 ·14	1 ·38	4 ·78	3 ·88
10	215635	3574	6 ·05	1 ·41	4 ·37	4 ·61
14	223678	3716	5 ·97	1 ·43	3 ·99	5 ·33
18	231695	3857	5 ·90	1 ·45	3 ·63	6 ·02
22	239672	3998	5 ·84	1 ·46	3 ·29	6 ·68
26	247585	4139	5 ·79	1 ·48	2 ·98	7 ·32
30	255406	4279	5 ·76	1 ·48	2 ·69	7 ·92
Apr. 3	−0 ·(1)263103	−0 ·(4)4418	+5 ·74	−1 ·49	− 2 ·44	+8 ·49

TABLE X.

Containing, The *Apparent* Right Ascension and Declination, and the Logarithm of the *True* Distance from the Earth, of HALLEY's Comet, from August 1·5, 1835, to March 31·5, 1836, Mean Time at Greenwich, deduced from approximate Elements of its orbit, on the supposition that those Elements continued invariable during the interval: and the Perturbations in Right Ascension and Declination produced by the disturbing Planets, on the assumption that the approximate Elements represent the actual orbit in which the Comet was moving at Mean Noon at Greenwich on July 30, 1835.

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1835.	° ′ ″	″	° ′ ″	″	
Aug. 1·5	81 21 56·0	− 0·2	+21 55 40·8	0·0	0·40743
2·5	81 33 55·2	0·3	21 59 49·1	0·0	0·40252
3·5	81 45 55·5	0·3	22 4 1·0	0·0	0·39750
4·5	81 57 56·7	− 0·3	22 8 16·8	0·0	0·39239
5·5	82 9 58·8	0·3	22 12 36·7	0·0	0·38717
6·5	82 22 2·1	0·4	22 17 1·0	0·0	0·38186
7·5	82 34 6·3	− 0·4	22 21 29·9	0·0	0·37643
8·5	82 46 11·8	0·2	22 26 3·7	0·0	0·37090
9·5	82 58 18·4	0·0	22 30 42·6	0·0	0·36525
10·5	83 10 26·3	+ 0·1	22 35 27·1	0·0	0·35949
11·5	83 22 35·7	0·2	22 40 17·3	0·0	0·35361
12·5	83 34 46·4	0·2	22 45 13·8	0·0	0·34760
13·5	83 46 58·7	+ 0·3	22 50 16·5	+ 0·1	0·34147
14·5	83 59 12·7	0·3	22 55 26·3	0·1	0·33521
15·5	84 11 28·6	0·4	23 0 43·3	0·1	0·32881
16·5	84 23 46·3	+ 0·4	23 6 8·1	+ 0·1	0·32228
17·5	84 36 6·0	0·5	23 11 41·3	0·2	0·31560
18·5	84 48 27·7	0·6	23 17 23·0	0·2	0·30878
19·5	85 0 51·8	+ 0·6	23 23 14·0	0·3	0·30180
20·5	85 13 18·4	0·6	23 29 15·0	0·2	0·29467
21·5	85 25 47·6	0·6	23 35 26·4	+ 0·1	0·28737
22·5	85 38 19·4	+ 0·6	23 41 48·9	+ 0·0	0·27991
23·5	85 50 54·7	0·7	23 48 23·6	− 0·1	0·27226
24·5	86 3 33·4	0·7	23 55 10·5	− 0·1	0·26444
25·5	86 16 15·7	+ 0·7	24 2 11·2	− 0·0	0·25643
26·5	86 29 2·3	0·7	24 9 26·2	+ 0·1	0·24823
27·5	86 41 53·4	0·8	24 16 56·8	0·1	0·23982
28·5	86 54 49·8	+ 0·8	24 24 43·6	0·1	0·23120
29·5	87 7 51·9	0·9	24 32 48·1	0·2	0·22237
30·5	87 21 0·4	0·9	24 41 11·5	0·3	0·21330
31·5	87 34 16·1	+ 1·0	+24 49 55·3	+ 0·3	0·20400

TABLE X.—*continued.*

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1835.	° ′ ″	″	° ′ ″	″	
Aug. 31 ·5	87 34 16 ·1	+ 1 ·0	+24 49 55 ·3	+ 0 ·3	0 ·20400
Sep. 1 ·5	87 47 39 ·7	1 ·0	24 59 0 ·9	0 ·3	0 ·19445
2 ·5	88 1 12 ·4	1 ·1	25 8 29 ·9	0 ·3	0 ·18463
3 ·5	88 14 54 ·8	+ 1 ·1	25 18 24 ·3	+ 0 ·3	0 ·17455
4 ·5	88 28 48 ·7	1 ·2	25 28 46 ·2	0 ·3	0 ·16418
5 ·5	88 42 54 ·9	1 ·2	25 39 37 ·0	0 ·2	0 ·15351
6 ·5	88 57 15 ·4	+ 1 ·2	25 51 0 ·1	+ 0 ·1	0 ·14253
7 ·5	89 11 51 ·6	1 ·2	26 2 57 ·9	0 ·0	0 ·13121
8 ·5	89 26 45 ·6	1 ·3	26 15 33 ·4	— 0 ·1	0 ·11955
9 ·5	89 41 59 ·3	+ 1 ·3	26 28 49 ·6	0 ·0	0 ·10752
10 ·5	89 57 35 ·4	1 ·4	26 42 50 ·8	+ 0 ·1	0 ·09509
11 ·5	90 13 36 ·7	1 ·5	26 57 41 ·0	0 ·2	0 ·08226
12 ·5	90 30 6 ·5	+ 1 ·6	27 13 24 ·6	+ 0 ·4	0 ·06900
13 ·5	90 47 8 ·2	1 ·7	27 30 7 ·2	0 ·3	0 ·05527
14 ·5	91 4 46 ·6	1 ·8	27 47 55 ·1	0 ·2	0 ·04105
15 ·5	91 23 6 ·7	+ 1 ·9	28 6 54 ·7	+ 0 ·1	0 ·02630
16 ·5	91 42 13 ·9	2 ·0	28 27 14 ·3	0 ·0	0 ·01100
17 ·5	92 2 15 ·0	2 ·1	28 49 2 ·6	— 0 ·1	9 ·99509
18 ·5	92 23 18 ·9	+ 2 ·3	29 12 30 ·3	— 0 ·1	9 ·97855
19 ·5	92 45 34 ·8	2 ·4	29 37 49 ·2	0 ·2	9 ·96131
20 ·5	93 9 14 ·2	2 ·6	30 5 13 ·1	0 ·2	9 ·94334
21 ·5	93 34 31 ·6	+ 2 ·8	30 34 59 ·0	— 0 ·2	9 ·92456
22 ·5	94 1 43 ·5	2 ·9	31 7 25 ·5	0 ·3	9 ·90492
23 ·5	94 31 11 ·8	3 ·1	31 42 56 ·7	0 ·4	9 ·88433
24 ·5	95 3 21 ·7	+ 3 ·3	32 21 58 ·2	— 0 ·4	9 ·86273
25 ·5	95 38 46 ·9	3 ·6	33 5 4 ·0	0 ·6	9 ·84000
26 ·5	96 18 8 ·7	3 ·8	33 52 53 ·4	0 ·8	9 ·81606
27 ·5	97 2 20 ·2	+ 4 ·1	34 46 13 ·2	— 1 ·1	9 ·79078
28 ·5	97 52 32 ·8	4 ·4	35 46 4 ·5	1 ·4	9 ·76403
29 ·5	98 50 19 ·0	4 ·4	36 53 37 ·6	1 ·8	9 ·73566
30 ·5	99 57 45 ·9	+ 4 ·2	38 10 22 ·2	— 2 ·2	9 ·70552
Oct. 1 ·5	101 17 50 ·6	3 ·9	39 38 10 ·3	2 ·7	9 ·67343
1 ·75	101 40 17 ·1	3 ·8	40 2 6 ·2	2 ·9	9 ·66508
2 ·0	102 3 51 ·5	+ 3 ·7	40 26 54 ·5	— 3 ·1	9 ·65660
2 ·25	102 28 38 ·3	3 ·7	40 52 37 ·5	3 ·3	9 ·64797
2 ·5	102 54 44 ·1	3 ·6	41 19 18 ·2	3 ·5	9 ·63921
2 ·75	103 22 14 ·9	+ 3 ·7	+41 46 58 ·9	— 3 ·5	9 ·63030

TABLE X.—*continued.*

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1835.	° ′ ″	″	° ′ ″	″	
Oct. 2·75	103 22 14·9	+ 3·7	+41 46 58·9	− 3·5	9·63030
3·0	103 51 18·8	3·9	42 15 43·0	3·6	9·62125
3·25	104 22 3·3	4·2	42 45 33·3	3·6	9·61204
3·5	104 54 38·1	+ 4·5	43 16 33·2	− 3·7	9·60269
3·75	105 29 12·6	4·8	43 48 45·8	3·7	9·59318
4·0	106 5 59·1	5·1	44 22 14·6	3·8	9·58352
4·25	106 45 9·7	+ 5·4	44 57 2·8	− 3·9	9·57370
4·5	107 26 58·9	5·9	45 33 14·0	4·1	9·56373
4·75	108 11 42·3	6·7	46 10 51·2	4·3	9·55360
5·0	108 59 39·4	+ 7·4	46 49 58·1	− 4·5	9·54331
5·25	109 51 10·2	7·9	47 30 37·7	4·7	9·53288
5·5	110 46 39·7	8·4	48 12 53·4	4·9	9·52229
5·75	111 46 34·7	+ 8·9	48 56 47·6	− 5·1	9·51156
6·0	112 51 26·8	9·3	49 42 22·2	5·4	9·50068
6·125	113 25 54·8	9·5	50 5 47·5	5·5	9·49519
6·25	114 1 50·6	+ 9·6	50 29 37·9	− 5·7	9·48967
6·375	114 39 20·0	9·7	50 53 53·8	5·8	9·48412
6·5	115 18 27·7	9·8	51 18 34·7	6·0	9·47854
6·625	115 59 20·1	+ 9·7	51 43 40·5	− 6·2	9·47293
6·75	116 42 4·2	9·6	52 9 10·8	6·4	9·46729
6·875	117 26 47·0	9·5	52 35 4·9	6·6	9·46163
7·0	118 13 35·4	+ 9·4	53 1 22·4	− 6·8	9·45595
7·125	119 2 37·8	9·2	53 28 2·3	7·0	9·45024
7·25	119 54 2·8	8·9	53 55 3·6	7·3	9·44452
7·375	120 48 0·0	+ 8·5	54 22 24·8	− 7·5	9·43878
7·5	121 44 38·8	8·1	54 50 4·6	7·8	9·43304
7·625	122 44 9·8	7·6	55 18 1·0	8·1	9·42728
7·75	123 46 44·3	+ 7·1	55 46 11·6	− 8·5	9·42152
7·875	124 52 34·5	6·5	56 14 33·8	8·9	9·41575
8·0	126 1 52·6	5·9	56 43 4·6	9·3	9·40999
8·125	127 14 51·9	+ 5·2	57 11 40·3	− 9·6	9·40424
8·25	128 31 46·1	4·5	57 40 16·8	10·0	9·39850
8·375	129 52 50·1	3·7	58 8 49·3	10·3	9·39278
8·5	131 18 19·0	+ 2·9	58 37 12·4	−10·6	9·38708
8·625	132 48 27·5	2·0	59 5 20·1	10·9	9·38141
8·75	134 23 31·7	+ 1·0	59 33 5·6	11·1	9·37578
8·875	136 3 47·3	− 0·1	+60 0 20·8	−11·3	9·37019

TABLE X.—*continued.*

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1835.	° ′ ″	″	° ′ ″	″	
Oct. 8·875	136 3 47·3	− 0·1	+60 0 20·8	−11·3	9·37019
9·0	137 49 29·2	1·2	60 26 57·1	11·5	9·36465
9·125	139 40 51·9	2·4	60 52 45·1	11·7	9·35916
9·25	141 38 8·3	− 3·7	61 17 34·1	−11·8	9·35375
9·375	143 41 29·6	5·2	61 41 12·6	11·9	9·34841
9·5	145 51 4·5	6·8	62 3 27·9	12·0	9·34316
9·625	148 6 58·2	− 8·7	62 24 6·6	−12·0	9·33800
9·75	150 29 12·1	10·9	62 42 54·3	11·9	9·33295
9·875	152 57 41·7	13·2	62 59 36·0	11·8	9·32801
10·0	155 32 17·6	−15·5	63 13 55·9	−11·7	9·32320
10·125	158 12 42·3	17·8	63 25 38·5	11·5	9·31853
10·25	160 58 32·1	19·9	63 34 27·9	11·2	9·31401
10·375	163 49 14·9	−21·8	63 40 9·0	−11·0	9·30964
10·5	166 44 11·9	23·5	63 42 26·9	10·7	9·30545
10·625	169 42 36·5	24·9	63 41 8·4	10·2	9·30144
10·75	172 43 34·9	−26·1	63 36 1·6	− 9·7	9·29764
10·875	175 46 9·8	27·2	63 26 57·3	9·2	9·29404
11·0	178 49 19·3	28·1	63 13 48·0	8·6	9·29066
11·125	181 52 2·0	−28·9	62 56 29·0	− 8·1	9·28751
11·25	184 53 16·5	29·5	62 34 58·9	7·6	9·28460
11·375	187 52 4·2	29·9	62 9 18·8	7·2	9·28196
11·5	190 47 30·9	−30·2	61 39 32·4	− 6·8	9·27957
11·625	193 38 49·1	30·2	61 5 46·0	6·5	9·27746
11·75	196 25 18·4	30·0	60 28 8·4	6·2	9·27562
11·875	199 6 26·3	−29·7	59 46 50·7	− 6·0	9·27407
12·0	201 41 47·1	29·3	59 2 5·4	5·7	9·27283
12·125	204 11 3·3	28·8	58 14 6·3	5·5	9·27188
12·25	206 34 4·0	−28·3	57 23 8·4	− 5·3	9·27124
12·375	208 50 44·7	27·7	56 29 27·3	5·1	9·27090
12·5	211 1 5·8	27·1	55 33 19·1	5·0	9·27087
12·625	213 5 12·0	−26·6	54 34 59·9	− 4·8	9·27116
12·75	215 3 10·7	26·2	53 34 45·8	4·7	9·27175
12·875	216 55 13·2	25·8	52 32 53·0	4·6	9·27264
13·0	218 41 32·0	−25·4	51 29 36·9	− 4·5	9·27384
13·125	220 22 21·3	24·9	50 25 12·1	4·4	9·27533
13·25	221 57 56·6	24·5	49 19 53·0	4·3	9·27712
13·375	223 28 33·2	−24·0	+48 13 52·9	− 4·2	9·27919

TABLE X.—*continued.*

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1835.	° ′ ″	″	° ′ ″	″	
Oct. 13·375	223 28 33·2	−24·0	+48 13 52·9	− 4·2	9·27919
13·5	224 54 26·9	23·6	47 7 24·8	4·1	9·28154
13·625	226 15 52·2	23·1	46 0 40·4	4·0	9·28415
13·75	227 33 5·1	−22·7	44 53 50·9	− 3·9	9·28702
13·875	228 46 19·8	22·2	43 47 6·3	3·8	9·29014
14·0	229 55 50·7	21·8	42 40 35·8	3·7	9·29349
14·125	231 1 50·6	−21·4	41 34 27·4	− 3·5	9·29706
14·25	232 4 33·0	21·0	40 28 49·2	3·3	9·30085
14·375	233 4 9·5	20·6	39 23 48·2	3·1	9·30484
14·5	234 0 51·6	−20·2	38 19 30·2	− 2·9	9·30902
14·625	234 54 49·6	20·0	37 16 0·1	2·6	9·31337
14·75	235 46 13·5	19·8	36 13 23·0	2·2	9·31789
14·875	236 35 12·6	−19·6	35 11 42·9	− 1·8	9·32257
15·0	237 21 56·0	19·4	34 11 3·3	1·3	9·32738
15·125	238 6 31·1	19·2	33 11 26·5	0·9	9·33232
15·25	238 49 6·0	−19·0	32 12 55·3	− 0·4	9·33738
15·375	239 29 48·0	18·8	31 15 31·5	+ 0·1	9·34255
15·5	240 8 43·4	18·7	30 19 16·5	0·6	9·34782
15·625	240 45 58·1	−18·5	29 24 10·7	+ 1·0	9·35318
15·75	241 21 38·2	18·3	28 30 15·4	1·4	9·35861
15·875	241 55 49·1	18·1	27 37 31·1	1·9	9·36411
16·0	242 28 35·2	−17·8	26 45 57·8	+ 2·3	9·36967
16·25	243 30 12·2	17·3	25 6 22·0	3·0	9·38095
16·5	244 27 4·3	16·7	23 31 24·8	3·7	9·39239
16·75	245 19 39·3	−16·2	22 0 59·3	+ 4·2	9·40392
17·0	246 8 24·0	15·6	20 34 57·2	4·6	9·41551
17·25	246 53 40·1	15·1	19 13 7·9	4·9	9·42712
17·5	247 35 48·7	−14·5	17 55 21·4	+ 5·2	9·43871
17·75	248 15 6·0	14·0	16 41 25·1	5·3	9·45026
18·0	248 51 48·0	13·4	15 31 8·7	5·4	9·46174
18·25	249 26 7·8	−12·9	14 24 20·1	+ 5·4	9·47313
18·5	249 58 18·7	12·4	13 20 48·6	5·3	9·48442
18·75	250 28 30·2	12·0	12 20 22·9	5·1	9·49559
19·0	250 56 53·1	−11·6	11 22 52·5	+ 4·9	9·50663
19·25	251 23 34·9	11·3	10 28 7·3	4·6	9·51753
19·5	251 48 44·8	11·0	9 35 57·6	4·3	9·52828
19·75	252 12 28·5	−10·7	+ 8 46 14·2	+ 4·0	9·53889

TABLE X.—*continued.*

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1835.	° ′ ″	″	° ′ ″	″	
Oct. 19 ·75	252 12 28 ·5	—10 ·7	+ 8 46 14 ·2	+ 4 ·0	9 ·53889
20 ·0	252 34 53 ·3	10 ·4	7 58 48 ·5	3 ·7	9 ·54935
20 ·25	252 56 4 ·1	10 ·1	7 13 32 ·1	3 ·5	9 ·55964
20 ·5	253 16 6 ·3	— 9 ·9	6 30 17 ·8	+ 3 ·2	9 ·56979
20 ·75	253 35 4 ·7	9 ·7	5 48 57 ·6	3 ·0	9 ·57977
21 ·0	253 53 3 ·9	9 ·5	5 9 25 ·6	2 ·7	9 ·58960
21 ·25	254 10 6 ·9	— 9 ·3	4 31 34 ·9	+ 2 ·5	9 ·59928
21 ·5	254 26 18 ·4	9 ·1	3 55 20 ·3	2 ·3	9 ·60879
21 ·75	254 41 40 ·6	8 ·9	3 20 35 ·8	2 ·1	9 ·61816
22 ·0	254 56 17 ·2	— 8 ·8	2 47 16 ·7	+ 1 ·8	9 ·62737
22 ·25	255 10 10 ·0	8 ·7	2 15 17 ·8	1 ·6	9 ·63643
22 ·5	255 23 22 ·9	8 ·6	1 44 34 ·9	1 ·5	9 ·64534
22 ·75	255 35 56 ·6	— 8 ·5	1 15 3 ·3	+ 1 ·3	9 ·65411
23 ·0	255 47 54 ·4	8 ·4	0 46 39 ·5	1 ·1	9 ·66274
23 ·25	255 59 17 ·4	8 ·3	+ 0 19 19 ·6	1 ·0	9 ·67122
23 ·5	256 10 8 ·6	— 8 ·2	— 0 6 59 ·5	+ 0 ·9	9 ·67957
23 ·75	256 20 28 ·6	8 ·1	0 32 21 ·2	0 ·8	9 ·68778
24 ·0	256 30 19 ·7	8 ·1	0 56 48 ·6	0 ·7	9 ·69587
24 ·25	256 39 42 ·5	— 8 ·0	1 20 24 ·5	+ 0 ·6	9 ·70382
24 ·5	256 48 39 ·4	7 ·9	1 43 11 ·4	0 ·5	9 ·71165
24 ·75	256 57 10 ·0	7 ·8	2 5 12 ·3	0 ·4	9 ·71935
25 ·0	257 5 17 ·1	— 7 ·7	2 26 29 ·0	+ 0 ·3	9 ·72694
25 ·25	257 13 0 ·5	7 ·6	2 47 3 ·9	0 ·3	9 ·73441
25 ·5	257 20 22 ·6	7 ·5	3 6 59 ·0	0 ·2	9 ·74176
25 ·75	257 27 23 ·2	— 7 ·4	3 26 15 ·9	+ 0 ·1	9 ·74900
26 ·0	257 34 4 ·2	7 ·4	3 44 56 ·7	0 ·1	9 ·75613
26 ·25	257 40 25 ·1	7 ·3	4 3 3 ·1	0 ·0	9 ·76315
26 ·5	257 46 28 ·2	— 7 ·2	4 20 36 ·6	0 ·0	9 ·77007
26 ·75	257 52 12 ·7	7 ·1	4 37 39 ·3	0 ·0	9 ·77688
27 ·0	257 57 40 ·5	7 ·0	4 54 11 ·9	0 ·0	9 ·78360
27 ·25	258 2 51 ·1	— 7 ·0	5 10 16 ·0	— 0 ·1	9 ·79021
27 ·5	258 7 46 ·6	6 ·9	5 25 52 ·9	0 ·1	9 ·79673
27 ·75	258 12 25 ·9	6 ·8	5 41 3 ·7	0 ·1	9 ·80316
28 ·0	258 16 51 ·0	— 6 ·7	5 55 49 ·5	— 0 ·1	9 ·80949
28 ·25	258 21 0 ·9	6 ·6	6 10 11 ·7	0 ·1	9 ·81573
28 ·5	258 24 58 ·4	6 ·6	6 24 11 ·0	0 ·2	9 ·82189
29 ·5	258 38 35 ·7	— 6 ·3	— 7 16 38 ·9	— 0 ·2	9 ·84567

TABLE X.—*continued.*

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1835.	° ′ ″	″	° ′ ″	″	
Oct. 29 ·5	258 38 35 ·7	— 6 ·3	— 7 16 38 ·9	— 0 ·2	9 ·84567
30 ·5	258 49 3 ·3	6 ·1	8 4 10 ·8	0 ·3	9 ·86818
31 ·5	258 56 42 ·3	5 ·9	8 47 30 ·7	0 ·3	9 ·88953
Nov. 1 ·5	259 1 49 ·3	— 5 ·7	9 27 15 ·0	— 0 ·4	9 ·90979
2 ·5	259 4 38 ·4	5 ·5	10 3 53 ·6	0 ·4	9 ·92905
3 ·5	259 5 21 ·5	5 ·4	10 37 51 ·2	0 ·5	9 ·94737
4 ·5	259 4 8 ·8	— 5 ·4	11 9 28 ·9	— 0 ·5	9 ·96482
5 ·5	259 1 9 ·3	5 ·4	11 39 4 ·4	0 ·6	9 ·98144
6 ·5	258 56 31 ·2	5 ·4	12 6 52 ·4	0 ·7	9 ·99729
7 ·5	258 50 21 ·2	— 5 ·5	12 33 5 ·8	— 0 ·8	0 ·01240
8 ·5	258 42 46 ·2	5 ·7	12 57 55 ·4	0 ·9	0 ·02681
9 ·5	258 33 52 ·0	5 ·8	13 21 30 ·5	1 ·0	0 ·04056
10 ·5	258 23 44 ·7	— 6 ·0	13 43 59 ·2	— 1 ·1	0 ·05367
11 ·5	258 12 30 ·0	6 ·0	14 5 28 ·2	1 ·2	0 ·06617
12 ·5	258 0 12 ·9	5 ·8	14 26 4 ·1	1 ·1	0 ·07809
13 ·5	257 46 59 ·2	— 5 ·5	14 45 51 ·6	— 1 ·1	0 ·08945
14 ·5	257 32 53 ·8	5 ·1	15 4 55 ·4	1 ·0	0 ·10027
15 ·5	257 18 2 ·2	5 ·1	15 23 19 ·3	1 ·0	0 ·11057
16 ·5	257 2 29 ·4	— 5 ·4	15 41 7 ·0	— 1 ·1	0 ·12037
17 ·5	256 46 20 ·4	5 ·8	15 58 21 ·5	1 ·2	0 ·12968
18 ·5	256 29 39 ·8	6 ·3	16 15 5 ·2	1 ·3	0 ·13853
19 ·5	256 12 32 ·3	— 6 ·7	16 31 20 ·7	— 1 ·4	0 ·14692
20 ·5	255 55 2 ·8	6 ·8	16 47 9 ·9	1 ·4	0 ·15489
21 ·5	255 37 15 ·2	6 ·8	17 2 34 ·9	1 ·4	0 ·16243
22 ·5	255 19 13 ·5	— 6 ·8	17 17 37 ·2	— 1 ·5	0 ·16957
23 ·5	255 1 1 ·4	6 ·9	17 32 18 ·4	1 ·5	0 ·17633
24 ·5	254 42 42 ·5	7 ·2	17 46 39 ·7	1 ·6	0 ·18271
25 ·5	254 24 19 ·8	— 7 ·6	18 0 42 ·5	— 1 ·7	0 ·18873
26 ·5	254 5 56 ·4	7 ·9	18 14 27 ·7	1 ·8	0 ·19441
27 ·5	253 47 34 ·3	8 ·2	18 27 56 ·6	1 ·9	0 ·19975
28 ·5	253 29 16 ·6	— 8 ·4	18 41 10 ·0	— 1 ·9	0 ·20478
29 ·5	253 11 4 ·6	8 ·5	18 54 9 ·1	2 ·0	0 ·20951
30 ·5	252 52 59 ·9	8 ·5	19 6 54 ·6	2 ·0	0 ·21394
Dec. 1 ·5	252 35 4 ·1	— 8 ·5	19 19 27 ·2	— 2 ·0	0 ·21809
2 ·5	252 17 18 ·5	8 ·5	19 31 47 ·9	2 ·0	0 ·22198
3 ·5	251 59 43 ·7	8 ·5	19 43 57 ·2	2 ·1	0 ·22561
4 ·5	251 42 20 ·6	— 8 ·6	—19 55 56 ·1	— 2 ·1	0 ·22899

TABLE X.—*continued.*

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1835.	° ′ ″	″	° ′ ″	″	
Dec. 4·5	251 42 20·6	− 8·6	−19 55 56·1	− 2·1	0·22899
5·5	251 25 8·5	8·6	20 7 45·2	2·1	0·23213
6·5	251 8 9·5	8·7	20 19 24·8	2·1	0·23505
7·5	250 51 22·4	− 8·9	20 30 55·9	− 2·0	0·23775
8·5	250 34 47·1	9·2	20 42 19·1	2·0	0·24023
9·5	250 18 23·7	9·5	20 53 34·7	2·0	0·24252
10·5	250 2 11·8	− 9·7	21 4 43·4	− 2·1	0·24461
11·5	249 46 10·5	9·7	21 15 45·8	2·3	0·24652
12·5	249 30 19·5	9·6	21 26 42·2	2·4	0·24824
13·5	249 14 37·6	− 9·6	21 37 33·0	− 2·6	0·24979
14·5	248 59 5·0	9·6	21 48 18·9	2·7	0·25116
15·5	248 43 40·2	9·6	21 59 0·1	2·8	0·25238
16·5	248 28 22·4	− 9·7	22 9 37·2	− 2·9	0·25344
17·5	248 13 9·9	9·9	22 20 10·6	2·9	0·25434
18·5	247 58 3·0	10·2	22 30 40·6	2·9	0·25510
19·5	247 42 59·5	−10·5	22 41 7·5	− 3·0	0·25571
20·5	247 27 58·8	10·8	22 51 31·7	3·1	0·25618
21·5	247 12 59·8	11·0	23 1 53·6	3·2	0·25651
22·5	246 58 0·8	−11·1	23 12 13·2	− 3·2	0·25672
23·5	246 43 0·3	11·2	23 22 31·5	3·3	0·25679
24·5	246 27 58·1	11·3	23 32 48·1	3·4	0·25674
25·5	246 12 52·2	−11·4	23 43 3·5	− 3·5	0·25656
26·5	245 57 41·8	11·6	23 53 17·9	3·7	0·25627
27·5	245 42 24·9	11·8	24 3 31·8	3·7	0·25586
28·5	245 27 1·4	−12·0	24 13 45·0	− 3·8	0·25534
29·5	245 11 28·8	12·2	24 23 58·1	3·9	0·25470
30·5	244 55 46·8	12·3	24 34 11·2	3·9	0·25396
31·5	244 39 53·4	−12·4	24 44 24·5	− 4·0	0·25311
1836.					
Jan. 1·5	244 23 47·3	12·5	24 54 38·2	4·1	0·25215
2·5	244 7 27·0	12·6	25 4 52·4	4·2	0·25109
3·5	243 50 51·3	−12·8	25 15 7·6	− 4·2	0·24994
4·5	243 33 59·2	13·0	25 25 23·3	4·3	0·24868
5·5	243 16 47·6	13·2	25 35 40·2	4·4	0·24733
6·5	242 59 17·4	−13·5	25 45 58·2	− 4·5	0·24589
7·5	242 41 24·9	13·8	25 56 17·5	4·5	0·24435
8·5	242 23 9·7	14·1	26 6 38·2	4·6	0·24272
9·5	242 4 29·1	−14·3	−26 17 0·2	− 4·7	0·24100

TABLE X.—*continued.*

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1836.	° ′ ″	″	° ′ ″	″	
Jan. 9 ·5	242 4 29 ·1	—14 ·3	—26 17 0 ·2	— 4 ·7	0 ·24100
10 ·5	241 45 22 ·6	14 ·3	26 27 23 ·6	4 ·8	0 ·23919
11 ·5	241 25 48 ·0	14 ·3	26 37 48 ·5	4 ·8	0 ·23730
12 ·5	241 5 43 ·8	—14 ·4	26 48 14 ·8	— 4 ·9	0 ·23532
13 ·5	240 45 7 ·5	14 ·4	26 58 42 ·6	5 ·0	0 ·23326
14 ·5	240 23 57 ·2	14 ·5	27 9 11 ·7	5 ·1	0 ·23112
15 ·5	240 2 11 ·8	—14 ·5	27 19 42 ·1	— 5 ·2	0 ·22890
16 ·5	239 39 48 ·7	14 ·6	27 30 13 ·8	5 ·3	0 ·22661
17 ·5	239 16 45 ·7	14 ·7	27 40 46 ·5	5 ·4	0 ·22424
18 ·5	238 53 1 ·1	—14 ·8	27 51 20 ·0	— 5 ·5	0 ·22179
19 ·5	238 28 33 ·1	15 ·1	28 1 54 ·1	5 ·6	0 ·21927
20 ·5	238 3 19 ·4	15 ·4	28 12 28 ·6	5 ·7	0 ·21669
21 ·5	237 37 17 ·0	—15 ·7	28 23 3 ·1	— 5 ·8	0 ·21404
22 ·5	237 10 24 ·1	16 ·1	28 33 37 ·3	5 ·9	0 ·21132
23 ·5	236 42 39 ·7	16 ·5	28 44 10 ·8	5 ·9	0 ·20854
24 ·5	236 14 0 ·3	—16 ·9	28 54 43 ·1	— 6 ·0	0 ·20570
25 ·5	235 44 23 ·3	17 ·3	29 5 13 ·5	6 ·1	0 ·20281
26 ·5	235 13 47 ·7	17 ·8	29 15 42 ·1	6 ·2	0 ·19986
27 ·5	234 42 10 ·8	—18 ·1	29 26 7 ·6	— 6 ·2	0 ·19687
28 ·5	234 9 29 ·7	18 ·5	29 36 29 ·7	6 ·3	0 ·19382
29 ·5	233 35 42 ·6	18 ·9	29 46 47 ·2	6 ·4	0 ·19073
30 ·5	233 0 46 ·5	—19 ·3	29 56 59 ·9	— 6 ·5	0 ·18760
31 ·5	232 24 39 ·9	19 ·5	30 7 6 ·4	6 ·5	0 ·18443
Feb. 1 ·5	231 47 19 ·2	19 ·7	30 17 5 ·6	6 ·6	0 ·18124
2 ·5	231 8 42 ·8	—19 ·9	30 26 56 ·6	— 6 ·6	0 ·17801
3 ·5	230 28 48 ·8	20 ·1	30 36 38 ·2	6 ·6	0 ·17475
4 ·5	229 47 34 ·2	20 ·6	30 46 8 ·9	6 ·7	0 ·17148
5 ·5	229 4 56 ·7	—21 ·4	30 55 27 ·1	— 6 ·8	0 ·16819
6 ·5	228 20 54 ·5	22 ·4	31 4 32 ·0	6 ·9	0 ·16489
7 ·5	227 35 23 ·9	23 ·7	31 13 21 ·0	6 ·9	0 ·16158
8 ·5	226 48 25 ·4	—24 ·6	31 21 52 ·8	— 6 ·9	0 ·15828
9 ·5	225 59 54 ·8	25 ·3	31 30 5 ·5	7 ·0	0 ·15498
10 ·5	225 9 50 ·6	25 ·8	31 37 56 ·9	7 ·0	0 ·15170
11 ·5	224 18 12 ·2	—26 ·2	31 45 24 ·5	— 7 ·0	0 ·14844
12 ·5	223 24 56 ·9	26 ·4	31 52 26 ·5	6 ·9	0 ·14520
13 ·5	222 30 4 ·4	26 ·7	31 59 0 ·4	6 ·9	0 ·14200
14 ·5	221 33 34 ·5	—27 ·2	—32 5 3 ·3	— 6 ·9	0 ·13884

TABLE X.—*continued.*

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1836.	° ′ ″	″	° ′ ″	″	
Feb. 14·5	221 33 34·5	—27·2	—32 5 3·3	— 6·9	0·13884
15·5	220 35 25·0	27·7	32 10 32·8	6·8	0·13573
16·5	219 35 37·1	28·5	32 15 26·0	6·7	0·13268
17·5	218 34 11·3	—29·3	32 19 40·2	— 6·6	0·12971
18·5	217 31 6·0	30·2	32 23 12·1	6·5	0·12680
19·5	216 26 24·8	31·2	32 25 59·2	6·3	0·12399
20·5	215 20 8·6	—32·2	32 27 58·4	— 6·2	0·12127
21·5	214 12 19·2	33·2	32 29 6·7	6·0	0·11866
22·5	213 2 59·1	34·2	32 29 21·0	5·8	0·11617
23·5	211 52 12·3	—35·2	32 28 38·5	— 5·6	0·11380
24·5	210 40 3·2	36·4	32 26 57·0	5·2	0·11157
25·5	209 26 34·3	37·5	32 24 13·0	4·8	0·10948
26·5	208 11 52·7	—38·6	32 20 24·5	— 4·4	0·10755
27·5	206 56 2·9	39·6	32 15 29·2	4·0	0·10579
28·5	205 39 9·3	40·2	32 9 24·3	3·6	0·10420
29·5	204 21 21·8	—40·6	32 2 9·2	— 3·3	0·10279
Mar. 1·5	203 2 45·6	41·0	31 53 41·6	2·9	0·10158
2·5	201 43 28·8	41·3	31 44 0·7	2·6	0·10058
3·5	200 23 37·5	—41·5	31 33 5·7	— 2·2	0·09978
4·5	199 3 22·5	41·6	31 20 55·9	1·9	0·09919
5·5	197 42 51·0	42·0	31 7 31·6	1·5	0·09884
6·5	196 22 10·7	—42·5	30 52 53·8	— 1·1	0·09871
7·5	195 1 32·4	43·0	30 37 2·7	— 0·6	0·09881
8·5	193 41 3·3	43·6	30 19 59·4	0·0	0·09915
9·5	192 20 51·3	—44·2	30 1 45·6	+ 0·6	0·09974
10·5	191 1 6·0	44·8	29 42 24·4	1·2	0·10058
11·5	189 41 57·2	45·4	29 21 58·3	1·8	0·10165
12·5	188 23 30·6	—45·7	29 0 29·6	+ 2·3	0·10298
13·5	187 5 54·9	46·0	28 38 2·0	2·9	0·10455
14·5	185 49 17·9	46·1	28 14 39·4	3·5	0·10637
15·5	184 33 45·0	—45·9	27 50 25·0	+ 3·8	0·10843
16·5	183 19 24·5	45·4	27 25 24·0	4·0	0·11073
17·5	182 6 21·9	44·8	26 59 40·6	4·2	0·11327
18·5	180 54 41·4	—44·4	26 33 18·5	+ 4·5	0·11603
19·5	179 44 28·5	44·3	26 6 22·9	4·9	0·11903
20·5	178 35 47·1	44·2	25 38 58·3	5·4	0·12223
21·5	177 28 41·6	—44·3	—25 11 9·4	+ 6·0	0·12565

TABLE X.—*continued.*

Date.	Apparent Right Ascension.	Perturbations.	Apparent Declination.	Perturbations.	Log. of True Dist. from the Earth.
1836.	° ′ ″	″	° ′ ″	″	
Mar. 21 ·5	177 28 41 ·6	−44 ·3	−25 11 9 ·4	+ 6 ·0	0 ·12565
22 ·5	176 23 13 ·5	44 ·4	24 43 0 ·7	6 ·3	0 ·12927
23 ·5	175 19 26 ·0	44 ·2	24 14 36 ·6	6 ·3	0 ·13309
24 ·5	174 17 21 ·8	−44 ·0	23 46 1 ·5	+ 6 ·3	0 ·13709
25 ·5	173 17 1 ·1	43 ·6	23 17 19 ·6	6 ·4	0 ·14127
26 ·5	172 18 24 ·9	43 ·3	22 48 34 ·7	6 ·5	0 ·14561
27 ·5	171 21 34 ·4	−43 ·0	22 19 50 ·8	+ 6 ·9	0 ·15011
28 ·5	170 26 29 ·4	42 ·7	21 51 11 ·6	7 ·4	0 ·15475
29 ·5	169 33 9 ·3	42 ·4	21 22 39 ·8	7 ·8	0 ·15953
30 ·5	168 41 33 ·0	−42 ·2	20 54 18 ·5	+ 8 ·3	0 ·16445
31 ·5	167 51 38 ·9	−42 ·2	−20 26 9 ·9	+ 8 ·8	0 ·16949

TABLE XI.

Containing 730 Equations of Condition for correcting the assumed Elements of the Orbit of HALLEY'S Comet, on July 30, 1835.

Date.	Equations of Condition dependent upon Right Ascensions.
1835.	″ ″ ″ ″ ″ ″
Aug. 1·5	+ 5·1 P — 55·8 Q + 301·0 R + 42·6 S + 1·0 U — 1·0 V = E
2·5	+ 5·1 P — 56·1 Q + 302·4 R + 42·6 S + 1·1 U — 0·9 V = E
3·5	+ 5·0 P — 56·5 Q + 303·8 R + 42·6 S + 1·1 U — 0·9 V = E
4·5	+ 4·9 P — 56·7 Q + 305·1 R + 42·7 S + 1·2 U — 0·8 V = E
5·5	+ 4·8 P — 56·9 Q + 306·4 R + 42·7 S + 1·1 U — 0·8 V = E
6·5	+ 4·7 P — 57·2 Q + 307·9 R + 42·8 S + 1·1 U — 0·8 V = E
7·5	+ 4·7 P — 57·3 Q + 309·5 R + 42·8 S + 1·4 U — 0·6 V = E
8·5	+ 4·5 P — 57·9 Q + 311·0 R + 42·9 S + 1·2 U — 0·6 V = E
9·5	+ 4·6 P — 58·0 Q + 312·7 R + 43·0 S + 1·4 U — 0·6 V = E
10·5	+ 4·4 P — 58·2 Q + 314·4 R + 43·0 S + 1·4 U — 0·5 V = E
11·5	+ 4·2 P — 58·5 Q + 315·9 R + 43·0 S + 1·3 U — 0·6 V = E
12·5	+ 4·2 P — 58·8 Q + 317·8 R + 43·1 S + 1·5 U — 0·5 V = E
13·5	+ 4·0 P — 59·2 Q + 319·7 R + 43·1 S + 1·4 U — 0·4 V = E
14·5	+ 3·9 P — 59·5 Q + 321·5 R + 43·2 S + 1·5 U — 0·4 V = E
15·5	+ 3·7 P — 59·8 Q + 323·3 R + 43·4 S + 1·5 U — 0·3 V = E
16·5	+ 3·6 P — 60·2 Q + 325·3 R + 43·4 S + 1·6 U — 0·2 V = E
17·5	+ 3·3 P — 60·6 Q + 327·3 R + 43·4 S + 1·7 U — 0·3 V = E
18·5	+ 3·2 P — 60·9 Q + 329·6 R + 43·5 S + 1·6 U — 0·1 V = E
19·5	+ 3·0 P — 61·4 Q + 331·7 R + 43·6 S + 1·5 U * = E
20·5	+ 2·8 P — 61·8 Q + 333·8 R + 43·7 S + 1·6 U + 0·1 V = E
21·5	+ 2·6 P — 62·3 Q + 336·1 R + 43·7 S + 1·6 U + 0·1 V = E
22·5	+ 2·4 P — 62·6 Q + 338·5 R + 43·8 S + 1·7 U + 0·2 V = E
23·5	+ 2·3 P — 63·1 Q + 340·9 R + 44·0 S + 1·5 U + 0·4 V = E
24·5	+ 1·9 P — 63·4 Q + 343·5 R + 44·0 S + 1·7 U + 0·4 V = E
25·5	+ 1·6 P — 64·0 Q + 346·1 R + 44·2 S + 1·9 U + 0·5 V = E
26·5	+ 1·5 P — 64·5 Q + 348·8 R + 44·3 S + 1·9 U + 0·5 V = E
27·5	+ 1·4 P — 64·9 Q + 351·8 R + 44·6 S + 2·0 U + 0·7 V = E
28·5	+ 0·8 P — 65·5 Q + 354·5 R + 44·5 S + 2·1 U + 0·8 V = E
29·5	+ 0·6 P — 66·0 Q + 357·6 R + 44·6 S + 2·1 U + 1·0 V = E
30·5	+ 0·2 P — 66·7 Q + 360·7 R + 44·7 S + 2·1 U + 1·0 V = E
31·5	— 0·3 P — 67·3 Q + 363·9 R + 44·9 S + 2·2 U + 1·1 V = E

TABLE XI.—*continued.*

Containing 730 Equations of Condition for correcting the assumed Elements of the Orbit of Halley's Comet, on July 30, 1835.

Date.	Equations of Condition dependent upon Declinations.
1835.	″ ″ ″ ″ ″ ″
Aug. 1·5	− 4·2 P + 16·2 Q − 88·9 R − 11·5 S + 14·5 U − 4·3 V = E′
2·5	− 4·3 P + 16·4 Q − 90·3 R − 11·5 S + 14·4 U − 4·2 V = E′
3·5	− 4·5 P + 16·8 Q − 91·8 R − 11·6 S + 14·5 U − 3·9 V = E′
4·5	− 4·5 P + 17·0 Q − 93·3 R − 11·8 S + 14·6 U − 3·7 V = E′
5·5	− 4·6 P + 17·3 Q − 94·6 R − 11·8 S + 14·8 U − 3·5 V = E′
6·5	− 4·8 P + 17·5 Q − 96·4 R − 12·1 S + 14·7 U − 3·3 V = E′
7·5	− 5·0 P + 17·9 Q − 98·1 R − 12·1 S + 14·8 U − 3·1 V = E′
8·5	− 5·0 P + 18·3 Q − 99·6 R − 12·6 S + 15·0 U − 2·9 V = E′
9·5	− 4·9 P + 18·5 Q − 101·4 R − 12·5 S + 15·1 U − 2·6 V = E′
10·5	− 5·4 P + 18·8 Q − 103·2 R − 12·8 S + 15·0 U − 2·4 V = E′
11·5	− 5·4 P + 19·2 Q − 104·9 R − 12·8 S + 15·2 U − 2·2 V = E′
12·5	− 5·6 P + 19·4 Q − 107·0 R − 13·0 S + 15·2 U − 2·0 V = E′
13·5	− 5·5 P + 19·9 Q − 108·9 R − 13·1 S + 15·4 U − 1·6 V = E′
14·5	− 5·6 P + 20·2 Q − 110·9 R − 13·4 S + 15·5 U − 1·3 V = E′
15·5	− 5·7 P + 20·6 Q − 113·0 R − 13·7 S + 15·6 U − 1·1 V = E′
16·5	− 6·0 P + 21·1 Q − 115·2 R − 13·9 S + 15·6 U − 0·9 V = E′
17·5	− 6·0 P + 21·4 Q − 117·6 R − 13·8 S + 15·8 U − 0·6 V = E′
18·5	− 6·2 P + 21·8 Q − 120·2 R − 14·1 S + 16·0 U − 0·3 V = E′
19·5	− 6·3 P + 22·4 Q − 122·2 R − 14·4 S + 16·0 U * = E′
20·5	− 6·8 P + 22·7 Q − 124·9 R − 14·8 S + 16·1 U + 0·3 V = E′
21·5	− 6·8 P + 23·2 Q − 127·7 R − 15·0 S + 16·2 U + 0·6 V = E′
22·5	− 6·9 P + 23·8 Q − 130·2 R − 14·9 S + 16·5 U + 0·9 V = E′
23·5	− 7·5 P + 24·1 Q − 133·5 R − 15·7 S + 16·3 U + 1·0 V = E′
24·5	− 7·3 P + 24·8 Q − 136·2 R − 15·7 S + 16·7 U + 1·6 V = E′
25·5	− 7·7 P + 25·3 Q − 139·5 R − 16·1 S + 16·9 U + 2·0 V = E′
26·5	− 8·1 P + 25·9 Q − 142·7 R − 16·7 S + 17·0 U + 2·3 V = E′
27·5	− 8·4 P + 26·3 Q − 146·5 R − 16·8 S + 17·0 U + 2·7 V = E′
28·5	− 8·6 P + 27·2 Q − 149·9 R − 17·2 S + 17·4 U + 3·1 V = E′
29·5	− 8·9 P + 27·8 Q − 153·6 R − 17·0 S + 17·5 U + 3·6 V = E′
30·5	− 9·0 P + 28·7 Q − 157·8 R − 17·5 S + 17·6 U + 4·0 V = E′
31·5	− 9·5 P + 29·4 Q − 162·1 R − 18·0 S + 17·9 U + 4·5 V = E′

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Right Ascensions.
1835.	″ ″ ″ ″ ″ ″
Aug. 31·5	− 0·3 P − 67·3 Q + 363·9 R + 44·9 S + 2·2 U + 1·1 V = E
Sep. 1·5	− 0·5 P − 67·8 Q + 367·2 R + 45·1 S + 2·4 U + 1·3 V = E
2·5	− 0·9 P − 68·6 Q + 370·7 R + 45·2 S + 2·3 U + 1·4 V = E
3·5	− 1·3 P − 69·1 Q + 374·5 R + 45·4 S + 2·4 U + 1·5 V = E
4·5	− 1·8 P − 69·8 Q + 378·2 R + 45·5 S + 2·5 U + 1·6 V = E
5·5	− 2·2 P − 70·5 Q + 382·1 R + 45·7 S + 2·6 U + 1·8 V = E
6·5	− 2·9 P − 71·3 Q + 386·2 R + 45·8 S + 2·6 U + 2·1 V = E
7·5	− 3·4 P − 72·1 Q + 390·5 R + 45·9 S + 2·7 U + 2·2 V = E
8·5	− 4·1 P − 73·0 Q + 394·7 R + 46·0 S + 2·7 U + 2·3 V = E
9·5	− 4·8 P − 73·8 Q + 399·3 R + 46·3 S + 3·0 U + 2·6 V = E
10·5	− 5·5 P − 74·7 Q + 404·1 R + 46·4 S + 3·0 U + 2·8 V = E
11·5	− 6·4 P − 75·6 Q + 409·3 R + 46·8 S + 3·1 U + 2·9 V = E
12·5	− 7·3 P − 76·5 Q + 414·3 R + 46·8 S + 3·2 U + 3·2 V = E
13·5	− 8·2 P − 77·3 Q + 419·8 R + 47·2 S + 3·4 U + 3·5 V = E
14·5	− 9·4 P − 78·5 Q + 425·4 R + 47·3 S + 3·4 U + 3·7 V = E
15·5	− 10·5 P − 79·7 Q + 431·0 R + 47·5 S + 3·5 U + 4·0 V = E
16·5	− 11·9 P − 80·8 Q + 437·0 R + 47·7 S + 3·7 U + 4·4 V = E
17·5	− 13·3 P − 81·7 Q + 443·8 R + 48·0 S + 3·9 U + 4·7 V = E
18·5	− 15·0 P − 83·1 Q + 450·4 R + 48·1 S + 3·9 U + 5·1 V = E
19·5	− 16·8 P − 84·5 Q + 457·0 R + 48·2 S + 4·3 U + 5·4 V = E
20·5	− 18·8 P − 85·7 Q + 463·9 R + 48·5 S + 4·4 U + 5·9 V = E
21·5	− 21·1 P − 87·2 Q + 471·3 R + 48·6 S + 4·6 U + 6·4 V = E
22·5	− 23·6 P − 89·1 Q + 478·6 R + 48·8 S + 4·9 U + 7·0 V = E
23·5	− 26·6 P − 90·0 Q + 485·9 R + 48·8 S + 5·1 U + 7·6 V = E
24·5	− 29·9 P − 90·9 Q + 493·6 R + 48·9 S + 5·4 U + 8·2 V = E
25·5	− 33·9 P − 92·5 Q + 500·3 R + 48·8 S + 5·9 U + 8·8 V = E
26·5	− 38·3 P − 94·3 Q + 506·6 R + 48·6 S + 6·1 U + 9·8 V = E
27·5	− 43·6 P − 94·7 Q + 512·6 R + 48·1 S + 6·6 U + 10·7 V = E
28·5	− 49·8 P − 96·0 Q + 516·4 R + 48·0 S + 7·1 U + 11·8 V = E
29·5	− 57·4 P − 96·6 Q + 518·0 R + 46·7 S + 7·7 U + 12·9 V = E
30·5	− 66·3 P − 96·5 Q + 516·3 R + 45·1 S + 8·5 U + 14·2 V = E
Oct. 1·5	− 77·6 P − 95·3 Q + 506·8 R + 42·8 S + 9·2 U + 15·9 V = E
1·75	− 80·6 P − 94·7 Q + 504·1 R + 42·3 S + 9·7 U + 16·4 V = E
2·0	− 84·0 P − 94·1 Q + 499·6 R + 41·4 S + 9·7 U + 16·9 V = E
2·25	− 87·5 P − 93·3 Q + 495·0 R + 40·6 S + 10·0 U + 17·4 V = E
2·5	− 91·5 P − 92·3 Q + 489·5 R + 39·6 S + 10·1 U + 18·0 V = E
2·75	− 95·2 P − 91·0 Q + 483·2 R + 38·8 S + 10·5 U + 18·6 V = E

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Declinations.
1835.	″ ″ ″ ″ ″ ″
Aug. 31·5	− 9·5 P + 29·4 Q − 162·1 R − 18·0 S + 17·9 U + 4·5 V = E′
Sep. 1·5	− 10·0 P + 30·2 Q − 166·2 R − 18·2 S + 18·0 U + 4·9 V = E′
2·5	− 10·1 P + 31·2 Q − 171·2 R − 18·8 S + 18·3 U + 5·4 V = E′
3·5	− 10·5 P + 32·0 Q − 176·1 R − 19·3 S + 18·6 U + 6·0 V = E′
4·5	− 11·2 P + 32·8 Q − 181·6 R − 19·8 S + 18·7 U + 6·4 V = E′
5·5	− 11·4 P + 34·0 Q − 187·0 R − 20·2 S + 19·1 U + 7·1 V = E′
6·5	− 11·8 P + 35·2 Q − 193·0 R − 20·8 S + 19·3 U + 7·7 V = E′
7·5	− 12·4 P + 36·1 Q − 199·4 R − 21·2 S + 19·7 U + 8·4 V = E′
8·5	− 12·9 P + 37·3 Q − 206·2 R − 22·1 S + 19·8 U + 8·9 V = E′
9·5	− 13·5 P + 38·8 Q − 213·6 R − 22·5 S + 20·1 U + 9·7 V = E′
10·5	− 14·2 P + 40·1 Q − 221·3 R − 23·3 S + 20·4 U + 10·2 V = E′
11·5	− 14·9 P + 41·4 Q − 229·9 R − 23·8 S + 20·8 U + 11·1 V = E′
12·5	− 15·6 P + 43·2 Q − 238·9 R − 24·8 S + 21·1 U + 12·0 V = E′
13·5	− 16·5 P + 45·1 Q − 248·7 R − 25·7 S + 21·6 U + 12·8 V = E′
14·5	− 17·3 P + 46·8 Q − 259·4 R − 26·5 S + 22·0 U + 13·8 V = E′
15·5	− 18·3 P + 49·0 Q − 271·1 R − 27·5 S + 22·3 U + 14·7 V = E′
16·5	− 19·4 P + 51·4 Q − 283·6 R − 28·6 S + 22·8 U + 15·8 V = E′
17·5	− 20·6 P + 53·9 Q − 297·5 R − 29·6 S + 23·2 U + 16·8 V = E′
18·5	− 21·9 P + 56·7 Q − 313·2 R − 31·2 S + 23·8 U + 18·1 V = E′
19·5	− 23·4 P + 59·6 Q − 329·5 R − 32·4 S + 24·4 U + 19·4 V = E′
20·5	− 25·0 P + 63·1 Q − 348·3 R − 34·1 S + 25·1 U + 20·8 V = E′
21·5	− 26·9 P + 66·8 Q − 369·3 R − 36·0 S + 25·6 U + 22·2 V = E′
22·5	− 29·0 P + 71·4 Q − 392·4 R − 38·1 S + 26·4 U + 23·9 V = E′
23·5	− 31·5 P + 75·7 Q − 419·0 R − 40·4 S + 27·0 U + 25·9 V = E′
24·5	− 34·0 P + 81·5 Q − 448·7 R − 42·8 S + 28·0 U + 27·6 V = E′
25·5	− 37·2 P + 87·5 Q − 483·0 R − 45·8 S + 29·0 U + 29·6 V = E′
26·5	− 40·9 P + 94·1 Q − 522·6 R − 49·2 S + 29·8 U + 31·8 V = E′
27·5	− 45·0 P + 103·2 Q − 566·9 R − 53·1 S + 30·9 U + 34·2 V = E′
28·5	− 49·9 P + 112·3 Q − 621·0 R − 57·6 S + 32·1 U + 36·8 V = E′
29·5	− 55·6 P + 123·4 Q − 683·8 R − 63·0 S + 33·3 U + 39·8 V = E′
30·5	− 62·3 P + 137·2 Q − 757·2 R − 69·4 S + 34·7 U + 42·9 V = E′
Oct. 1·5	− 70·2 P + 153·1 Q − 847·4 R − 76·9 S + 36·0 U + 46·3 V = E′
1·75	− 72·3 P + 157·6 Q − 872·2 R − 79·1 S + 36·6 U + 47·2 V = E′
2·0	− 74·7 P + 162·3 Q − 898·2 R − 81·4 S + 37·0 U + 48·1 V = E′
2·25	− 77·1 P + 167·4 Q − 925·6 R − 83·7 S + 37·4 U + 49·1 V = E′
2·5	− 79·5 P + 172·5 Q − 954·5 R − 86·1 S + 37·5 U + 50·0 V = E′
2·75	− 82·0 P + 178·3 Q − 984·8 R − 88·4 S + 38·0 U + 51·0 V = E′

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Right Ascensions.						
1835.	″	″	″	″	″	″	
Oct. 2·75	− 95·2 P	− 91·0 Q	+ 483·2 R	+ 38·8 S	+ 10·5 U	+ 18·6 V	= E
3·0	− 99·5 P	− 89·6 Q	+ 475·7 R	+ 37·6 S	+ 10·7 U	+ 19·1 V	= E
3·25	− 104·0 P	− 87·8 Q	+ 467·1 R	+ 37·0 S	+ 11·1 U	+ 19·7 V	= E
3·5	− 108·8 P	− 86·0 Q	+ 456·9 R	+ 35·0 S	+ 11·4 U	+ 20·2 V	= E
3·75	− 113·8 P	− 84·0 Q	+ 445·4 R	+ 33·6 S	+ 11·9 U	+ 21·0 V	= E
4·0	− 119·3 P	− 81·8 Q	+ 431·8 R	+ 31·8 S	+ 12·2 U	+ 21·6 V	= E
4·25	− 125·1 P	− 79·5 Q	+ 416·4 R	+ 29·8 S	+ 12·7 U	+ 22·2 V	= E
4·5	− 131·4 P	− 76·3 Q	+ 398·7 R	+ 27·3 S	+ 13·1 U	+ 22·9 V	= E
4·75	− 138·0 P	− 72·7 Q	+ 378·8 R	+ 25·1 S	+ 13·7 U	+ 23·7 V	= E
5·0	− 145·2 P	− 68·7 Q	+ 355·9 R	+ 22·3 S	+ 14·0 U	+ 24·6 V	= E
5·25	− 152·7 P	− 63·9 Q	+ 330·4 R	+ 19·6 S	+ 14·7 U	+ 25·5 V	= E
5·5	− 161·4 P	− 58·4 Q	+ 300·9 R	+ 15·9 S	+ 15·1 U	+ 26·4 V	= E
5·75	− 170·3 P	− 52·3 Q	+ 268·0 R	+ 12·3 S	+ 15·7 U	+ 27·4 V	= E
6·0	− 180·0 P	− 45·3 Q	+ 230·0 R	+ 7·8 S	+ 16·3 U	+ 28·4 V	= E
6·125	− 185·0 P	− 41·4 Q	+ 208·8 R	+ 5·5 S	+ 16·6 U	+ 28·9 V	= E
6·25	− 190·2 P	− 37·3 Q	+ 186·2 R	+ 3·1 S	+ 17·0 U	+ 29·4 V	= E
6·375	− 195·7 P	− 33·0 Q	+ 161·9 R	+ 0·5 S	+ 17·2 U	+ 29·9 V	= E
6·5	− 201·4 P	− 28·3 Q	+ 135·9 R	− 2·2 S	+ 17·5 U	+ 30·4 V	= E
6·625	− 207·2 P	− 23·3 Q	+ 108·0 R	− 5·1 S	+ 18·0 U	+ 31·0 V	= E
6·75	− 213·3 P	− 17·8 Q	+ 78·1 R	− 8·2 S	+ 18·5 U	+ 31·6 V	= E
6·875	− 219·8 P	− 11·9 Q	+ 46·0 R	− 11·5 S	+ 18·8 U	+ 32·1 V	= E
7·0	− 226·5 P	− 5·7 Q	+ 11·7 R	− 15·0 S	+ 19·1 U	+ 32·7 V	= E
7·125	− 233·2 P	+ 0·8 Q	− 24·8 R	− 18·6 S	+ 19·6 U	+ 33·4 V	= E
7·25	− 240·3 P	+ 7·8 Q	− 63·8 R	− 22·4 S	+ 20·0 U	+ 34·1 V	= E
7·375	− 247·8 P	+ 15·2 Q	− 105·7 R	− 26·6 S	+ 20·4 U	+ 34·7 V	= E
7·5	− 255·6 P	+ 23·3 Q	− 150·4 R	− 31·0 S	+ 20·8 U	+ 35·3 V	= E
7·625	− 263·5 P	+ 32·2 Q	− 198·0 R	− 35·6 S	+ 21·3 U	+ 36·0 V	= E
7·75	− 271·6 P	+ 41·8 Q	− 249·0 R	− 40·5 S	+ 21·9 U	+ 36·6 V	= E
7·875	− 280·3 P	+ 51·9 Q	− 303·7 R	− 45·6 S	+ 22·3 U	+ 37·2 V	= E
8·0	− 289·4 P	+ 62·7 Q	− 362·1 R	− 51·1 S	+ 22·7 U	+ 37·8 V	= E
8·125	− 298·7 P	+ 74·3 Q	− 424·4 R	− 57·0 S	+ 23·4 U	+ 38·4 V	= E
8·25	− 308·1 P	+ 86·7 Q	− 490·9 R	− 63·3 S	+ 24·0 U	+ 39·0 V	= E
8·375	− 318·1 P	+ 99·9 Q	− 561·6 R	− 70·1 S	+ 24·4 U	+ 39·7 V	= E
8·5	− 328·6 P	+ 114·0 Q	− 636·9 R	− 77·2 S	+ 24·8 U	+ 40·3 V	= E
8·625	− 339·0 P	+ 129·0 Q	− 716·9 R	− 84·6 S	+ 25·5 U	+ 40·9 V	= E
8·75	− 349·5 P	+ 144·9 Q	− 801·8 R	− 92·4 S	+ 26·1 U	+ 41·5 V	= E
8·875	− 360·3 P	+ 161·8 Q	− 892·6 R	− 100·6 S	+ 26·6 U	+ 42·1 V	= E

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Declinations					
1835.	″	″	″	″	″	″
Oct. 2·75	− 82·0 P	+ 178·3 Q	− 984·3 R	− 83·4 S	+ 38·0 U	+ 51·0 V = E′
3·0	− 84·7 P	+ 184·2 Q	− 1016·9 R	− 91·0 S	+ 38·5 U	+ 51·8 V = E′
3·25	− 87·5 P	+ 190·6 Q	− 1050·5 R	− 93·7 S	+ 38·8 U	+ 52·8 V = E′
3·5	− 90·4 P	+ 197·1 Q	− 1086·1 R	− 96·7 S	+ 39·1 U	+ 53·6 V = E′
3·75	− 93·3 P	+ 203·9 Q	− 1123·4 R	− 99·8 S	+ 39·5 U	+ 54·6 V = E′
4·0	− 96·5 P	+ 211·0 Q	− 1162·9 R	− 103·2 S	+ 39·9 U	+ 55·6 V = E′
4·25	− 99·7 P	+ 218·0 Q	− 1204·5 R	− 106·7 S	+ 40·4 U	+ 56·6 V = E′
4·5	− 102·8 P	+ 226·2 Q	− 1248·2 R	− 110·6 S	+ 40·8 U	+ 57·7 V = E′
4·75	− 106·1 P	+ 234·6 Q	− 1294·0 R	− 114·4 S	+ 41·1 U	+ 58·6 V = E′
5·0	− 109·6 P	+ 243·2 Q	− 1341·9 R	− 118·4 S	+ 41·5 U	+ 59·4 V = E′
5·25	− 112·8 P	+ 252·7 Q	− 1392·0 R	− 122·4 S	+ 41·9 U	+ 60·4 V = E′
5·5	− 116·3 P	+ 262·5 Q	− 1444·8 R	− 126·8 S	+ 42·0 U	+ 61·0 V = E′
5·75	− 119·7 P	+ 272·7 Q	− 1499·5 R	− 131·3 S	+ 42·4 U	+ 61·8 V = E′
6·0	− 123·1 P	+ 283·0 Q	− 1557·3 R	− 135·6 S	+ 42·5 U	+ 62·3 V = E′
6·125	− 124·6 P	+ 288·5 Q	− 1587·2 R	− 137·9 S	+ 42·7 U	+ 62·5 V = E′
6·25	− 126·1 P	+ 294·0 Q	− 1617·7 R	− 140·4 S	+ 42·8 U	+ 62·7 V = E′
6·375	− 127·4 P	+ 299·7 Q	− 1648·9 R	− 142·9 S	+ 42·9 U	+ 63·0 V = E′
6·5	− 128·7 P	+ 305·5 Q	− 1680·6 R	− 145·6 S	+ 42·9 U	+ 63·2 V = E′
6·625	− 130·0 P	+ 311·4 Q	− 1712·9 R	− 148·2 S	+ 42·9 U	+ 63·4 V = E′
6·75	− 131·2 P	+ 317·4 Q	− 1745·6 R	− 150·8 S	+ 42·9 U	+ 63·6 V = E′
6·875	− 132·2 P	+ 323·5 Q	− 1778·6 R	− 153·3 S	+ 43·0 U	+ 63·7 V = E′
7·0	− 133·1 P	+ 329·6 Q	− 1812·0 R	− 155·9 S	+ 43·0 U	+ 63·8 V = E′
7·125	− 133·8 P	+ 335·6 Q	− 1845·6 R	− 158·5 S	+ 43·0 U	+ 63·9 V = E′
7·25	− 134·3 P	+ 341·7 Q	− 1879·5 R	− 161·1 S	+ 42·9 U	+ 63·9 V = E′
7·375	− 134·5 P	+ 348·1 Q	− 1913·6 R	− 163·5 S	+ 42·8 U	+ 63·8 V = E′
7·5	− 134·6 P	+ 354·4 Q	− 1947·7 R	− 165·9 S	+ 42·6 U	+ 63·6 V = E′
7·625	− 134·2 P	+ 360·6 Q	− 1981·5 R	− 168·3 S	+ 42·4 U	+ 63·5 V = E′
7·75	− 133·5 P	+ 366·8 Q	− 2015·0 R	− 170·7 S	+ 42·2 U	+ 63·3 V = E′
7·875	− 132·4 P	+ 372·8 Q	− 2048·0 R	− 173·1 S	+ 42·0 U	+ 62·9 V = E′
8·0	− 130·9 P	+ 378·6 Q	− 2080·2 R	− 175·4 S	+ 41·7 U	+ 62·4 V = E′
8·125	− 128·9 P	+ 384·2 Q	− 2111·5 R	− 177·5 S	+ 41·3 U	+ 62·1 V = E′
8·25	− 126·5 P	+ 389·6 Q	− 2141·4 R	− 179·4 S	+ 40·8 U	+ 61·8 V = E′
8·375	− 123·5 P	+ 394·6 Q	− 2169·4 R	− 181·0 S	+ 40·4 U	+ 60·9 V = E′
8·5	− 119·8 P	+ 399·2 Q	− 2195·3 R	− 182·3 S	+ 39·9 U	+ 59·9 V = E′
8·625	− 115·3 P	+ 403·3 Q	− 2218·7 R	− 183·6 S	+ 39·3 U	+ 59·0 V = E′
8·75	− 110·0 P	+ 406·9 Q	− 2239·0 R	− 184·5 S	+ 38·7 U	+ 58·1 V = E′
8·875	− 103·7 P	+ 409·9 Q	− 2255·7 R	− 184·8 S	+ 38·1 U	+ 57·1 V = E′

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Right Ascensions.
1835.	″ ″ ″ ″ ″ ″
Oct. 8·875	−360·3 P +161·8 Q − 892·6 R −100·6 S +26·6 U + 42·1 V = E
9·0	−371·3 P +179·7 Q − 987·6 R −109·3 S +27·1 U + 42·6 V = E
9·125	−382·3 P +198·5 Q −1088·6 R −118·4 S +27·7 U + 43·1 V = E
9·25	−393·3 P +218·3 Q −1195·0 R −127·9 S +28·3 U + 43·5 V = E
9·375	−404·3 P +239·3 Q −1306·7 R −137·6 S +28·9 U + 43·9 V = E
9·5	−415·0 P +261·0 Q −1423·4 R −147·7 S +29·5 U + 44·2 V = E
9·625	−425·4 P +283·6 Q −1545·2 R −158·1 S +30·0 U + 44·3 V = E
9·75	−435·4 P +306·8 Q −1671·3 R −168·8 S +30·5 U + 44·3 V = E
9·875	−445·0 P +330·8 Q −1801·3 R −179·7 S +30·8 U + 44·1 V = E
10·0	−453·8 P +355·2 Q −1934·2 R −190·8 S +31·1 U + 43·8 V = E
10·125	−461·7 P +380·1 Q −2069·3 R −202·0 S +31·3 U + 43·4 V = E
10·25	−468·5 P +404·9 Q −2205·3 R −213·1 S +31·5 U + 42·8 V = E
10·375	−474·5 P +429·7 Q −2341·2 R −224·1 S +31·4 U + 42·0 V = E
10·5	−479·0 P +454·1 Q −2475·4 R −234·7 S +31·3 U + 41·0 V = E
10·625	−481·1 P +478·0 Q −2606·2 R −244·6 S +31·0 U + 39·8 V = E
10·75	−481·4 P +501·0 Q −2732·4 R −253·8 S +30·6 U + 38·4 V = E
10·875	−480·5 P +522·6 Q −2852·3 R −262·4 S +80·1 U + 36·9 V = E
11·0	−477·9 P +542·7 Q −2964·9 R −270·2 S +29·6 U + 35·2 V = E
11·125	−473·5 P +561·2 Q −3068·8 R −277·2 S +29·0 U + 33·4 V = E
11·25	−467·4 P +577·8 Q −3163·4 R −283·4 S +28·3 U + 31·4 V = E
11·375	−459·7 P +592·3 Q −3247·4 R −288·6 S +27·6 U + 29·4 V = E
11·5	−450·4 P +604·7 Q −3320·5 R −292·6 S +26·8 U + 27·3 V = E
11·625	−439·7 P +615·0 Q −3382·0 R −295·1 S +25·8 U + 25·0 V = E
11·75	−427·9 P +623·2 Q −3432·1 R −296·6 S +24·4 U + 22·7 V = E
11·875	−415·3 P +629·2 Q −3470·6 R −297·6 S +23·3 U + 20·3 V = E
12·0	−402·0 P +633·2 Q −3498·3 R −297·9 S +22·3 U + 18·0 V = E
12·125	−387·7 P +635·3 Q −3515·4 R −296·9 S +21·3 U + 15·7 V = E
12·25	−373·1 P +635·7 Q −3523·0 R −295·2 S +20·2 U + 13·5 V = E
12·375	−358·3 P +634·5 Q −3521·7 R −292·8 S +19·1 U + 11·3 V = E
12·5	−343·2 P +631·7 Q −3512·3 R −289·8 S +18·0 U + 9·1 V = E
12·625	−327·8 P +627·4 Q −3495·6 R −286·4 S +16·8 U + 7·1 V = E
12·75	−312·3 P +621·8 Q −3472·2 R −282·5 S +15·7 U + 5·2 V = E
12·875	−297·4 P +615·4 Q −3442·8 R −278·2 S +14·7 U + 3·4 V = E
13·0	−282·7 P +608·2 Q −3408·1 R −273·6 S +13·8 U + 1·6 V = E
13·125	−268·2 P +600·5 Q −3369·1 R −268·7 S +13·1 U − 0·1 V = E
13·25	−254·0 P +592·2 Q −3326·3 R −263·5 S +12·3 U − 1·7 V = E
13·375	−240·4 P +583·3 Q −3280·3 R −258·0 S +11·3 U − 3·2 V = E

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Declinations.						
1835.	″	″	″	″	″	″	
Oct. 8·875	−103·7 P	+409·9 Q	−2255·7 R	−184·8 S	+38·1 U	+ 57·1 V	= E′
9·0	− 96·4 P	+412·1 Q	−2268·1 R	−184·6 S	+37·4 U	+ 56·1 V	= E′
9·125	− 88·0 P	+413·3 Q	−2275·3 R	−184·1 S	+36·7 U	+ 54·8 V	= E′
9·25	− 78·4 P	+413·4 Q	−2277·3 R	−183·0 S	+35·9 U	+ 53·5 V	= E′
9·375	− 67·6 P	+412·4 Q	−2272·8 R	−181·1 S	+35·0 U	+ 52·1 V	= E′
9·5	− 55·4 P	+410·1 Q	−2261·1 R	−178·4 S	+34·0 U	+ 50·7 V	= E′
9·625	− 41·8 P	+406·2 Q	−2241·3 R	−175·0 S	+33·1 U	+ 49·1 V	= E′
9·75	− 26·7 P	+400·6 Q	−2212·5 R	−170·8 S	+32·1 U	+ 47·4 V	= E′
9·875	− 10·3 P	+393·3 Q	−2173·9 R	−165·8 S	+30·7 U	+ 45·8 V	= E′
10·0	+ 7·7 P	+384·1 Q	−2124·9 R	−159·8 S	+29·2 U	+ 44·1 V	= E′
10·125	+ 27·2 P	+372·7 Q	−2064·9 R	−152·8 S	+28·0 U	+ 42·4 V	= E′
10·25	+ 47·8 P	+359·3 Q	−1993·6 R	−144·9 S	+26·8 U	+ 40·7 V	= E′
10·375	+ 69·6 P	+343·7 Q	−1910·7 R	−136·1 S	+25·3 U	+ 39·0 V	= E′
10·5	+ 92·5 P	+326·0 Q	−1816·3 R	−126·2 S	+23·8 U	+ 37·4 V	= E′
10·625	+116·7 P	+306·1 Q	−1710·2 R	−115·1 S	+22·6 U	+ 36·1 V	= E′
10·75	+141·5 P	+284·3 Q	−1593·5 R	−103·2 S	+21·4 U	+ 34·9 V	= E′
10·875	+166·6 P	+260·6 Q	−1466·9 R	− 90·7 S	+20·2 U	+ 33·9 V	= E′
11·0	+191·8 P	+235·4 Q	−1331·7 R	− 77·5 S	+19·0 U	+ 33·1 V	= E′
11·125	+217·1 P	+208·9 Q	−1189·0 R	− 63·7 S	+18·2 U	+ 32·7 V	= E′
11·25	+242·0 P	+181·4 Q	−1040·8 R	− 49·6 S	+17·4 U	+ 32·5 V	= E′
11·375	+265·9 P	+153·1 Q	− 888·8 R	− 35·6 S	+16·5 U	+ 32·5 V	= E′
11·5	+288·9 P	+124·6 Q	− 734·8 R	− 21·5 S	+15·6 U	+ 32·7 V	= E′
11·625	+310·9 P	+ 96·2 Q	− 580·3 R	− 7·5 S	+15·3 U	+ 33·5 V	= E′
11·75	+331·6 P	+ 68·1 Q	− 427·0 R	+ 6·4 S	+15·0 U	+ 34·5 V	= E′
11·875	+350·6 P	+ 40·8 Q	− 277·0 R	+ 20·0 S	+14·9 U	+ 35·7 V	= E′
12·0	+368·0 P	+ 14·4 Q	− 131·3 R	+ 32·4 S	+14·8 U	+ 37·2 V	= E′
12·125	+383·6 P	− 11·0 Q	+ 8·4 R	+ 44·3 S	+14·8 U	+ 38·8 V	= E′
12·25	+397·7 P	− 35·0 Q	+ 141·5 R	+ 55·6 S	+15·0 U	+ 40·8 V	= E′
12·375	+409·5 P	− 57·2 Q	+ 267·0 R	+ 66·2 S	+15·4 U	+ 43·2 V	= E′
12·5	+419·7 P	− 77·6 Q	+ 384·5 R	+ 76·2 S	+16·0 U	+ 45·8 V	= E′
12·625	+429·0 P	− 96·0 Q	+ 493·5 R	+ 85·3 S	+16·7 U	+ 48·2 V	= E′
12·75	+436·8 P	−112·7 Q	+ 593·7 R	+ 93·4 S	+17·4 U	+ 51·0 V	= E′
12·875	+442·4 P	−127·8 Q	+ 684·8 R	+100·7 S	+18·1 U	+ 53·8 V	= E′
13·0	+446·4 P	−141·2 Q	+ 767·2 R	+106·9 S	+18·8 U	+ 56·6 V	= E′
13·125	+449·1 P	−153·1 Q	+ 840·8 R	+112·4 S	+19·6 U	+ 59·6 V	= E′
13·25	+450·5 P	−163·5 Q	+ 906·0 R	+117·2 S	+20·5 U	+ 62·5 V	= E′
13·375	+450·7 P	−172·3 Q	+ 962·9 R	+121·4 S	+21·4 U	+ 65·3 V	= E′

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Right Ascensions.
1835.	″ ″ ″ ″ ″ ″
Oct. 13·375	−240·4 P +583·3 Q −3280·3 R −258·0 S +11·3 U − 3·2 V = E
13·5	−227·2 P +574·2 Q −3231·5 R −252·4 S +10·3 U − 4·6 V = E
13·625	−214·2 P +565·2 Q −3180·3 R −246·8 S + 9·6 U − 5·9 V = E
13·75	−201·6 P +556·1 Q −3127·2 R −241·1 S + 8·9 U − 7·1 V = E
13·875	−189·6 P +546·7 Q −3072·4 R −235·4 S + 8·2 U − 8·1 V = E
14·0	−178·0 P +537·0 Q −3016·5 R −229·7 S + 7·6 U − 9·0 V = E
14·125	−166·8 P +527·1 Q −2959·8 R −224·0 S + 7·0 U − 10·0 V = E
14·25	−156·2 P +517·2 Q −2902·7 R −218·3 S + 6·5 U − 11·0 V = E
14·375	−146·2 P +507·3 Q −2845·5 R −212·6 S + 5·8 U − 11·7 V = E
14·5	−136·6 P +497·3 Q −2788·2 R −207·0 S + 5·2 U − 12·3 V = E
14·625	−127·3 P +487·2 Q −2730·8 R −201·4 S + 4·7 U − 13·0 V = E
14·75	−118·3 P +477·0 Q −2673·6 R −195·9 S + 4·3 U − 13·6 V = E
14·875	−109·8 P +466·6 Q −2616·8 R −190·6 S + 3·9 U − 14·1 V = E
15·0	−101·7 P +456·3 Q −2560·6 R −185·5 S + 3·6 U − 14·6 V = E
15·125	− 93·9 P +446·2 Q −2505·1 R −180·4 S + 3·2 U − 15·1 V = E
15·25	− 86·6 P +436·3 Q −2450·5 R −175·3 S + 2·9 U − 15·6 V = E
15·375	− 79·8 P +426·7 Q −2396·8 R −170·4 S + 2·4 U − 16·0 V = E
15·5	− 73·3 P +417·3 Q −2344·1 R −165·6 S + 2·0 U − 16·3 V = E
15·625	− 67·0 P +408·1 Q −2292·3 R −160·9 S + 1·6 U − 16·7 V = E
15·75	− 60·9 P +399·1 Q −2241·4 R −156·2 S + 1·3 U − 17·0 V = E
15·875	− 55·3 P +390·4 Q −2191·7 R −151·8 S + 1·0 U − 17·3 V = E
16·0	− 49·9 P +381·9 Q −2142·9 R −147·5 S + 0·8 U − 17·5 V = E
16·25	− 39·8 P +365·3 Q −2048·3 R −139·0 S + 0·3 U − 17·9 V = E
16·5	− 30·8 P +349·6 Q −1958·7 R −131·4 S − 0·2 U − 18·3 V = E
16·75	− 22·3 P +334·5 Q −1873·1 R −123·8 S − 0·5 U − 18·5 V = E
17·0	− 15·2 P +320·0 Q −1791·9 R −117·0 S − 0·8 U − 18·5 V = E
17·25	− 8·2 P +306·3 Q −1714·4 R −110·5 S − 1·0 U − 18·6 V = E
17·5	− 2·6 P +293·3 Q −1641·7 R −104·3 S − 1·3 U − 18·6 V = E
17·75	+ 3·0 P +280·9 Q −1572·6 R − 98·3 S − 1·4 U − 18·6 V = E
18·0	+ 7·9 P +269·4 Q −1507·4 R − 92·8 S − 1·6 U − 18·4 V = E
18·25	+ 12·7 P +258·4 Q −1445·2 R − 87·4 S − 1·7 U − 18·4 V = E
18·5	+ 16·2 P +248·1 Q −1387·0 R − 82·9 S − 1·8 U − 18·2 V = E
18·75	+ 20·3 P +238·3 Q −1331·9 R − 78·1 S − 1·9 U − 18·1 V = E
19·0	+ 23·3 P +228·8 Q −1280·0 R − 74·1 S − 1·8 U − 18·0 V = E
19·25	+ 26·6 P +220·1 Q −1230·3 R − 69·8 S − 2·0 U − 17·9 V = E
19·5	+ 29·1 P +211·9 Q −1183·7 R − 66·4 S − 2·4 U − 17·6 V = E
19 75	+ 31·8 P +204·0 Q −1139·1 R − 62·5 S − 2·2 U − 17·6 V = E

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Declinations.
1835.	
Oct. 13·375	+450″·7 P −172″·3 Q + 962″·9 R +121″·4 S +21″·4 U + 65″·3 V = E′
13·5	+449·6 P −130·0 Q +1012·0 R +124·9 S +22·3 U + 68·1 V = E′
13·625	+447·6 P −187·3 Q +1053·6 R +127·7 S +23·2 U + 70·9 V = E′
13·75	+444·5 P −193·7 Q +1088·0 R +129·8 S +24·0 U + 73·6 V = E′
13·875	+440·3 P −198·8 Q +1115·6 R +131·1 S +24·6 U + 76·1 V = E′
14·0	+435·5 P −202·8 Q +1137·0 R +131·9 S +25·2 U + 78·5 V = E′
14·125	+430·2 P −205·8 Q +1153·1 R +132·5 S +26·1 U + 80·9 V = E′
14·25	+424·2 P −207·9 Q +1163·9 R +132·6 S +27·0 U + 83·2 V = E′
14·375	+417·3 P −209·4 Q +1169·6 R +132·1 S +27·5 U + 85·3 V = E′
14·5	+410·0 P −210·2 Q +1171·0 R +131·2 S +28·0 U + 87·3 V = E′
14·625	+402·8 P −209·8 Q +1168·6 R +130·2 S +28·7 U + 89·2 V = E′
14·75	+395·4 P −208·6 Q +1162·6 R +128·9 S +29·3 U + 91·0 V = E′
14·875	+387·6 P −206·6 Q +1153·3 R +127·4 S +29·7 U + 92·5 V = E′
15·0	+379·5 P −203·9 Q +1141·4 R +125·6 S +30·1 U + 93·9 V = E′
15·125	+371·5 P −201·0 Q +1127·5 R +123·8 S +30·6 U + 95·3 V = E′
15·25	+363·4 P −197·9 Q +1111·5 R +121·8 S +31·1 U + 96·5 V = E′
15·375	+354·9 P −194·5 Q +1093·5 R +119·4 S +31·3 U + 97·5 V = E′
15·5	+346·5 P −190·9 Q +1074·1 R +116·9 S +31·4 U + 98·3 V = E′
15·625	+338·2 P −187·0 Q +1054·0 R +114·6 S +31·7 U + 99·1 V = E′
15·75	+330·0 P −182·9 Q +1032·9 R +112·2 S +31·9 U + 99·8 V = E′
15·875	+321·6 P −178·8 Q +1010·9 R +109·6 S +31·9 U +100·3 V = E′
16·0	+313·2 P −174·6 Q + 988·0 R +106·9 S +31·9 U +100·7 V = E′
16·25	+297·3 P −165·9 Q + 941·0 R +101·8 S +32·1 U +101·5 V = E′
16·5	+281·7 P −157·3 Q + 892·8 R + 96·6 S +31·8 U +101·7 V = E′
16·75	+266·9 P −149·0 Q + 843·8 R + 91·6 S +31·9 U +101·8 V = E′
17·0	+252·5 P −141·0 Q + 795·4 R + 86·5 S +31·5 U +101·5 V = E′
17·25	+239·4 P −132·9 Q + 748·8 R + 81·9 S +31·5 U +101·2 V = E′
17·5	+226·3 P −125·4 Q + 703·6 R + 77·2 S +30·6 U +100·4 V = E′
17·75	+214·4 P −117·6 Q + 661·0 R + 73·0 S +30·4 U + 99·7 V = E′
18·0	+202·8 P −110·1 Q + 620·1 R + 68·6 S +29·7 U + 98·7 V = E′
18·25	+192·3 P −102·9 Q + 581·7 R + 64·8 S +29·2 U + 97·6 V = E′
18·5	+182·2 P − 96·3 Q + 545·0 R + 61·0 S +28·4 U + 96·4 V = E′
18·75	+172·9 P − 89·9 Q + 510·4 R + 57·6 S +27·9 U + 95·3 V = E′
19·0	+164·2 P − 84·0 Q + 477·8 R + 54·3 S +27·4 U + 94·2 V = E′
19·25	+155·9 P − 78·6 Q + 446·8 R + 51·0 S +26·7 U + 92·8 V = E′
19·5	+148·1 P − 73·7 Q + 417·7 R + 48·2 S +26·1 U + 91·5 V = E′
19·75	+141·6 P − 69·1 Q + 390·6 R + 45·6 S +25·5 U + 90·1 V = E′

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Right Ascensions.
1835.	″ ″ ″ ″ ″ ″
Oct. 19·75	+ 31·8 P + 204·0 Q − 1139·1 R − 62·5 S − 2·2 U − 17·6 V = E
20·0	+ 33·8 P + 196·3 Q − 1097·5 R − 59·5 S − 2·6 U − 17·6 V = E
20·25	+ 35·9 P + 189·3 Q − 1057·5 R − 56·2 S − 2·6 U − 17·4 V = E
20·5	+ 37·9 P + 182·6 Q − 1019·4 R − 53·2 S − 2·4 U − 17·0 V = E
20·75	+ 39·8 P + 176·2 Q − 983·4 R − 50·1 S − 2·3 U − 16·6 V = E
21·0	+ 41·2 P + 170·1 Q − 949·8 R − 47·5 S − 2·4 U − 16·5 V = E
21·25	+ 42·8 P + 164·3 Q − 916·9 R − 44·7 S − 2·1 U − 16·3 V = E
21·5	+ 44·0 P + 158·9 Q − 886·4 R − 42·4 S − 2·4 U − 16·0 V = E
21·75	+ 45·5 P + 153·6 Q − 856·9 R − 39·8 S − 2·3 U − 15·9 V = E
22·0	+ 46·4 P + 149·0 Q − 829·0 R − 37·9 S − 2·5 U − 15·7 V = E
22·25	+ 47·9 P + 144·4 Q − 802·0 R − 35·6 S − 2·1 U − 15·5 V = E
22·5	+ 48·3 P + 139·9 Q − 777·0 R − 34·0 S − 2·4 U − 15·3 V = E
22·75	+ 49·6 P + 135·5 Q − 752·8 R − 31·7 S − 2·2 U − 15·1 V = E
23·0	+ 50·3 P + 131·4 Q − 729·9 R − 30·2 S − 2·2 U − 14·8 V = E
23·25	+ 51·2 P + 127·2 Q − 707·8 R − 28·2 S − 2·0 U − 14·6 V = E
23·5	+ 51·6 P + 123·4 Q − 687·0 R − 26·8 S − 2·1 U − 14·4 V = E
23·75	+ 52·6 P + 120·2 Q − 666·7 R − 24·9 S − 1·9 U − 14·1 V = E
24·0	+ 53·0 P + 116·6 Q − 647·6 R − 23·6 S − 2·1 U − 14·0 V = E
24·25	+ 53·8 P + 113·2 Q − 629·0 R − 22·0 S − 1·8 U − 14·0 V = E
24·5	+ 54·1 P + 110·2 Q − 611·8 R − 20·9 S − 2·2 U − 13·9 V = E
24·75	+ 54·9 P + 107·1 Q − 594·7 R − 19·2 S − 1·9 U − 13·6 V = E
25·0	+ 55·1 P + 104·2 Q − 578·8 R − 18·3 S − 2·2 U − 13·5 V = E
25·25	+ 55·7 P + 101·5 Q − 562·8 R − 16·8 S − 1·8 U − 13·3 V = E
25·5	+ 56·0 P + 98·8 Q − 548·3 · − 15·9 S − 2·1 U − 13·1 V = E
25·75	+ 56·6 P + 96·0 Q − 533·8 R − 14·4 S − 1·8 U − 13·0 V = E
26·0	+ 56·7 P + 93·4 Q − 520·4 R − 13·6 S − 2·1 U − 12·9 V = E
26·25	+ 57·4 P + 90·9 Q − 507·1 R − 12·3 S − 1·7 U − 12·4 V = E
26·5	+ 57·3 P + 88·5 Q − 494·8 R − 11·8 S − 2·0 U − 12·6 V = E
26·75	+ 58·0 P + 86·3 Q − 482·3 R − 10·5 S − 1·7 U − 12·4 V = E
27·0	+ 58·1 P + 84·3 Q − 470·8 R − 9·8 S − 1·9 U − 12·3 V = E
27·25	+ 58·7 P + 82·4 Q − 459·0 R − 8·6 S − 1·5 U − 11·9 V = E
27·5	+ 58·5 P + 80·4 Q − 448·6 R − 8·1 S − 1·9 U − 12·0 V = E
27·75	+ 59·2 P + 78·7 Q − 437·7 R − 6·9 S − 1·6 U − 11·5 V = E
28·0	+ 59·1 P + 77·1 Q − 427·9 R − 6·4 S − 1·8 U − 11·6 V = E
28·25	+ 59·8 P + 75·6 Q − 417·7 R − 5·2 S − 1·2 U − 11·2 V = E
28·5	+ 59·4 P + 73·6 Q − 409·0 R − 5·0 S − 1·9 U − 11·4 V = E
29·5	+ 60·4 P + 67·5 Q − 375·4 R − 2·2 S − 1·6 U − 10·8 V = E

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Declinations.
1835.	″ ″ ″ ″ ″ ″
Oct. 19·75	+ 141·6 P − 69·1 Q + 390·6 R + 45·6 S + 25·5 U + 90·1 V = E′
20·0	+ 134·0 P − 64·7 Q + 364·7 R + 43·0 S + 24·7 U + 88·8 V = E′
20·25	+ 127·8 P − 60·7 Q + 340·8 R + 40·8 S + 24·2 U + 87·4 V = E′
20·5	+ 121·8 P − 57·0 Q + 318·0 R + 38·5 S + 23·3 U + 86·0 V = E′
20·75	+ 116·3 P − 53·3 Q + 297·1 R + 36·5 S + 22·8 U + 84·6 V = E′
21·0	+ 110·9 P − 49·7 Q + 277·4 R + 34·3 S + 21·9 U + 83·8 V = E′
21·25	+ 106·3 P − 46·2 Q + 259·4 R + 32·8 S + 21·5 U + 82·1 V = E′
21·5	+ 101·5 P − 43·0 Q + 242·3 R + 30·9 S + 20·8 U + 80·8 V = E′
21·75	+ 97·4 P − 39·8 Q + 226·3 R + 29·5 S + 20·4 U + 79·6 V = E′
22·0	+ 93·1 P − 37·0 Q + 211·4 R + 27·8 S + 19·7 U + 78·3 V = E′
22·25	+ 89·3 P − 34·3 Q + 197·6 R + 26·6 S + 19·2 U + 77·3 V = E′
22·5	+ 85·6 P − 31·9 Q + 184·3 R + 25·1 S + 18·6 U + 75·9 V = E′
22·75	+ 82·3 P − 29·9 Q + 172·0 R + 23·9 S + 18·1 U + 74·7 V = E′
23·0	+ 79·0 P − 27·9 Q + 160·3 R + 22·8 S + 17·5 U + 73·6 V = E′
23·25	+ 76·2 P − 26·0 Q + 149·5 R + 21·8 S + 17·1 U + 72·6 V = E′
23·5	+ 73·2 P − 24·3 Q + 139·2 R + 20·7 S + 16·5 U + 71·5 V = E′
23·75	+ 70·5 P − 22·7 Q + 129·5 R + 19·7 S + 16·0 U + 70·3 V = E′
24·0	+ 67·6 P − 21·2 Q + 120·3 R + 18·8 S + 15·5 U + 69·3 V = E′
24·25	+ 65·6 P − 19·2 Q + 111·8 R + 18·1 S + 15·2 U + 68·3 V = E′
24·5	+ 63·3 P − 17·9 Q + 103·7 R + 17·2 S + 14·6 U + 67·3 V = E′
24·75	+ 61·1 P − 16·6 Q + 96·3 R + 16·5 S + 14·3 U + 66·4 V = E′
25·0	+ 59·0 P − 15·3 Q + 89·3 R + 15·7 S + 13·8 U + 65·4 V = E′
25·25	+ 57·1 P − 14·1 Q + 82·8 R + 15·1 S + 13·4 U + 64·3 V = E′
25·5	+ 55·3 P − 13·1 Q + 76·7 R + 14·6 S + 13·0 U + 63·7 V = E′
25·75	+ 53·6 P − 12·2 Q + 70·7 R + 14·0 S + 12·6 U + 62·7 V = E′
26·0	+ 51·9 P − 11·4 Q + 65·2 R + 13·4 S + 12·2 U + 61·9 V = E′
26·25	+ 50·3 P − 10·6 Q + 59·8 R + 12·9 S + 11·8 U + 61·0 V = E′
26·5	+ 48·6 P − 10·0 Q + 54·7 R + 12·2 S + 11·3 U + 60·0 V = E′
26·75	+ 47·4 P − 9·0 Q + 50·3 R + 12·0 S + 11·2 U + 59·5 V = E′
27·0	+ 46·1 P − 8·1 Q + 45·9 R + 11·6 S + 10·8 U + 58·7 V = E′
27·25	+ 44·8 P − 7·5 Q + 41·8 R + 11·1 S + 10·4 U + 57·9 V = E′
27·5	+ 43·5 P − 6·7 Q + 37·9 R + 10·7 S + 10·1 U + 57·2 V = E′
27·75	+ 42·3 P − 6·1 Q + 34·3 R + 10·4 S + 9·7 U + 56·4 V = E′
28·0	+ 41·1 P − 5·4 Q + 30·7 R + 9·9 S + 9·3 U + 55·7 V = E′
28·25	+ 40·1 P − 4·7 Q + 27·6 R + 9·7 S + 9·2 U + 55·0 V = E′
28·5	+ 39·0 P − 4·1 Q + 24·3 R + 9·3 S + 8·7 U + 53·8 V = E′
29 5	+ 35·4 P − 2·1 Q + 13·4 R + 8·2 S + 7·6 U + 51·7 V = E′

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Right Ascensions.
1835.	″ ″ ″ ″ ″ ″
Oct. 29·5	+ 60·4 P + 67·5 Q − 375·4 R − 2·2 S − 1·6 U − 10·8 V = E
30·5	+ 61·2 P + 62·2 Q − 345·9 R + 0·3 S − 1·4 U − 10·4 V = E
31·5	+ 61·9 P + 57·4 Q − 320·7 R + 2·4 S − 1·4 U − 9·9 V = E
Nov. 1·5	+ 62·5 P + 53·8 Q − 298·8 R + 4·3 S − 1·3 U − 9·5 V = E
2·5	+ 63·0 P + 50·2 Q − 279·9 R + 6·0 S − 1·0 U − 9·2 V = E
3·5	+ 63·7 P + 47·5 Q − 263·3 R + 7·6 S − 1·0 U − 8·7 V = E
4·5	+ 64·2 P + 44·3 Q − 249·8 R + 8·9 S − 1·0 U − 8·2 V = E
5·5	+ 64·7 P + 42·8 Q − 237·8 R + 10·1 S − 0·8 U − 7·7 V = E
6·5	+ 64·9 P + 41·0 Q − 228·2 R + 11·1 S − 0·8 U − 7·5 V = E
7·5	+ 65·6 P + 39·5 Q − 220·1 R + 12·3 S − 0·8 U − 7·3 V = E
8·5	+ 66·1 P + 38·2 Q − 213·5 R + 13·3 S − 0·7 U − 6·9 V = E
9·5	+ 66·5 P + 37·5 Q − 208·5 R + 14·1 S − 0·6 U − 6·7 V = E
10·5	+ 66·8 P + 36·7 Q − 204·9 R + 15·0 S − 0·6 U − 6·3 V = E
11·5	+ 67·0 P + 36·1 Q − 202·2 R + 15·6 S − 0·4 U − 6·1 V = E
12·5	+ 67·7 P + 35·8 Q − 200·6 R + 16·8 S − 0·5 U − 5·9 V = E
13·5	+ 67·7 P + 35·8 Q − 200·0 R + 17·0 S − 0·1 U − 5·5 V = E
14·5	+ 68·0 P + 36·1 Q − 200·3 R + 17·6 S − 0·4 U − 5·4 V = E
15·5	+ 68·1 P + 36·0 Q − 201·2 R + 18·3 S − 0·2 U − 4·9 V = E
16·5	+ 68·0 P + 36·1 Q − 203·1 R + 18·7 S − 0·3 U − 4·9 V = E
17·5	+ 68·1 P + 36·7 Q − 205·3 R + 19·0 S − 0·2 U − 4·5 V = E
18·5	+ 68·1 P + 37·2 Q − 208·1 R + 19·9 S − 0·1 U − 4·3 V = E
19·5	+ 68·0 P + 37·7 Q − 211·2 R + 20·2 S * − 4·0 V = E
20·5	+ 67·9 P + 38·2 Q − 214·9 R + 21·1 S − 0·1 U − 3·8 V = E
21·5	+ 67·6 P + 38·8 Q − 218·9 R + 20·9 S − 0·1 U − 3·7 V = E
22·5	+ 67·3 P + 39·3 Q − 222·9 R + 21·2 S − 0·2 U − 3·3 V = E
23·5	+ 67·0 P + 40·3 Q − 226·9 R + 21·7 S * − 3·2 V = E
24·5	+ 66·7 P + 41·2 Q − 231·3 R + 22·0 S + 0·2 U − 3·1 V = E
25·5	+ 66·4 P + 42·0 Q − 235·7 R + 22·3 S + 0·1 U − 2·9 V = E
26·5	+ 65·8 P + 42·6 Q − 240·2 R + 22·6 S * − 2·8 V = E
27·5	+ 65·4 P + 43·4 Q − 244·1 R + 22·9 S * − 2·6 V = E
28·5	+ 64·9 P + 44·0 Q − 248·7 R + 23·2 S * − 2·4 V = E
29·5	+ 64·4 P + 44·7 Q − 252·7 R + 23·5 S + 0·1 U − 2·2 V = E
30·5	+ 63·9 P + 45·4 Q − 256·9 R + 23·8 S + 0·1 U − 2·1 V = E
Dec. 1·5	+ 63·5 P + 46·3 Q − 261·0 R + 24·3 S + 0·2 U − 1·8 V = E
2·5	+ 62·9 P + 47·1 Q − 264·7 R + 24·3 S + 0·1 U − 1·7 V = E
3·5	+ 62·2 P + 47·7 Q − 268·6 R + 24·6 S * − 1·6 V = E
4·5	+ 61·7 P + 48·4 Q − 272·7 R + 24·8 S + 0·2 U − 1·4 V = E

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Declinations.
1835.	″ ″ ″ ″ ″ ″
Oct. 29·5	+ 35·4 P − 2·1 Q + 13·4 R + 8·2 S + 7·6 U + 51·7 V = E′
30·5	+ 32·3 P − 0·8 Q + 4·6 R + 7·4 S + 6·6 U + 49·3 V = E′
31·5	+ 29·6 P + 0·5 Q − 2·6 R + 6·7 S + 5·6 U + 47·1 V = E′
Nov. 1·5	+ 27·4 P + 1·6 Q − 8·4 R + 6·1 S + 4·7 U + 44·9 V = E′
2·5	+ 25·5 P + 2·5 Q − 13·1 R + 5·7 S + 4·1 U + 43·1 V = E′
3·5	+ 23·7 P + 3·0 Q − 17·1 R + 5·2 S + 3·1 U + 41·2 V = E′
4·5	+ 22·4 P + 3·7 Q − 20·4 R + 4·9 S + 2·5 U + 39·6 V = E′
5·5	+ 21·2 P + 4·2 Q − 23·1 R + 4·7 S + 1·9 U + 38·0 V = E′
6·5	+ 19·7 P + 4·6 Q − 25·6 R + 4·6 S + 1·2 U + 36·4 V = E′
7·5	+ 19·1 P + 5·0 Q − 27·6 R + 4·3 S + 0·7 U + 35·0 V = E′
8·5	+ 18·3 P + 5·3 Q − 29·4 R + 4·2 S + 0·1 U + 33·6 V = E′
9·5	+ 17·5 P + 5·5 Q − 31·1 R + 4·0 S − 0·4 U + 32·3 V = E′
10·5	+ 16·3 P + 5·8 Q − 32·6 R + 3·9 S − 0·9 U + 31·0 V = E′
11·5	+ 16·0 P + 5·8 Q − 34·1 R + 3·7 S − 1·5 U + 29·6 V = E′
12·5	+ 15·6 P + 6·2 Q − 35·0 R + 3·9 S − 1·7 U + 28·6 V = E′
13·5	+ 15·1 P + 6·4 Q − 36·3 R + 3·8 S − 1·8 U + 27·5 V = E′
14·5	+ 14·6 P + 6·8 Q − 37·3 R + 3·8 S − 2·5 U + 26·4 V = E′
15·5	+ 14·2 P + 6·8 Q − 38·2 R + 3·8 S − 2·9 U + 25·3 V = E′
16·5	+ 13·9 P + 7·1 Q − 39·3 R + 3·9 S − 3·1 U + 24·3 V = E′
17·5	+ 13·6 P + 7·3 Q − 40·0 R + 3·9 S − 3·4 U + 23·3 V = E′
18·5	+ 13·2 P + 7·4 Q − 41·2 R + 3·6 S − 3·8 U + 22·3 V = E′
19·5	+ 12·8 P + 7·5 Q − 42·2 R + 3·8 S − 4·1 U + 21·3 V = E′
20·5	+ 12·4 P + 7·7 Q − 43·2 R + 3·8 S − 4·4 U + 20·2 V = E′
21·5	+ 12·1 P + 7·8 Q − 44·1 R + 3·7 S − 4·7 U + 19·3 V = E′
22·5	+ 11·9 P + 7·7 Q − 44·7 R + 3·8 S − 4·6 U + 18·4 V = E′
23·5	+ 11·6 P + 8·2 Q − 45·7 R + 3·9 S − 5·1 U + 17·5 V = E′
24·5	+ 11·3 P + 8·3 Q − 46·6 R + 3·9 S − 5·4 U + 16·6 V = E′
25·5	+ 11·0 P + 8·5 Q − 47·4 R + 3·9 S − 5·6 U + 15·7 V = E′
26·5	+ 10·7 P + 8·4 Q − 48·3 R + 3·9 S − 5·8 U + 14·8 V = E′
27·5	+ 10·5 P + 8·7 Q − 48·9 R + 3·9 S − 6·1 U + 14·0 V = E′
28·5	+ 10·5 P + 8·8 Q − 49·7 R + 4·0 S − 6·2 U + 13·1 V = E′
29·5	+ 10·0 P + 8·9 Q − 50·3 R + 4·0 S − 6·4 U + 12·3 V = E′
30·5	+ 9·8 P + 9·1 Q − 51·0 R + 4·1 S − 6·6 U + 11·5 V = E′
Dec. 1·5	+ 9·6 P + 9·2 Q − 51·8 R + 4·2 S − 6·8 U + 10·7 V = E′
2·5	+ 9·4 P + 9·3 Q − 52·4 R + 4·1 S − 7·0 U + 9·9 V = E′
3·5	+ 9·1 P + 9·4 Q − 53·1 R + 4·1 S − 7·2 U + 9·0 V = E′
4·5	+ 8·9 P + 9·5 Q − 53·8 R + 4·2 S − 7·2 U + 8·3 V = E′

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Right Ascensions.
1835.	″ ″ ″ ″ ″ ″
Dec. 4·5	+ 61·7 P + 48·4 Q − 272·7 R + 24·8 S + 0·2 U − 1·4 V = E
5·5	+ 61·2 P + 49·3 Q − 275·4 R + 25·2 S + 0·2 U − 1·1 V = E
6·5	+ 60·6 P + 49·5 Q − 279·1 R + 25·4 S + 0·1 U − 1·1 V = E
7·5	+ 59·9 P + 50·1 Q − 282·7 R + 25·6 S + 0·1 U − 1·0 V = E
8·5	+ 59·5 P + 50·8 Q − 285·6 R + 25·9 S + 0·1 U − 0·8 V = E
9·5	+ 58·9 P + 51·5 Q − 289·1 R + 26·1 S + 0·1 U − 0·7 V = E
10·5	+ 58·3 P + 51·6 Q − 292·1 R + 26·3 S + 0·1 U − 0·7 V = E
11·5	+ 57·9 P + 52·4 Q − 295·1 R + 26·6 S + 0·2 U − 0·5 V = E
12·5	+ 57·5 P + 52·7 Q − 297·7 R + 26·8 S + 0·2 U − 0·3 V = E
13·5	+ 56·9 P + 53·3 Q − 300·6 R + 27·1 S + 0·1 U − 0·3 V = E
14·5	+ 56·5 P + 54·0 Q − 303·5 R + 27·3 S + 0·2 U − 0·1 V = E
15·5	+ 55·9 P + 54·4 Q − 306·1 R + 27·5 S + 0·2 U * = E
16·5	+ 55·5 P + 54·7 Q − 308·7 R + 27·8 S + 0·1 U + 0·1 V = E
17·5	+ 55·1 P + 55·4 Q − 311·2 R + 28·1 S + 0·2 U + 0·3 V = E
18·5	+ 54·7 P + 56·0 Q − 313·6 R + 28·3 S + 0·1 U + 0·3 V = E
19·5	+ 54·2 P + 56·6 Q − 316·6 R + 28·4 S * + 0·4 V = E
20·5	+ 53·9 P + 56·8 Q − 318·9 R + 28·8 S * + 0·5 V = E
21·5	+ 53·5 P + 57·0 Q − 321·9 R + 29·0 S * + 0·6 V = E
22·5	+ 52·9 P + 57·3 Q − 324·2 R + 29·1 S − 0·3 U + 0·4 V = E
23·5	+ 52·9 P + 58·3 Q − 326·6 R + 29·6 S * + 0·7 V = E
24·5	+ 52·5 P + 59·0 Q − 328·9 R + 29·9 S + 0·1 U + 0·9 V = E
25·5	+ 52·2 P + 58·9 Q − 331·5 R + 30·1 S + 0·1 U + 1·1 V = E
26·5	+ 51·9 P + 59·7 Q − 334·1 R + 30·5 S * + 1·0 V = E
27·5	+ 51·6 P + 60·0 Q − 336·7 R + 30·7 S * + 1·1 V = E
28·5	+ 51·3 P + 60·5 Q − 339·2 R + 31·0 S − 0·1 U + 1·2 V = E
29·5	+ 51·1 P + 61·0 Q − 341·8 R + 31·4 S − 0·1 U + 1·3 V = E
30·5	+ 50·9 P + 61·5 Q − 345·2 R + 31·7 S − 0·1 U + 1·4 V = E
31·5	+ 50·7 P + 62·3 Q − 347·6 R + 32·0 S * + 1·5 V = E
1836.	
Jan. 1·5	+ 50·4 P + 62·0 Q − 351·2 R + 32·3 S − 0·1 U + 1·5 V = E
2·5	+ 50·3 P + 62·6 Q − 353·5 R + 32·6 S − 0·2 U + 1·6 V = E
3·5	+ 50·1 P + 63·6 Q − 356·0 R + 33·0 S − 0·2 U + 1·7 V = E
4·5	+ 49·9 P + 63·7 Q − 359·5 R + 33·3 S − 0·2 U + 1·8 V = E
5·5	+ 49·8 P + 65·0 Q − 362·0 R + 33·7 S − 0·2 U + 1·8 V = E
6·5	+ 49·7 P + 65·2 Q − 365·8 R + 34·1 S − 0·1 U + 2·0 V = E
7·5	+ 49·5 P + 65·6 Q − 368·6 R + 34·5 S − 0·3 U + 2·0 V = E
8·5	+ 49·4 P + 65·7 Q − 372·4 R + 34·9 S − 0·3 U + 1·9 V = E
9·5	+ 49·3 P + 67·1 Q − 375·1 R + 35·3 S − 0·3 U + 2·1 V = E

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Declinations.						
1835.	″	″	″	″	″	″	
Dec. 4.5	+ 8.9 P	+ 9.5 Q	− 53.8 R	+ 4.2 S	− 7.2 U	+ 8.3 V	= E′
5.5	+ 8.8 P	+ 9.8 Q	− 53.5 R	+ 4.3 S	− 7.3 U	+ 7.5 V	= E′
6.5	+ 8.5 P	+ 9.8 Q	− 55.0 R	+ 4.3 S	− 7.6 U	+ 6.7 V	= E′
7.5	+ 8.4 P	+ 9.9 Q	− 55.5 R	+ 4.4 S	− 7.7 U	+ 6.0 V	= E′
8.5	+ 8.2 P	+ 10.0 Q	− 56.1 R	+ 4.5 S	− 7.9 U	+ 5.3 V	= E′
9.5	+ 8.1 P	+ 10.2 Q	− 56.7 R	+ 4.4 S	− 8.0 U	+ 4.5 V	= E′
10.5	+ 7.8 P	+ 10.1 Q	− 57.7 R	+ 4.5 S	− 8.2 U	+ 3.7 V	= E′
11.5	+ 7.8 P	+ 10.4 Q	− 57.8 R	+ 4.6 S	− 8.3 U	+ 3.1 V	= E′
12.5	+ 7.6 P	+ 10.5 Q	− 58.4 R	+ 4.7 S	− 8.4 U	+ 2.4 V	= E′
13.5	+ 7.4 P	+ 10.6 Q	− 59.1 R	+ 4.7 S	− 8.6 U	+ 1.6 V	= E′
14.5	+ 7.3 P	+ 10.7 Q	− 59.7 R	+ 4.7 S	− 8.7 U	+ 0.9 V	= E′
15.5	+ 7.1 P	+ 10.7 Q	− 60.2 R	+ 4.9 S	− 8.8 U	+ 0.1 V	= E′
16.5	+ 7.0 P	+ 10.9 Q	− 60.7 R	+ 5.1 S	− 9.0 U	− 0.6 V	= E′
17.5	+ 6.8 P	+ 11.0 Q	− 61.5 R	+ 5.0 S	− 9.1 U	− 1.3 V	= E′
18.5	+ 6.8 P	+ 11.2 Q	− 62.0 R	+ 5.1 S	− 9.2 U	− 2.0 V	= E′
19.5	+ 6.6 P	+ 11.3 Q	− 62.6 R	+ 5.2 S	− 9.3 U	− 2.6 V	= E′
20.5	+ 6.5 P	+ 11.4 Q	− 63.2 R	+ 5.2 S	− 9.5 U	− 3.3 V	= E′
21.5	+ 6.4 P	+ 11.5 Q	− 64.0 R	+ 5.4 S	− 9.5 U	− 4.0 V	= E′
22.5	+ 6.0 P	+ 11.4 Q	− 64.8 R	+ 5.2 S	− 10.0 U	− 5.0 V	= E′
23.5	+ 6.1 P	+ 11.7 Q	− 65.3 R	+ 5.4 S	− 9.9 U	− 5.4 V	= E′
24.5	+ 6.1 P	+ 12.1 Q	− 65.8 R	+ 5.6 S	− 10.1 U	− 6.1 V	= E′
25.5	+ 6.0 P	+ 12.0 Q	− 66.4 R	+ 5.7 S	− 10.0 U	− 6.8 V	= E′
26.5	+ 5.8 P	+ 12.1 Q	− 67.2 R	+ 5.9 S	− 10.2 U	− 7.5 V	= E′
27.5	+ 5.7 P	+ 12.3 Q	− 67.6 R	+ 6.0 S	− 10.3 U	− 8.1 V	= E′
28.5	+ 5.5 P	+ 12.3 Q	− 68.5 R	+ 6.0 S	− 10.5 U	− 9.0 V	= E′
29.5	+ 5.4 P	+ 12.4 Q	− 69.2 R	+ 6.2 S	− 10.6 U	− 9.7 V	= E′
30.5	+ 5.4 P	+ 12.6 Q	− 69.9 R	+ 6.3 S	− 10.7 U	− 10.3 V	= E′
31.5	+ 5.0 P	+ 12.7 Q	− 70.6 R	+ 6.3 S	− 10.8 U	− 11.0 V	= E′
1836.							
Jan. 1.5	+ 5.2 P	+ 13.0 Q	− 71.1 R	+ 6.5 S	− 10.9 U	− 11.7 V	= E′
2.5	+ 5.0 P	+ 12.9 Q	− 72.0 R	+ 6.5 S	− 11.0 U	− 12.5 V	= E′
3.5	+ 5.1 P	+ 13.3 Q	− 72.4 R	+ 6.9 S	− 11.0 U	− 13.0 V	= E′
4.5	+ 4.9 P	+ 13.2 Q	− 73.3 R	+ 6.9 S	− 11.2 U	− 13.9 V	= E′
5.5	+ 4.7 P	+ 13.4 Q	− 73.9 R	+ 7.0 S	− 11.5 U	− 14.6 V	= E′
6.5	+ 4.6 P	+ 13.5 Q	− 74.7 R	+ 7.1 S	− 11.6 U	− 15.4 V	= E′
7.5	+ 4.5 P	+ 13.6 Q	− 75.4 R	+ 7.2 S	− 11.8 U	− 16.2 V	= E′
8.5	+ 4.1 P	+ 13.7 Q	− 76.0 R	+ 7.4 S	− 11.8 U	− 16.8 V	= E′
9.5	+ 4.4 P	+ 14.0 Q	− 76.6 R	+ 7.5 S	− 12.0 U	− 17.5 V	= E′

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Right Ascensions.
1836.	
Jan. 9·5	+ 49″·3 P + 67″·1 Q − 375″·1 R + 35″·3 S − 0″·3 U + 2″·1 V = E
10·5	+ 49·2 P + 67·9 Q − 378·7 R + 35·7 S − 0·4 U + 2·1 V = E
11·5	+ 49·1 P + 68·7 Q − 382·3 R + 36·2 S − 0·4 U + 2·1 V = E
12·5	+ 49·0 P + 68·7 Q − 386·8 R + 36·8 S − 0·4 U + 2·2 V = E
13·5	+ 49·0 P + 69·3 Q − 390·7 R + 37·3 S − 0·4 U + 2·3 V = E
14·5	+ 49·0 P + 70·3 Q − 394·8 R + 37·7 S − 0·5 U + 2·5 V = E
15·5	+ 49·0 P + 70·7 Q − 399·1 R + 38·2 S − 0·4 U + 2·4 V = E
16·5	+ 48·8 P + 71·2 Q − 403·3 R + 38·8 S − 0·4 U + 2·3 V = E
17·5	+ 48·3 P + 72·9 Q − 407·6 R + 39·3 S − 0·5 U + 2·5 V = E
18·5	+ 48·9 P + 73·7 Q − 411·8 R + 40·1 S − 0·4 U + 2·6 V = E
19·5	+ 48·8 P + 74·2 Q − 416·7 R + 40·4 S − 0·7 U + 2·6 V = E
20·5	+ 48·8 P + 74·6 Q − 421·9 R + 40·1 S − 0·6 U + 2·5 V = E
21·5	+ 48·9 P + 75·7 Q − 426·9 R + 41·8 S − 0·7 U + 2·6 V = E
22·5	+ 48·8 P + 76·6 Q − 431·9 R + 42·3 S − 0·8 U + 2·5 V = E
23·5	+ 48·7 P + 77·7 Q − 437·5 R + 43·1 S − 0·5 U + 2·6 V = E
24·5	+ 48·8 P + 79·0 Q − 443·0 R + 43·7 S − 1·1 U + 2·4 V = E
25·5	+ 49·0 P + 80·4 Q − 448·3 R + 44·7 S − 0·9 U + 2·7 V = E
26·5	+ 49·0 P + 81·2 Q − 453·8 R + 45·4 S − 1·1 U + 2·6 V = E
27·5	+ 49·0 P + 82·0 Q − 460·4 R + 46·2 S − 1·0 U + 2·6 V = E
28·5	+ 49·0 P + 83·1 Q − 466·9 R + 47·0 S − 1·0 U + 2·5 V = E
29·5	+ 49·0 P + 83·9 Q − 473·2 R + 47·8 S − 1·2 U + 2·4 V = E
30·5	+ 49·1 P + 85·4 Q − 480·3 R + 48·9 S − 1·4 U + 2·5 V = E
31·5	+ 49·3 P + 86·4 Q − 487·4 R + 49·7 S − 1·3 U + 2·4 V = E
1·5	+ 49·3 P + 87·6 Q − 493·3 R + 50·9 S − 1·3 U + 2·5 V = E
2·5	+ 49·2 P + 89·3 Q − 501·2 R + 51·6 S − 1·7 U + 2·2 V = E
3·5	+ 49·3 P + 91·1 Q − 507·3 R + 52·7 S − 1·5 U + 2·2 V = E
4·5	+ 49·3 P + 91·5 Q − 516·2 R + 53·7 S − 1·7 U + 2·1 V = E
5·5	+ 49·3 P + 93·7 Q − 523·3 R + 54·8 S − 2·1 U + 2·0 V = E
6·5	+ 49·4 P + 94·6 Q − 531·5 R + 55·9 S − 2·0 U + 1·8 V = E
7·5	+ 49·3 P + 95·9 Q − 539·1 R + 57·0 S − 2·2 U + 1·5 V = E
8·5	+ 49·4 P + 97·6 Q − 548·9 R + 58·3 S − 2·2 U + 1·5 V = E
9·5	+ 49·4 P + 99·3 Q − 556·4 R + 59·3 S − 2·5 U + 1·2 V = E
10·5	+ 49·3 P + 101·0 Q − 564·8 R + 61·0 S − 2·4 U + 1·2 V = E
11·5	+ 49·3 P + 101·7 Q − 574·0 R + 62·1 S − 2·6 U + 0·9 V = E
12·5	+ 49·3 P + 104·1 Q − 582·4 R + 63·4 S − 2·8 U + 0·7 V = E
13·5	+ 49·2 P + 105·9 Q − 591·0 R + 65·0 S − 2·9 U + 0·5 V = E
14·5	+ 49·0 P + 107·1 Q − 600·3 R + 66·0 S − 3·1 U + 0·3 V = E

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Declinations.						
1836.	″	″	″	″	″	″	
Jan. 9·5	+ 4·4 P	+ 14·0 Q	− 76·6 R	+ 7·5 S	− 12·0 U	− 17·5 V	= E′
10·5	+ 4·2 P	+ 14·0 Q	− 77·4 R	+ 7·6 S	− 12·2 U	− 18·3 V	= E′
11·5	+ 4·1 P	+ 14·1 Q	− 78·1 R	+ 7·7 S	− 12·3 U	− 19·1 V	= E′
12·5	+ 3·9 P	+ 14·2 Q	− 78·8 R	+ 7·9 S	− 12·4 U	− 19·9 V	= E′
13·5	+ 3·9 P	+ 14·4 Q	− 79·4 R	+ 8·1 S	− 12·5 U	− 20·6 V	= E′
14·5	+ 3·7 P	+ 14·5 Q	− 80·0 R	+ 8·1 S	− 12·9 U	− 21·2 V	= E′
15·5	+ 3·5 P	+ 14·6 Q	− 80·6 R	+ 8·2 S	− 12·9 U	− 22·2 V	= E′
16·5	+ 3·4 P	+ 14·7 Q	− 81·2 R	+ 8·4 S	− 13·0 U	− 23·0 V	= E′
17·5	+ 2·8 P	+ 14·9 Q	− 81·7 R	+ 8·8 S	− 13·1 U	− 23·8 V	= E′
18·5	+ 3·1 P	+ 15·0 Q	− 82·2 R	+ 8·8 S	− 13·3 U	− 24·6 V	= E′
19·5	+ 3·0 P	+ 15·1 Q	− 82·7 R	+ 8·8 S	− 13·5 U	− 25·4 V	= E′
20·5	+ 2·9 P	+ 15·2 Q	− 83·1 R	+ 9·0 S	− 13·8 U	− 26·3 V	= E′
21·5	+ 2·7 P	+ 15·2 Q	− 83·4 R	+ 9·1 S	− 13·7 U	− 27·1 V	= E′
22·5	+ 2·5 P	+ 15·3 Q	− 83·9 R	+ 9·2 S	− 14·0 U	− 28·0 V	= E′
23·5	+ 2·4 P	+ 15·4 Q	− 84·1 R	+ 9·5 S	− 14·1 U	− 28·8 V	= E′
24·5	+ 2·1 P	+ 15·4 Q	− 84·3 R	+ 9·5 S	− 14·2 U	− 29·7 V	= E′
25·5	+ 1·7 P	+ 15·3 Q	− 84·6 R	+ 9·4 S	− 14·6 U	− 30·8 V	= E′
26·5	+ 1·7 P	+ 15·5 Q	− 84·4 R	+ 9·7 S	− 14·5 U	− 31·5 V	= E′
27·5	+ 1·6 P	+ 15·6 Q	− 84·4 R	+ 9·8 S	− 14·7 U	− 32·4 V	= E′
28·5	+ 1·3 P	+ 15·5 Q	− 84·2 R	+ 9·9 S	− 14·8 U	− 33·3 V	= E′
29·5	+ 0·9 P	+ 15·4 Q	− 84·1 R	+ 9·9 S	− 15·0 U	− 34·4 V	= E′
30·5	+ 0·8 P	+ 15·5 Q	− 83·6 R	+ 10·2 S	− 15·1 U	− 35·2 V	= E′
31·5	+ 0·6 P	+ 15·5 Q	− 83·0 R	+ 10·3 S	− 15·3 U	− 36·0 V	= E′
Feb. 1·5	+ 0·3 P	+ 15·3 Q	− 82·5 R	+ 10·2 S	− 15·5 U	− 37·2 V	= E′
2·5	*	+ 15·2 Q	− 81·6 R	+ 10·3 S	− 15·7 U	− 38·0 V	= E′
3·5	− 0·3 P	+ 15·1 Q	− 80·6 R	+ 10·4 S	− 15·7 U	− 39·1 V	= E′
4·5	− 0·7 P	+ 14·8 Q	− 79·5 R	+ 10·3 S	− 16·0 U	− 40·1 V	= E′
5·5	− 1·0 P	+ 14·4 Q	− 78·4 R	+ 10·1 S	− 16·5 U	− 41·2 V	= E′
6·5	− 1·3 P	+ 14·4 Q	− 76·6 R	+ 10·3 S	− 16·3 U	− 42·1 V	= E′
7·5	− 1·7 P	+ 14·1 Q	− 75·0 R	+ 10·1 S	− 16·6 U	− 43·2 V	= E′
8·5	− 2·1 P	+ 13·6 Q	− 72·8 R	+ 10·0 S	− 16·8 U	− 44·3 V	= E′
9·5	− 2·4 P	+ 13·4 Q	− 70·6 R	+ 10·0 S	− 16·9 U	− 45·2 V	= E′
10·5	− 2·8 P	+ 13·0 Q	− 68·1 R	+ 10·0 S	− 16·9 U	− 46·3 V	= E′
11·5	− 3·2 P	+ 12·4 Q	− 65·5 R	+ 9·6 S	− 17·3 U	− 47·5 V	= E′
12·5	− 3·6 P	+ 11·8 Q	− 62·5 R	+ 9·4 S	− 17·5 U	− 48·6 V	= E′
13·5	− 4·2 P	+ 11·3 Q	− 59·0 R	+ 9·3 S	− 17·5 U	− 49·5 V	= E′
14·5	− 4·5 P	+ 10·6 Q	− 55·5 R	+ 9·2 S	− 17·7 U	− 50·6 V	= E′

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Right Ascensions.						
1836.	″	″	″	″	″	″	
Feb. 14·5	+ 49·0 P	+ 107·1 Q	− 600·8 R	+ 66·0 S	− 3·1 U	+ 0·3 V	= E
15·5	+ 49·0 P	+ 109·0 Q	− 608·7 R	+ 67·7 S	− 3·0 U	*	= E
16·5	+ 48·7 P	+ 111·0 Q	− 618·2 R	+ 69·2 S	− 3·5 U	− 0·5 V	= E
17·5	+ 48·1 P	+ 112·7 Q	− 628·1 R	+ 70·7 S	− 3·5 U	− 0·8 V	= E
18·5	+ 48·4 P	+ 114·2 Q	− 636·5 R	+ 72·3 S	− 3·7 U	− 1·0 V	= E
19·5	+ 48·0 P	+ 116·2 Q	− 645·1 R	+ 73·6 S	− 3·9 U	− 1·5 V	= E
20·5	+ 47·9 P	+ 117·3 Q	− 654·5 R	+ 75·2 S	− 4·0 U	− 1·9 V	= E
21·5	+ 47·5 P	+ 119·0 Q	− 663·1 R	+ 76·8 S	− 4·3 U	− 2·4 V	= E
22·5	+ 47·2 P	+ 120·8 Q	− 671·0 R	+ 78·4 S	− 4·5 U	− 2·8 V	= E
23·5	+ 46·7 P	+ 122·9 Q	− 679·2 R	+ 79·7 S	− 4·7 U	− 3·3 V	= E
24·5	+ 46·3 P	+ 124·1 Q	− 687·6 R	+ 81·3 S	− 5·0 U	− 3·8 V	= E
25·5	+ 45·8 P	+ 125·3 Q	− 694·9 R	+ 82·6 S	− 5·2 U	− 4·5 V	= E
26·5	+ 45·2 P	+ 126·9 Q	− 702·4 R	+ 84·2 S	− 5·2 U	− 4·9 V	= E
27·5	+ 44·7 P	+ 127·8 Q	− 709·1 R	+ 85·7 S	− 6·1 U	− 5·3 V	= E
28·5	+ 44·2 P	+ 129·1 Q	− 715·3 R	+ 87·4 S	− 6·2 U	− 5·9 V	= E
29·5	+ 43·5 P	+ 129·9 Q	− 721·1 R	+ 88·6 S	− 6·4 U	− 6·4 V	= E
Mar. 1·5	+ 42·8 P	+ 131·2 Q	− 726·8 R	+ 89·9 S	− 6·6 U	− 7·0 V	= E
2·5	+ 42·0 P	+ 132·1 Q	− 731·5 R	+ 91·2 S	− 6·3 U	− 7·7 V	= E
3·5	+ 41·2 P	+ 133·4 Q	− 734·9 R	+ 92·4 S	− 6·2 U	− 8·1 V	= E
4·5	+ 40·3 P	+ 134·5 Q	− 738·9 R	+ 93·1 S	− 6·6 U	− 9·0 V	= E
5·5	+ 39·7 P	+ 134·8 Q	− 741·5 R	+ 94·8 S	− 6·5 U	− 9·2 V	= E
6·5	+ 38·6 P	+ 135·5 Q	− 742·9 R	+ 95·4 S	− 7·0 U	− 10·0 V	= E
7·5	+ 37·5 P	+ 135·0 Q	− 744·4 R	+ 96·2 S	− 7·1 U	− 10·9 V	= E
8·5	+ 36·7 P	+ 135·1 Q	− 745·3 R	+ 97·2 S	− 7·2 U	− 11·4 V	= E
9·5	+ 35·7 P	+ 135·4 Q	− 745·4 R	+ 98·1 S	− 7·4 U	− 12·0 V	= E
10·5	+ 34·7 P	+ 136·5 Q	− 743·9 R	+ 98·8 S	− 7·4 U	− 12·4 V	= E
11·5	+ 33·6 P	+ 135·3 Q	− 742·5 R	+ 99·3 S	− 7·7 U	− 13·2 V	= E
12·5	+ 32·5 P	+ 135·0 Q	− 740·1 R	+ 99·7 S	− 7·8 U	− 13·8 V	= E
13·5	+ 31·5 P	+ 134·7 Q	− 737·0 R	+ 100·1 S	− 8·1 U	− 14·2 V	= E
14·5	+ 30·5 P	+ 133·8 Q	− 733·2 R	+ 100·2 S	− 7·9 U	− 14·8 V	= E
15·5	+ 29·4 P	+ 133·6 Q	− 728·5 R	+ 100·5 S	− 7·1 U	− 15·3 V	= E
16·5	+ 28·3 P	+ 132·8 Q	− 723·1 R	+ 100·6 S	− 7·9 U	− 15·6 V	= E
17·5	+ 27·3 P	+ 131·4 Q	− 717·8 R	+ 100·5 S	− 8·0 U	− 16·1 V	= E
18·5	+ 26·2 P	+ 130·8 Q	− 711·2 R	+ 100·5 S	− 8·0 U	− 16·7 V	= E
19·5	+ 25·2 P	+ 129·7 Q	− 704·9 R	+ 100·2 S	− 8·2 U	− 17·1 V	= E
20·5	+ 24·2 P	+ 128·5 Q	− 697·8 R	+ 100·2 S	− 8·4 U	− 17·4 V	= E
21·5	+ 23·2 P	+ 126·9 Q	− 690·6 R	+ 99·8 S	− 8·1 U	− 17·9 V	= E

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Declinations.						
1836.	″	″	″	″	″	″	
Feb. 14·5	− 4·5 P	+ 10·6 Q	− 55·5 R	+ 9·2 S	− 17·7 U	− 50·6 V	= E′
15·5	− 4·9 P	+ 10·1 Q	− 51·4 R	+ 8·6 S	− 17·8 U	− 51·7 V	= E′
16·5	− 5·3 P	+ 9·3 Q	− 46·6 R	+ 8·2 S	− 18·0 U	− 52·8 V	= E′
17·5	− 6·0 P	+ 8·4 Q	− 42·2 R	+ 7·9 S	− 18·1 U	− 53·7 V	= E′
18·5	− 6·3 P	+ 7·5 Q	− 37·5 R	+ 7·5 S	− 17·8 U	− 54·9 V	= E′
19·5	− 6·8 P	+ 6·5 Q	− 32·1 R	+ 6·9 S	− 18·3 U	− 56·1 V	= E′
20·5	− 7·3 P	+ 5·6 Q	− 26·1 R	+ 6·4 S	− 18·5 U	− 57·2 V	= E′
21·5	− 7·8 P	+ 4·5 Q	− 19·9 R	+ 5·8 S	− 18·6 U	− 58·2 V	= E′
22·5	− 8·4 P	+ 3·3 Q	− 13·6 R	+ 5·1 S	− 18·8 U	− 59·3 V	= E′
23·5	− 8·9 P	+ 1·8 Q	− 7·0 R	+ 4·2 S	− 18·9 U	− 60·6 V	= E′
24·5	− 9·2 P	+ 0·9 Q	+ 0·6 R	+ 3·5 S	− 19·0 U	− 61·3 V	= E′
25·5	− 9·8 P	− 0·4 Q	+ 8·1 R	+ 2·8 S	− 18·9 U	− 62·2 V	= E′
26·5	− 10·3 P	− 1·9 Q	+ 15·9 R	+ 1·8 S	− 19·1 U	− 63·4 V	= E′
27·5	− 10·8 P	− 3·3 Q	+ 23·9 R	+ 0·9 S	− 18·4 U	− 64·4 V	= E′
28·5	− 11·3 P	− 4·8 Q	+ 32·1 R	*	− 18·6 U	− 65·4 V	= E′
29·5	− 11·6 P	− 6·2 Q	+ 40·7 R	− 1·1 S	− 18·5 U	− 66·2 V	= E′
Mar. 1·5	− 12·1 P	− 7·8 Q	+ 49·6 R	− 2·1 S	− 18·6 U	− 67·2 V	= E′
2·5	− 12·4 P	− 9·3 Q	+ 58·6 R	− 3·4 S	− 19·2 U	− 68·0 V	= E′
3·5	− 12·9 P	− 10·9 Q	+ 67·4 R	− 4·3 S	− 19·2 U	− 68·7 V	= E′
4·5	− 13·2 P	− 12·8 Q	+ 76·3 R	− 5·7 S	− 19·4 U	− 69·7 V	= E′
5·5	− 14·0 P	− 14·9 Q	+ 85·0 R	− 7·3 S	− 19·7 U	− 70·9 V	= E′
6·5	− 13·7 P	− 16·1 Q	+ 94·2 R	− 7·9 S	− 19·2 U	− 71·2 V	= E′
7·5	− 14·0 P	− 17·5 Q	+ 102·8 R	− 9·1 S	− 19·3 U	− 71·9 V	= E′
8·5	− 14·2 P	− 18·8 Q	+ 112·0 R	− 10·2 S	− 18·9 U	− 72·2 V	= E′
9·5	− 14·4 P	− 20·3 Q	+ 120·2 R	− 11·8 S	− 18·2 U	− 73·1 V	= E′
10·5	− 14·5 P	− 22·6 Q	+ 128·4 R	− 12·8 S	− 19·0 U	− 73·6 V	= E′
11·5	− 14·5 P	− 23·6 Q	+ 136·3 R	− 13·9 S	− 18·7 U	− 74·1 V	= E′
12·5	− 14·6 P	− 25·0 Q	+ 143·8 R	− 15·1 S	− 18·9 U	− 74·6 V	= E′
13·5	− 14·8 P	− 26·6 Q	+ 150·7 R	− 16·3 S	− 18·8 U	− 75·1 V	= E′
14·5	− 14·5 P	− 27·5 Q	+ 157·8 R	− 17·3 S	− 18·6 U	− 75·2 V	= E′
15·5	− 14·5 P	− 29·0 Q	+ 163·8 R	− 18·4 S	− 18·5 U	− 75·7 V	= E′
16·5	− 14·3 P	− 30·1 Q	+ 169·7 R	− 19·5 S	− 18·4 U	− 76·0 V	= E′
17·5	− 14·1 P	− 30·8 Q	+ 175·2 R	− 20·3 S	− 18·3 U	− 76·0 V	= E′
18·5	− 14·0 P	− 32·1 Q	+ 179·9 R	− 21·3 S	− 18·1 U	− 76·1 V	= E′
19·5	− 13·8 P	− 33·0 Q	+ 184·3 R	− 22·2 S	− 18·0 U	− 76·4 V	= E′
20·5	− 13·6 P	− 33·5 Q	+ 188·3 R	− 23·0 S	− 17·8 U	− 76·5 V	= E′
21·5	− 13·3 P	− 34·2 Q	+ 191·6 R	− 23·9 S	− 17·8 U	− 76·6 V	= E′

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Right Ascensions.
1836.	
Mar. 21·5	$+ 23''\cdot2\ P + 126''\cdot9\ Q - 690''\cdot6\ R + 99''\cdot8\ S - 8''\cdot1\ U - 17''\cdot9\ V = E$
22·5	$+ 22\cdot3\ P + 126\cdot1\ Q - 682\cdot8\ R + 99\cdot2\ S - 8\cdot2\ U - 18\cdot2\ V = E$
23·5	$+ 21\cdot3\ P + 124\cdot7\ Q - 674\cdot4\ R + 98\cdot7\ S - 8\cdot0\ U - 18\cdot4\ V = E$
24·5	$+ 20\cdot3\ P + 123\cdot0\ Q - 666\cdot2\ R + 98\cdot3\ S - 8\cdot1\ U - 18\cdot8\ V = E$
25·5	$+ 19\cdot6\ P + 121\cdot5\ Q - 657\cdot4\ R + 97\cdot9\ S - 7\cdot8\ U - 18\cdot8\ V = E$
26·5	$+ 18\cdot5\ P + 120\cdot1\ Q - 648\cdot4\ R + 97\cdot1\ S - 8\cdot1\ U - 19\cdot4\ V = E$
27·5	$+ 17\cdot8\ P + 118\cdot2\ Q - 639\cdot6\ R + 96\cdot9\ S - 7\cdot9\ U - 19\cdot4\ V = E$
28·5	$+ 16\cdot8\ P + 116\cdot9\ Q - 630\cdot2\ R + 95\cdot7\ S - 7\cdot9\ U - 19\cdot7\ V = E$
29·5	$+ 16\cdot0\ P + 114\cdot9\ Q - 621\cdot4\ R + 94\cdot8\ S - 7\cdot9\ U - 19\cdot9\ V = E$
30·5	$+ 15\cdot1\ P + 113\cdot9\ Q - 611\cdot6\ R + 94\cdot1\ S - 7\cdot9\ U - 20\cdot3\ V = E$
31·5	$+ 14\cdot6\ P + 114\cdot0\ Q - 600\cdot5\ R + 93\cdot3\ S - 7\cdot9\ U - 20\cdot2\ V = E$

TABLE XI.—*continued.*

Date.	Equations of Condition dependent upon Declinations.
1836.	
Mar. 21 ·5	$-13''\cdot3\,P - 34''\cdot2\,Q + 191''\cdot6\,R - 23''\cdot9\,S - 17''\cdot8\,U - 76''\cdot6\,V = E'$
22 ·5	$-13\cdot0\,P - 34\cdot9\,Q + 194\cdot6\,R - 24\cdot3\,S - 17\cdot6\,U - 76\cdot5\,V = E'$
23 ·5	$-12\cdot6\,P - 35\cdot3\,Q + 196\cdot9\,R - 24\cdot9\,S - 17\cdot4\,U - 76\cdot4\,V = E'$
24 ·5	$-12\cdot3\,P - 35\cdot7\,Q + 199\cdot0\,R - 25\cdot6\,S - 17\cdot3\,U - 76\cdot5\,V = E'$
25 ·5	$-12\cdot0\,P - 36\cdot1\,Q + 200\cdot8\,R - 26\cdot2\,S - 17\cdot0\,U - 76\cdot3\,V = E'$
26 ·5	$-11\cdot6\,P - 36\cdot4\,Q + 201\cdot5\,R - 26\cdot6\,S - 16\cdot9\,U - 76\cdot2\,V = E'$
27 ·5	$-11\cdot3\,P - 36\cdot4\,Q + 202\cdot2\,R - 26\cdot9\,S - 16\cdot7\,U - 76\cdot0\,V = E'$
28 ·5	$-11\cdot0\,P - 36\cdot6\,Q + 202\cdot5\,R - 27\cdot1\,S - 16\cdot1\,U - 75\cdot7\,V = E'$
29 ·5	$-10\cdot6\,P - 36\cdot5\,Q + 202\cdot5\,R - 27\cdot6\,S - 16\cdot4\,U - 75\cdot6\,V = E'$
30 ·5	$-10\cdot3\,P - 36\cdot8\,Q + 201\cdot7\,R - 27\cdot8\,S - 16\cdot2\,U - 75\cdot3\,V = E'$
31 ·5	$-9\cdot8\,P - 37\cdot9\,Q + 199\cdot9\,R - 28\cdot0\,S - 16\cdot1\,U - 74\cdot9\,V = E'$

ON THE PERTURBATIONS OF URANUS.

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1. The irregularities in the motions of Uranus have for a long time engaged the attention of Astronomers. When the path of the planet became approximately known, it was found that, previously to its discovery by Sir W. Herschel in 1781, it had several times been observed as a fixed star by Flamsteed, Bradley, Mayer, and Lemonnier. Although these observations are doubtless very far inferior in accuracy to the modern ones, they must be considered valuable, in consequence of the great extension which they give to the observed arc of the planet's orbit. Bouvard, however, to whom we owe the Tables of Uranus at present in use, found that it was impossible to satisfy these observations, without attributing much larger errors to the modern observations than they admit of, and consequently founded his Tables exclusively on the latter. But in a very few years sensible errors began again to show themselves, and though the Tables were formed so recently as 1821, their error at the present time exceeds two minutes of space, and is still rapidly increasing. There appeared, therefore, no longer any sufficient reason for rejecting the ancient observations, especially since, with the exception of Flamsteed's first observation, which is more than twenty years anterior to any of the others, they are mutually confirmatory of each other.

2. Now that the discovery of another planet has confirmed in the most brilliant manner the conclusions of analysis, and enabled us with certainty to refer these irregularities to their true cause, it is unnecessary for me to enter at length upon the reasons which led me to reject the various other hypotheses which had been formed to account for them. It is sufficient to say, that they all appeared to be very improbable in themselves, and incapable of being tested by any exact calculation. Some had even supposed that at the great distance of Uranus from the Sun, the law of attraction becomes different from that of the inverse square of the distance. But the law of gravitation was too firmly established for this to be admitted, till every other hypothesis had failed, and I felt convinced that in this, as in every previous instance of the kind, the discrepancies which had for a time thrown doubts on the truth of the law, would eventually afford the most striking confirmation of it.

3. My attention was first directed to this subject several years since, by reading Mr. Airy's valuable Report on the recent progress of Astronomy. I find among my

papers the following memorandum, dated July 3, 1841:—"Formed a design, in the beginning of this week, of investigating, as soon as possible after taking my degree, the irregularities in the motion of Uranus which are yet unaccounted for; in order to find whether they may be attributed to the action of an undiscovered planet beyond it, and if possible, thence to determine approximately the elements of its orbit, &c., which would probably lead to its discovery." Accordingly, in 1843, I attempted a first solution of the problem, assuming the orbit to be a circle, with a radius equal to twice the mean distance of Uranus from the Sun. Some assumption as to the mean distance was clearly necessary in the first instance, and Bode's law appeared to render it probable that the above would not be far from the truth. This investigation was founded exclusively on the modern observations, and the errors of the Tables were taken from those given in the Equations of Condition of Bouvard's Tables as far as the year 1821, and subsequently from the observations given in the *Astronomische Nachrichten*, and from the Cambridge and Greenwich Observations. The result showed that a good general agreement between theory and observation might be obtained; but the larger differences occurring in years where the observations used were deficient in number, and the Greenwich Planetary Observations being then in process of reduction, I applied to Mr. Airy, through the kind intervention of Professor Challis, for the observations of some years in which the agreement appeared least satisfactory. The Astronomer Royal, in the kindest possible manner, sent me, in February 1844, the results of all the Greenwich Observations of Uranus.

4. Meanwhile the Royal Academy of Sciences of Göttingen had proposed the Theory of Uranus as the subject of their mathematical prize, and although the little time which I could spare from important duties in my college prevented me from attempting the complete examination of the theory, which a competition for the prize would have required, yet this fact, together with the possession of such a valuable series of observations, induced me to undertake a new solution of the problem. I now took into account the most important terms depending on the first power of the eccentricity of the disturbing planet, retaining the same assumption as before with respect to the mean distance. For the modern observations, the errors of the Tables were taken exclusively from the Greenwich Observations as far as the year 1830, with the exception of an observation by Bessel, in 1823; and subsequently from the Cambridge and Greenwich Observations, and those given in various numbers of the *Astronomische Nachrichten.* The errors of the Tables for the ancient Observations were taken from those given in the Equations of Condition of Bouvard's Tables. After obtaining several solutions differing little from each other, by gradually taking into account more and more terms of the series expressing the Perturbations, I communicated to Professor Challis, in September 1845, the final values which I had obtained for the mass, heliocentric longitude, and elements of the orbit of the assumed planet. The same results, slightly corrected, I communicated in the following month to the Astronomer Royal. The eccentricity coming out much larger than was probable, and later observations showing that the theory founded on the first hypothesis as to the mean distance, was still sensibly in error, I afterwards repeated my investigation, supposing the mean distance to be about $\frac{1}{30}$th part less than before. The result,

which I communicated to Mr. Airy, in the beginning of September of the present year, appeared more satisfactory than my former one, the eccentricity being smaller, and the errors of theory, compared with late observations, being less, and led me to infer that the distance should be still further diminished.

5. In November, 1845, M. Le Verrier presented to the Royal Academy of Sciences at Paris, a very complete and elaborate investigation of the Theory of Uranus, as disturbed by the action of Jupiter and Saturn, in which he pointed out several small inequalities which had previously been neglected; and in June, of the present year, he followed up this investigation by a memoir, in which he attributed the residual disturbances to the action of another planet at a distance from the Sun equal to twice that of Uranus, and found a longitude for the new planet agreeing very nearly with the result which I had obtained on the same hypothesis. On the 31st of August he presented to the Academy a more complete investigation, in which he determined the mass and the elements of the orbit of the new planet, and also obtained limiting values of the mean distance and heliocentric longitude. I mention these dates merely to show that my results were arrived at independently, and previously to the publication of those of M. Le Verrier, and not with the intention of interfering with his just claims to the honours of the discovery; for there is no doubt that his researches were first published to the world, and led to the actual discovery of the planet by Dr. Galle, so that the facts stated above cannot detract, in the slightest degree, from the credit due to M. Le Verrier.

6. In order not to have an inconvenient number of equations of condition, I divided the modern observations into groups, each including a period of three years, and as Mr. Airy had shown that the error of the Tabular Radius Vector was sometimes considerable, I either selected those observations which were made near opposition, or combined the others in such a manner that the result should be nearly free from the effects of this error. From the observations of each group, the error of the Tables in heliocentric longitude was found, corresponding to the time of mean opposition in the middle year of the group. Thus were formed 21 normal errors of the Tables, corresponding to as many equidistant periods between 1780 and 1840. The error for 1780 was found by interpolating between the errors of 1781, 1782, and 1783, and those given by the Ancient Observations of 1769 and 1771, and though not entitled to the same weight as the others, cannot, I think, be liable to much uncertainty. In my last calculations, I might have used more recent observations, but in order to obtain the effect due to the change of mean distance, it was necessary that the investigation should be founded on the same elements as before, and the later observations might be used as a test of the theory.

7. In order to satisfy myself that there was no important error in Bouvard's Tables, I recomputed all the principal inequalities produced by the action of Jupiter and Saturn, and found no difference of any consequence except in the equation depending on the mean longitude of Saturn minus twice that of Uranus, the error of which had been already pointed out by Bessel. The principal equation depending on the action of Jupiter, also required correction in consequence of the increased value which has been lately obtained for the mass of that planet. The corrections to be applied to Bouvard's Tables on these accounts, are the following:

$$+1''\!\cdot\!918 \sin\{\phi_1-2\phi_2-13^\circ\ 1'\!\cdot\!5\}$$
$$+1''\!\cdot\!085 \sin\{\phi-\phi_2\}$$

ϕ, ϕ_1, ϕ_2 being the mean longitudes of Jupiter, Saturn, and Uranus, respectively. In the Reduction of the Greenwich Observations, the latter correction was already taken into account. M. Hansen having also found some new inequalities in the motion of Uranus, depending on the square of the disturbing force, I re-computed the values of these, following the same method as that given by M. Delaunay in the *Conn. des Temps* for 1845, and my results agreed very closely with his, the terms to be added to the longitude being

$$+32''\!\cdot\!00 \sin\{3\phi_2-6\phi_1+2\phi+22^\circ\ 18'\!\cdot\!8\}$$
$$-\ 8\cdot\!35 \sin\{2\phi_2-6\phi_1+2\phi+39\ \ 10\cdot\!5\}$$
$$-\ 1\cdot\!49 \sin\{4\phi_2-6\phi_1+2\phi+34\ \ 48\cdot\!4\}$$

With respect to the inequalities of higher orders neglected by Bouvard, I considered that the most important of them would be, either those of long period, or those whose period was nearly equal to that of Uranus. During three-fourths of a revolution of the planet, the effects of the former class would be nearly confounded with those arising from a change in the epoch and mean motion, and those of the latter class with the effects produced by a constant change in the eccentricity and longitude of the Perihelion. The position of the planet to be determined would therefore be little affected by these terms, and the others would probably be much smaller than those which would necessarily be neglected in a first approximation to the perturbations produced by the new planet.

8. Taking into account the several corrections above-mentioned, the residual differences between the theoretical and observed heliocentric longitudes were the following:

Ancient Observations.

Year.	Observation—Theory.
1690	$+61''\cdot2$
1712	$+92\cdot7$
1715	$+73\cdot8$
1750	$-47\cdot6$
1753	$-39\cdot5$
1756	$-45\cdot7$
1764	$-34\cdot9$
1769	$-19\cdot3$
1771	$-\ 2\cdot3$

Modern Observations.

Year.	Observation—Theory.
1780	$+\ 3''\cdot46$
1783	$+\ 8\cdot45$
1786	$+12\cdot36$
1789	$+19\cdot02$
1792	$+18\cdot70$
1795	$+21\cdot38$
1798	$+20\cdot95$
1801	$+22\cdot21$
1804	$+24\cdot16$
1807	$+22\cdot07$
1810	$+23\cdot16$
1813	$+22\cdot00$
1816	$+22\cdot88$
1819	$+20\cdot69$
1822	$+20\cdot97$
1825	$+18\cdot16$
1828	$+10\cdot82$
1831	$-\ 3\cdot98$
1834	$-20\cdot80$
1837	$-42\cdot66$
1840	$-66\cdot64$

9. It is easily seen that the series expressing the correction of the *Mean* longitude in terms of the corrections applied to the elements of the orbit, is more convergent than that which gives the correction of the *true* longitude, and the same thing is true for the perturbations of the mean longitude, as compared with those of the true. The corrections found above were accordingly converted into corrections of mean longitude by multiplying each of them by the factor $\frac{r^2}{ab}$, r being the Rad. Vector, and a and b the semi-axes of the orbit. Hence these latter corrections were found to be the following:

Ancient Observations.

Year.	Observation—Theory.
	$''$
1690	+62 ·6
1712	+81 ·5
1715	+67 ·2
1750	—51 ·8
1753	—43 ·2
1756	—50 ·1
1764	—37 ·8
1769	—20 ·5
1771	— 2 ·4

Modern Observations.

Year.	Observation—Theory.
	$''$
1780	+ 3 ·42
1783	+ 8 ·19
1786	+11 ·74
1789	+17 ·75
1792	+17 ·22
1795	+19 ·52
1798	+19 ·06
1801	+20 ·24
1804	+22 ·19
1807	+20 ·52
1810	+21 ·89
1813	+21 ·19
1816	+22 ·50
1819	+20 ·78
1822	+21 ·50
1825	+18 ·97
1828	+11 ·50
1831	— 4 ·29
1834	—22 ·63
1837	—46 ·70
1840	—73 ·09

These numbers form the basis of the subsequent investigations.

10. Let $\delta\varepsilon$, δa, δe, and $\delta\varpi$ denote the corrections to be applied to the Tabular Elements of Uranus, then the correction of the mean longitude at any time t is

$$= \delta\varepsilon + 2e^2\delta\varpi + t\,\delta n - \left\{2\cos(nt+\varepsilon-\varpi) + \frac{e}{2}\cos 2(nt+\varepsilon-\varpi)\right\} e\,\delta\varpi$$

$$+ \left\{2\sin(nt+\varepsilon-\varpi) + \frac{e}{2}\sin 2(nt+\varepsilon-\varpi)\right\}\delta e$$

If we include the small term $2e^2\delta\varpi$ in the quantity $\delta\varepsilon$, this correction may be put under the following form:

$$\delta\varepsilon + t\,\delta n + \cos nt\,\delta x_1 + \sin nt\,\delta y_1 + \cos 2nt\,\delta x_2 + \sin 2nt\,\delta y_2$$

in which expression

$$\delta x_2 = \frac{1}{4}e\{\cos(\varepsilon-\varpi)\,\delta x_1 + \sin(\varepsilon-\varpi)\,\delta y_1\}$$

$$\delta y_2 = -\frac{1}{4}e\{\sin(\varepsilon-\varpi)\,\delta x_1 - \cos(\varepsilon-\varpi)\,\delta y_1\}$$

11. Also, adopting the notation of Pontécoulant's "Théorie Analytique," the perturbations of mean longitude

$$= \frac{m'}{2} \Sigma F_i \sin i (nt - n't + \varepsilon - \varepsilon')$$
$$+ m' e \Sigma G_i \sin \{ i (nt - n't + \varepsilon - \varepsilon') - (nt + \varepsilon - \varpi) \}$$
$$+ m' e' \Sigma H_i \sin \{ i (nt - n't + \varepsilon - \varepsilon') - (nt + \varepsilon - \varpi') \}$$

Where the accented letters belong to the disturbing planet, i takes all integral values, positive and negative, except zero, and if we put $i(n - n') = z$, the values of F G_i and H_i are the following:

$$F_i = \left\{ \frac{3 i n^4}{z^2 (z^2 - n^2)} + \frac{i n^2}{z^2 - n^2} \right\} a A_i + \frac{2 n^3}{z (z^2 - n^2)} a^2 \frac{d A_i}{d a}$$

$$G_i = \left\{ - \frac{3 i (i-1) n^4}{(z-n)^2 z (z-2n)} - \frac{i (i+1) n^2}{z (z-2n)} + \frac{i n^2}{z^2 - n^2} + \frac{3 i n^3}{z (z-n)(z-2n)} \right\} a A_i$$
$$+ \left\{ - \frac{3}{2} \frac{(i-1) n^4}{(z-n)^2 z (z-2n)} - \frac{1}{2} \frac{(i-1) n^2}{z (z-2n)} - \frac{1}{2} \frac{n^2}{z^2 - n^2} - \frac{2 i n^3}{z (z-n)(z-2n)} \right\} a^2 \frac{d A_i}{d a}$$
$$- \frac{n^3}{z (z-n)(z-2n)} a^3 \frac{d^2 A_i}{d a^2}.$$

$$H_i = \left\{ \frac{3}{2} \frac{(i-1)(2i-1) n^4}{(z-n)^2 z (z-2n)} + \frac{1}{2} \frac{(i-1)(2i-1) n^2}{z (z-2n)} \right\} a A_{i-1}$$
$$+ \left\{ \frac{3}{2} \frac{(i-1) n^4}{(z-n)^2 z (z-2n)} + \frac{1}{2} \frac{(i-1) n^2}{z (z-2n)} + \frac{2 i n^3}{z (z-n)(z-2n)} \right\} a^2 \frac{d A_{i-1}}{d a}$$
$$+ \frac{n^3}{z (z-n)(z-2n)} a^3 \frac{d^2 A_{i-1}}{d a^2}$$

12. Now, if we assume $\frac{a}{a'}$ or $\alpha = \sin 30^\circ = 0{\cdot}5$, the values of the fundamental quantities b, $\alpha \frac{db}{d\alpha}$, $\alpha^2 \frac{d^2 b}{d\alpha^2}$, will be

$\log. b_0 = 0{\cdot}33170$; $\log. \alpha \frac{db_0}{d\alpha} = 9{\cdot}53765$ $\log. \alpha^2 \frac{d^2 b_0}{d\alpha^2} = 9{\cdot}77848$

$\log. b_1 = 9{\cdot}74497$; $\log. \alpha \frac{db_1}{d\alpha} = 9{\cdot}83868$; $\log. \alpha^2 \frac{d^2 b_1}{d\alpha^2} = 9{\cdot}70857$

$\log. b_2 = 9{\cdot}32425$; $\log. \alpha \frac{db_2}{d\alpha} = 9{\cdot}68012$ $\log. \alpha^2 \frac{d^2 b_2}{d\alpha^2} = 9{\cdot}87776$

$\log. b_3 = 8{\cdot}91670$; $\log. \alpha \frac{db_3}{d\alpha} = 9{\cdot}46315$ $\log. \alpha^2 \frac{d^2 b_3}{d\alpha^2} = 9{\cdot}86253$

Hence the principal inequalities of mean longitude, produced by the action of a planet whose mass is $\frac{m'}{5000}$, that of the Sun being unity, and the eccentricity of whose orbit is $\frac{e'}{20}$ will be the following:

$$\begin{array}{lll}
-36''\cdot 99\, m' & \sin & \{nt-n't+\varepsilon-\varepsilon'\} \\
+58\cdot 97\, m' & \sin 2 & \{nt-n't+\varepsilon-\varepsilon'\} \\
+\ 5\cdot 80\, m' & \sin 3 & \{nt-n't+\varepsilon-\varepsilon'\} \\
+\ 2\cdot 06\, m' & \sin & \{n't+\varepsilon'-\varpi\} \\
-\ 4\cdot 30\, m'e' & \sin & \{n't+\varepsilon'-\varpi'\} \\
+31\cdot 25\, m' & \sin & \{nt-2n't+\varepsilon-2\varepsilon'+\varpi\} \\
-12\cdot 14\, m'e' & \sin & \{nt-2n't+\varepsilon-2\varepsilon'+\varpi'\} \\
+48\cdot 55\, m' & \sin & \{2nt-3n't+2\varepsilon-3\varepsilon'+\varpi\} \\
-93\cdot 01\, m'e' & \sin & \{2nt-3n't+2\varepsilon-3\varepsilon'+\varpi'\}
\end{array}$$

To these may be added the following, which are of two dimensions in terms of the eccentricities:

$$\begin{array}{lll}
+0''\cdot 57\, m' & \sin 3 & \{nt-n't+\varepsilon-\varepsilon'\} \\
-1\cdot 08\, m'e' & \sin & \{3(nt-n't+\varepsilon-\varepsilon')-\varpi+\varpi'\}
\end{array}$$

These expressions may be put under the following form:

$$\begin{array}{l}
h_1 \cos(n-n')t+h_2\cos 2(n-n')t+h_3\cos 3(n-n')t \\
+k_1 \sin(n-n')t+k_2\sin 2(n-n')t+k_3\sin 3(n-n')t \\
+p_1 \cos n't+p_2\cos(n-2n')t+p_3\cos(2n-3n')t \\
+q_1 \sin n't+q_2\sin(n-2n')t+q_3\sin(2n-3n')t
\end{array}$$

13. Let the time of the mean opposition in 1810 be taken as the epoch from which t is reckoned; this date, expressed in decimal parts of a year, will be 1810·328. Also, let 3 synodic periods of Uranus, = 3·0362 years, be taken for the unit of time; then the change of the mean anomaly in an unit of time will be $13^\circ\ 0'\cdot 5$; also $n=13^\circ\ 0'\cdot 6$, $n'=4^\circ\ 36'\cdot 0$ $\therefore$ $n-n'=8^\circ\ 24'\cdot 6$, $n-2n'=3^\circ\ 48'\cdot 6$, $2n-3n'=12^\circ\ 13'\cdot 2$. Hence the equations of condition given by the modern observations will be of the form

$$\begin{array}{rl}
c'' = & \delta\varepsilon+\delta x_1\cos\{13^\circ\ 0'\cdot 5\}t+\delta x_2\cos\{26^\circ\ 1'\cdot 0\}t \\
& +t\delta n+\delta y_1\sin\{13\ \ 0\cdot 5\}t+\delta y_2\sin\{26\ \ 1\cdot 0\}t \\
& \\
& +h_1\cos\{8^\circ\ 24'\cdot 6\}t+h_2\cos\{16^\circ\ 49'\cdot 2\}t+h_3\cos\{25^\circ\ 13'\cdot 8\}t \\
& +k_1\sin\{8\ 24\cdot 6\}t+k_2\sin\{16\ 49\cdot 2\}t+k_3\sin\{25\ 13\cdot 8\}t \\
& \\
& +p_1\cos\{4^\circ\ 36'\cdot 0\}t+p_2\cos\{3^\circ\ 48'\cdot 6\}t+p_3\cos\{12^\circ\ 13'\cdot 2\}t \\
& +q_1\sin\{4\ 36\cdot 0\}t+q_2\sin\{3\ 48\cdot 6\}t+q_3\sin\{12\ 13\cdot 2\}t
\end{array}$$

in which t assumes all integral values from -10 to $+10$ in succession, and the several values of c'' are contained in the table given in Article 9.

14. The final equations for the corrections of the elliptic elements will be found by multiplying each equation successively by the co-efficients of $\delta\epsilon$, δn, δx_1 and δy_1, which occur in it, and adding the several results.

Let the equations be treated in a similar manner with reference to the quantities $h_1, k_1, h_2, k_2, h_3, k_3, p_2, q_2, p_3, q_3$.

It will be seen that, in consequence of the arrangement which has been given to the equations of condition, the equations thus formed naturally separate themselves into two groups, one of which involves only $\delta\epsilon$, δx_1, δx_2, with the quantities h and p, while the other involves δn, δy_1, δy_2, with the quantities k and q.

Also, the co-efficients in these equations are easily calculated by the following formulæ, putting $t=10$ in their right hand members:

$$\Sigma\, 2 \cos m t = \frac{\sin m\,(t+\frac{1}{2})}{\sin \frac{1}{2} m}$$

$$\Sigma\, 2\, t \sin m t = \frac{(t+1)\sin m t - t \sin m\,(t+1)}{2 \sin^2 \frac{1}{2} m}$$

$$\Sigma\, 2 \cos m t \cos n t = \frac{1}{2}\left\{\frac{\sin (m-n)\,(t+\frac{1}{2})}{\sin \frac{1}{2}(m-n)} + \frac{\sin (m+n)\,(t+\frac{1}{2})}{\sin \frac{1}{2}(m+n)}\right\}$$

$$\Sigma\, 2 \sin m t \sin n t = \frac{1}{2}\left\{\frac{\sin (m-n)(t+\frac{1}{2})}{\sin \frac{1}{2}(m-n)} - \frac{\sin (m+n)\,(t+\frac{1}{2})}{\sin \frac{1}{2}(m+n)}\right\}$$

$$\Sigma\, 2 \cos^2 m t = t + \frac{1}{2} + \frac{1}{2}\frac{\sin m\,(2t+1)}{\sin m}$$

$$\Sigma\, 2 \sin^2 m t = t + \frac{1}{2} - \frac{1}{2}\frac{\sin m\,(2t+1)}{\sin m}$$

15. By performing the calculations, the equations of the first group are found to be the following:

$$
\begin{array}{lrl}
(\epsilon) & 151''\cdot48 = & 21\cdot0000\ \delta\epsilon + 6\cdot0670\ \delta x_1 - 4\cdot4358\ \delta x_2 \\
 & & +13\cdot6320\ h_1 + 0\cdot4043\ h_2 - 4\cdot5608\ h_3 \\
 & & +18\cdot6046\ p_1 + 19\cdot3384\ p_2 + 7\cdot3721\ p_3 \\
(x) & 246\cdot48 = & 6\cdot0670\ \delta\epsilon + 8\cdot2821\ \delta x_1 + 4\cdot1762\ \delta x_2 \\
 & & +\ 7\cdot4041\ h_1 + 8\cdot2523\ h_2 + 4\cdot6963\ h_3 \\
 & & +\ 6\cdot5389\ p_1 + 6\cdot3978\ p_2 + 8\cdot1831\ p_3 \\
(h_1) & 209\cdot74 = & 13\cdot6320\ \delta\epsilon + 7\cdot4041\ \delta x_1 - 0\cdot2337\ \delta x_2 \\
 & & +10\cdot7022\ h_1 + 4\cdot5356\ h_2 - 0\cdot0018\ h_3 \\
 & & +12\cdot7013\ p_1 + 12\cdot9883\ p_2 + 8\cdot0038\ p_3 \\
(h_2) & 242\cdot68 = & 0\cdot4043\ \delta\epsilon + 8\cdot2523\ \delta x_1 + 7\cdot5650\ \delta x_2 \\
 & & +\ 4\cdot5356\ h_1 + 10\cdot2960\ h_2 + 8\cdot1944\ h_3 \\
 & & +\ 1\cdot7866\ p_1 + 1\cdot3667\ p_2 + 7\cdot6671\ p_3 \\
(h_3) & 86\cdot67 = & -\ 4\cdot5608\ \delta\epsilon + 4\cdot6963\ \delta x_1 + 10\cdot5023\ \delta x_2 \\
 & & -\ 0\cdot0018\ h_1 + 8\cdot1944\ h_2 + 10\cdot7071\ h_3 \\
 & & -\ 3\cdot0812\ p_1 - 3\cdot5347\ p_2 + 3\cdot8855\ p_3
\end{array}
$$

$$
\begin{aligned}
(p_2)\quad 165''{\cdot}99 =\ & 19{\cdot}3384\,\delta\epsilon + 6{\cdot}3978\,\delta x_1 - 3{\cdot}4948\,\delta x_2 \\
& + 12{\cdot}9883\,h_1 + 1{\cdot}3667\,h_2 - 3{\cdot}5347\,h_3 \\
& + 17{\cdot}2795\,p_1 + 17{\cdot}9106\,p_2 + 7{\cdot}5423\,p_3 \\
(p_3)\quad 212{\cdot}56 =\ & 7{\cdot}3721\,\delta\epsilon + 8{\cdot}1831\,\delta x_1 + 3{\cdot}4071\,\delta x_2 \\
& + 8{\cdot}0038\,h_1 + 7{\cdot}6671\,h_2 + 3{\cdot}8855\,h_3 \\
& + 7{\cdot}6127\,p_1 + 7{\cdot}5423\,p_2 + 8{\cdot}2019\,p_3
\end{aligned}
$$

16. By means of (ϵ) eliminate $\delta\epsilon$ from each of the other equations, and these latter become

$$
\begin{aligned}
(x)\quad 202''{\cdot}72 =\ & 6{\cdot}5294\,\delta x_1 + 5{\cdot}4577\,\delta x_2 + 3{\cdot}4658\,h_1 + 8{\cdot}1355\,h_2 \\
& + 6{\cdot}0139\,h_3 + 1{\cdot}1640\,p_1 + 0{\cdot}8109\,p_2 + 6{\cdot}0533\,p_3 \\
(h_1)\quad 111{\cdot}41 =\ & 3{\cdot}4658\,\delta x_1 + 2{\cdot}6458\,\delta x_2 + 1{\cdot}8531\,h_1 + 4{\cdot}2731\,h_2 \\
& + 2{\cdot}9588\,h_3 + 0{\cdot}6243\,p_1 + 0{\cdot}4349\,p_2 + 3{\cdot}2183\,p_3 \\
(h_2)\quad 239{\cdot}76 =\ & 8{\cdot}1355\,\delta x_1 + 7{\cdot}6504\,\delta x_2 + 4{\cdot}2731\,h_1 + 10{\cdot}2882\,h_2 \\
& + 8{\cdot}2822\,h_3 + 1{\cdot}4284\,p_1 + 0{\cdot}9944\,p_2 + 7{\cdot}5252\,p_3 \\
(h_3)\quad 119{\cdot}57 =\ & 6{\cdot}0139\,\delta x_1 + 9{\cdot}5389\,\delta x_2 + 2{\cdot}9588\,h_1 + 8{\cdot}2822\,h_2 \\
& + 9{\cdot}7166\,h_3 + 0{\cdot}9593\,p_1 + 0{\cdot}6652\,p_2 + 5{\cdot}4866\,p_3 \\
(p_2)\quad 26{\cdot}50 =\ & 0{\cdot}8109\,\delta x_1 + 0{\cdot}5900\,\delta x_2 + 0{\cdot}4349\,h_1 + 0{\cdot}9944\,h_2 \\
& + 0{\cdot}6652\,h_3 + 0{\cdot}1470\,p_1 + 0{\cdot}1024\,p_2 + 0{\cdot}7535\,p_3 \\
(p_3)\quad 189{\cdot}38 =\ & 6{\cdot}0533\,\delta x_1 + 4{\cdot}9643\,\delta x_2 + 3{\cdot}2183\,h_1 + 7{\cdot}5252\,h_2 \\
& + 5{\cdot}4866\,h_3 + 1{\cdot}0815\,p_1 + 0{\cdot}7535\,p_2 + 5{\cdot}6139\,p_3
\end{aligned}
$$

17. Again, by means of (x) eliminate δx_1 from each of the other equations, and we find

$$
\begin{aligned}
(h_1)\quad 3''{\cdot}807 =\ & -0{\cdot}2512\,\delta x_2 + 0{\cdot}0135\,h_1 - 0{\cdot}0452\,h_2 - 0{\cdot}2334\,h_3 \\
& + 0{\cdot}0065\,p_1 + 0{\cdot}0045\,p_2 + 0{\cdot}0052\,p_3 \\
(h_2)\quad -12{\cdot}821 =\ & 0{\cdot}8502\,\delta x_2 - 0{\cdot}0452\,h_1 + 0{\cdot}1515\,h_2 + 0{\cdot}7890\,h_3 \\
& - 0{\cdot}0219\,p_1 - 0{\cdot}0160\,p_2 - 0{\cdot}0171\,p_3 \\
(h_3)\quad -67{\cdot}149 =\ & 4{\cdot}5120\,\delta x_2 - 0{\cdot}2334\,h_1 + 0{\cdot}7890\,h_2 + 4{\cdot}1775\,h_3 \\
& - 0{\cdot}1128\,p_1 - 0{\cdot}0817\,p_2 - 0{\cdot}0888\,p_3 \\
(p_2)\quad 1{\cdot}327 =\ & -0{\cdot}0878\,\delta x_2 + 0{\cdot}0045\,h_1 - 0{\cdot}0160\,h_2 - 0{\cdot}0817\,h_3 \\
& + 0{\cdot}0024\,p_1 + 0{\cdot}0017\,p_2 + 0{\cdot}0018\,p_3 \\
(p_3)\quad 1{\cdot}448 =\ & -0{\cdot}0955\,\delta x_2 + 0{\cdot}0052\,h_1 - 0{\cdot}0171\,h_2 - 0{\cdot}0888\,h_3 \\
& + 0{\cdot}0024\,p_1 + 0{\cdot}0018\,p_2 + 0{\cdot}0020\,p_3
\end{aligned}
$$

18. Similarly, the equations of the second group are found to be

$$
\begin{aligned}
(n)\quad -171''{\cdot}27 =\ & 77{\cdot}0000\,\delta n + 9{\cdot}3938\,\delta y_1 - 1{\cdot}2183\,\delta y_2 \\
& + 8{\cdot}8463\,k_1 + 7{\cdot}3034\,k_2 - 0{\cdot}5927\,k_3 \\
& + 5{\cdot}7519\,q_1 + 4{\cdot}8755\,q_2 + 9{\cdot}5583\,q_3 \\
(y)\quad -166{\cdot}33 =\ & 93{\cdot}9380\,\delta n + 12{\cdot}7179\,\delta y_1 + 1{\cdot}8907\,\delta y_2 \\
& + 11{\cdot}2022\,k_1 + 11{\cdot}0848\,k_2 + 2{\cdot}6731\,k_3 \\
& + 7{\cdot}0956\,q_1 + 5{\cdot}9913\,q_2 + 12{\cdot}7441\,q_3
\end{aligned}
$$

$$
\begin{aligned}
(k_1)\quad -182''\cdot 87 &= 88\cdot 1630\ \delta n + 11\cdot 2022\ \delta y_1 - 0\cdot 3210\ \delta y_2 \\
&\quad + 10\cdot 2978\ k_1 + 9\cdot 0964\ k_2 + 0\cdot 4061\ k_3 \\
&\quad + 6\cdot 6370\ q_1 + 5\cdot 6163\ q_2 + 11\cdot 3346\ q_3 \\
(k_2)\quad -89\cdot 07 &= 73\cdot 0340\ \delta n + 11\cdot 0848\ \delta y_1 + 4\cdot 8266\ \delta y_2 \\
&\quad + 9\cdot 0964\ k_1 + 10\cdot 7040\ k_2 + 5\cdot 4376\ k_3 \\
&\quad + 5\cdot 5855\ q_1 + 4\cdot 6976\ q_2 + 10\cdot 9375\ q_3 \\
(k_3)\quad +124\cdot 80 &= -5\cdot 9270\ \delta n + 2\cdot 6731\ \delta y_1 + 10\cdot 4253\ \delta y_2 \\
&\quad + 0\cdot 4061\ k_1 + 5\cdot 4376\ k_2 + 10\cdot 2929\ k_3 \\
&\quad - 0\cdot 2497\ q_1 - 0\cdot 2643\ q_2 + 2\cdot 1788\ q_3 \\
(q_2)\quad -107\cdot 02 &= 48\cdot 7550\ \delta n + 5\cdot 9913\ \delta y_1 - 0\cdot 6614\ \delta y_2 \\
&\quad + 5\cdot 6163\ k_1 + 4\cdot 6976\ k_2 - 0\cdot 2643\ k_3 \\
&\quad + 3\cdot 6475\ q_1 + 3\cdot 0894\ q_2 + 6\cdot 0897\ q_3 \\
(q_3)\quad -175\cdot 89 &= 95\cdot 5830\ \delta n + 12\cdot 7441\ \delta y_1 + 1\cdot 3845\ \delta y_2 \\
&\quad + 11\cdot 3346\ k_1 + 10\cdot 9375\ k_2 + 2\cdot 1788\ k_3 \\
&\quad + 7\cdot 2084\ q_1 + 6\cdot 0897\ q_2 + 12\cdot 7981\ q_3
\end{aligned}
$$

19. By means of (n), eliminate δn from each of the other equations, and we have

$$
\begin{aligned}
(y)\quad 42''\cdot 61 &= 1\cdot 2578\ \delta y_1 + 3\cdot 3771\ \delta y_2 + 0\cdot 4100\ k_1 + 2\cdot 1748\ k_2 \\
&\quad + 3\cdot 3962\ k_3 + 0\cdot 0785\ q_1 + 0\cdot 0433\ q_2 + 1\cdot 0833\ q_3 \\
(k_1)\quad 13\cdot 90 &= 0\cdot 4100\ \delta y_1 + 1\cdot 0787\ \delta y_2 + 0\cdot 1346\ k_1 + 0\cdot 7057\ k_2 \\
&\quad + 1\cdot 0871\ k_3 + 0\cdot 0288\ q_1 + 0\cdot 0150\ q_2 + 0\cdot 3534\ q_3 \\
(k_2)\quad 73\cdot 38 &= 2\cdot 1748\ \delta y_1 + 5\cdot 9822\ \delta y_2 + 0\cdot 7057\ k_1 + 3\cdot 7767\ k_2 \\
&\quad + 5\cdot 9998\ k_3 + 0\cdot 1298\ q_1 + 0\cdot 0732\ q_2 + 1\cdot 8715\ q_3 \\
(k_3)\quad 111\cdot 62 &= 3\cdot 3962\ \delta y_1 + 10\cdot 3315\ \delta y_2 + 1\cdot 0871\ k_1 + 5\cdot 9998\ k_2 \\
&\quad + 10\cdot 2473\ k_3 + 0\cdot 1930\ q_1 + 0\cdot 1110\ q_2 + 2\cdot 9145\ q_3 \\
(q_2)\quad 1\cdot 42 &= 0\cdot 0433\ \delta y_1 + 0\cdot 1100\ \delta y_2 + 0\cdot 0150\ k_1 + 0\cdot 0732\ k_2 \\
&\quad + 0\cdot 1110\ k_3 + 0\cdot 0055\ q_1 + 0\cdot 0023\ q_2 + 0\cdot 0375\ q_3 \\
(q_3)\quad 36\cdot 72 &= 1\cdot 0833\ \delta y_1 + 2\cdot 8969\ \delta y_2 + 0\cdot 3534\ k_1 + 1\cdot 8715\ k_2 \\
&\quad + 2\cdot 9145\ k_3 + 0\cdot 0684\ q_1 + 0\cdot 0375\ q_2 + 0\cdot 9330\ q_3
\end{aligned}
$$

20. Again, eliminating δy_1 by means of (y) we find

$$
\begin{aligned}
(k\quad 0''\cdot 009 &= -0\cdot 0221\ \delta y_2 + 0\cdot 0010\ k_1 - 0\cdot 0032\ k_2 - 0\cdot 0200\ k_3 \\
&\quad + 0\cdot 0032\ q_1 + 0\cdot 0009\ q_2 + 0\cdot 0003\ q_3 \\
(k_2)\quad -0\cdot 301 &= 0\cdot 1430\ \delta y_2 - 0\cdot 0032\ k_1 + 0\cdot 0162\ k_2 + 0\cdot 1274\ k_3 \\
&\quad - 0\cdot 0059\ q_1 - 0\cdot 0017\ q_2 - 0\cdot 0016\ q_3 \\
(k_3)\quad -3\cdot 443 &= 1\cdot 2129\ \delta y_2 - 0\cdot 0200\ k_1 + 0\cdot 1274\ k_2 + 1\cdot 0769\ k_3 \\
&\quad - 0\cdot 0189\ q_1 - 0\cdot 0059\ q_2 - 0\cdot 0105\ q_3 \\
(q_2)\quad -0\cdot 045 &= -0\cdot 0062\ \delta y_2 + 0\cdot 0009\ k_1 - 0\cdot 0017\ k_2 - 0\cdot 0059\ k_3 \\
&\quad + 0\cdot 0028\ q_1 + 0\cdot 0008\ q_2 + 0\cdot 0002\ q_3 \\
(q_3)\quad +0\cdot 017 &= -0\cdot 0116\ \delta y_2 + 0\cdot 0003\ k_1 - 0\cdot 0016\ k_2 - 0\cdot 0105\ k_3 \\
&\quad + 0\cdot 0008\ q_1 + 0\cdot 0002\ q_2 + 0\cdot 0000\ q_3
\end{aligned}
$$

21. From the equations remaining in the two groups after the elimination of $\delta\epsilon$, δn, δx_1, δy_1 it will be easy, when approximate values of the mass and mean longitude of the disturbing planet have been found, to deduce the final equations for determining these quantities more accurately by the method of minimum squares.

It may be observed, however, that the equations in each group are very nearly identical with each other, and therefore two final equations may be formed by simply adding together the several equations of each group, after giving the unknown quantities the same sign in them all. Thus we find

$$\begin{aligned} 86''\cdot552 &= -5\cdot7967\,\delta x_2 + 0\cdot3018\,h_1 - 1\cdot0188\,h_2 - 5\cdot3704\,h_3 \\ &\qquad + 0\cdot1460\,p_1 + 0\cdot1056\,p_2 + 0\cdot1149\,p_3 \\ 3\cdot725 &= -1\cdot3958\,\delta y_2 + 0\cdot0254\,k_1 - 0\cdot1501\,k_2 - 1\cdot2407\,k_3 \\ &\qquad + 0\cdot0316\,q_1 + 0\cdot0095\,q_2 + 0\cdot0127\,q_3 \end{aligned}$$

22. If in the expressions before given for δx_2 and δy_2 we substitute $e = 0\cdot046679$ and $\epsilon - \varpi = 50^\circ 15'\cdot8$, we obtain

$$\begin{aligned} \delta x_2 &= 0\cdot007460\,\delta x_1 + 0\cdot008974\,\delta y_1 \\ \delta y_2 &= -0\cdot008974\,\delta x_1 + 0\cdot007460\,\delta y_1 \end{aligned}$$

Substituting these values in the equations (x) and (y) and in those just found, it may be seen that by adding to the latter equations

$$0\cdot006768\,(x) + 0\cdot040287\,(y)$$

and $\quad -0\cdot001869\,(x) + 0\cdot008187\,(y)$ respectively,

δx_1 and δy_1 will be eliminated, and we shall obtain the following equations:

$$\begin{aligned} (1)\qquad 89''\cdot641 &= 0\cdot3252\,h_1 - 0\cdot9637\,h_2 - 5\cdot3297\,h_3 \\ &\quad + 0\cdot0165\,k_1 + 0\cdot0876\,k_2 + 0\cdot1368\,k_3 \\ &\quad + 0\cdot1539\,p_1 + 0\cdot1111\,p_2 + 0\cdot1559\,p_3 \\ &\quad + 0\cdot0032\,q_1 + 0\cdot0017\,q_2 + 0\cdot0436\,q_3 \\ (2)\qquad 3\cdot695 &= -0\cdot0065\,h_1 - 0\cdot0152\,h_2 - 0\cdot0112\,h_3 \\ &\quad + 0\cdot0288\,k_1 - 0\cdot1323\,k_2 - 1\cdot2129\,k_3 \\ &\quad - 0\cdot0022\,p_1 - 0\cdot0015\,p_2 - 0\cdot0113\,p_3 \\ &\quad + 0\cdot0323\,q_1 + 0\cdot0099\,q_2 + 0\cdot0215\,q_3 \end{aligned}$$

23. These equations would be sufficient for determining the mass of the disturbing planet and its longitude at the epoch, if the eccentricity of the orbit were neglected. We will now proceed to find equations from the Ancient Observations for determining the eccentricity and longitude of the Perihelion.

The equations of condition given by the Ancient Observations are the following:

$$\begin{aligned} 62''\cdot6 = \quad & \delta\epsilon - 0\cdot8776\,\delta x_1 + 0\cdot5402\,\delta x_2 + 0\cdot8712\,h_1 + 0\cdot5180\,h_2 \\ & -39\cdot31\,\delta n - 0\cdot4795\,\delta y_1 + 0\cdot8415\,\delta y_2 + 0\cdot4909\,k_1 + 0\cdot8554\,k_2 \\ & + 0\cdot0314\,h_3 - 0\cdot9999\,p_1 - 0\cdot8640\,p_2 - 0\cdot5055\,p_3 \\ & + 0\cdot9995\,k_3 + 0\cdot0145\,q_1 - 0\cdot5035\,q_2 - 0\cdot8628\,q_3 \end{aligned}$$

$$\begin{aligned}
84''{\cdot}5 = {} & \delta\epsilon + 0{\cdot}4975\,\delta x_1 - 0{\cdot}5050\,\delta x_2 + 0{\cdot}0288\,h_1 - 0{\cdot}9984\,h_2 \\
& -32{\cdot}30\,\delta n - 0{\cdot}8675\,\delta y_1 - 0{\cdot}8631\,\delta y_2 + 0{\cdot}9996\,k_1 + 0{\cdot}0573\,k_2 \\
& -0{\cdot}0860\,h_3 - 0{\cdot}8534\,p_1 - 0{\cdot}5456\,p_2 + 0{\cdot}8220\,p_3 \\
& -0{\cdot}9963\,k_3 - 0{\cdot}5213\,q_1 - 0{\cdot}8380\,q_2 - 0{\cdot}5695\,q_3
\end{aligned}$$

$$\begin{aligned}
67{\cdot}2 = {} & \delta\epsilon + 0{\cdot}6732\,\delta x_1 - 0{\cdot}0935\,\delta x_2 - 0{\cdot}1120\,h_1 - 0{\cdot}9749\,h_2 \\
& -31{\cdot}34\,\delta n - 0{\cdot}7394\,\delta y_1 - 0{\cdot}9956\,\delta y_2 + 0{\cdot}9937\,k_1 - 0{\cdot}2227\,k_2 \\
& +0{\cdot}3305\,h_3 - 0{\cdot}8105\,p_1 - 0{\cdot}4912\,p_2 + 0{\cdot}9206\,p_3 \\
& -0{\cdot}9438\,k_3 - 0{\cdot}5857\,q_1 - 0{\cdot}8711\,q_2 - 0{\cdot}3905\,q_3
\end{aligned}$$

$$\begin{aligned}
-51{\cdot}8 = {} & \delta\epsilon - 0{\cdot}2616\,\delta x_1 - 0{\cdot}8631\,\delta x_2 - 0{\cdot}9649\,h_1 + 0{\cdot}8618\,h_2 \\
& -19{\cdot}59\,\delta n + 0{\cdot}9652\,\delta y_1 - 0{\cdot}5050\,\delta y_2 - 0{\cdot}2627\,k_1 + 0{\cdot}5073\,k_2 \\
& -0{\cdot}6982\,h_3 - 0{\cdot}0023\,p_1 + 0{\cdot}2650\,p_2 - 0{\cdot}5090\,p_3 \\
& -0{\cdot}7159\,k_3 - 1{\cdot}0000\,q_1 - 0{\cdot}9642\,q_2 + 0{\cdot}8607\,q_3
\end{aligned}$$

$$\begin{aligned}
-43{\cdot}2 = {} & \delta\epsilon - 0{\cdot}4741\,\delta x_1 - 0{\cdot}5505\,\delta x_2 - 0{\cdot}9154\,h_1 + 0{\cdot}6758\,h_2 \\
& -18{\cdot}58\,\delta n + 0{\cdot}8805\,\delta y_1 - 0{\cdot}8348\,\delta y_2 - 0{\cdot}4025\,k_1 + 0{\cdot}7371\,k_2 \\
& -0{\cdot}3220\,h_3 + 0{\cdot}0787\,p_1 + 0{\cdot}3291\,p_2 - 0{\cdot}6814\,p_3 \\
& -0{\cdot}9467\,k_3 - 0{\cdot}9969\,q_1 - 0{\cdot}9443\,q_2 + 0{\cdot}7319\,q_3
\end{aligned}$$

$$\begin{aligned}
-50{\cdot}1 = {} & \delta\epsilon - 0{\cdot}6430\,\delta x_1 - 0{\cdot}1731\,\delta x_2 - 0{\cdot}8543\,h_1 + 0{\cdot}4599\,h_2 \\
& -17{\cdot}68\,\delta n + 0{\cdot}7659\,\delta y_1 - 0{\cdot}9849\,\delta y_2 - 0{\cdot}5198\,k_1 + 0{\cdot}8879\,k_2 \\
& +0{\cdot}0686\,h_3 + 0{\cdot}1510\,p_1 + 0{\cdot}3848\,p_2 - 0{\cdot}8085\,p_3 \\
& -0{\cdot}9976\,k_3 - 0{\cdot}9885\,q_1 - 0{\cdot}9230\,q_2 + 0{\cdot}5885\,q_3
\end{aligned}$$

$$\begin{aligned}
-37{\cdot}8 = {} & \delta\epsilon - 0{\cdot}9492\,\delta x_1 + 0{\cdot}8021\,\delta x_2 - 0{\cdot}6189\,h_1 - 0{\cdot}2340\,h_2 \\
& -15{\cdot}25\,\delta n + 0{\cdot}3145\,\delta y_1 - 0{\cdot}5972\,\delta y_2 - 0{\cdot}7855\,k_1 + 0{\cdot}9722\,k_2 \\
& +0{\cdot}9085\,h_3 + 0{\cdot}3396\,p_1 + 0{\cdot}5287\,p_2 - 0{\cdot}9939\,p_3 \\
& -0{\cdot}4179\,k_3 - 0{\cdot}9406\,q_1 - 0{\cdot}8488\,q_2 + 0{\cdot}1100\,q_3
\end{aligned}$$

$$\begin{aligned}
-20{\cdot}5 = {} & \delta\epsilon - 0{\cdot}9985\,\delta x_1 + 0{\cdot}9942\,\delta x_2 - 0{\cdot}4128\,h_1 - 0{\cdot}6591\,h_2 \\
& -13{\cdot}60\,\delta n - 0{\cdot}0538\,\delta y_1 + 0{\cdot}1074\,\delta y_2 - 0{\cdot}9108\,k_1 + 0{\cdot}7520\,k_2 \\
& +0{\cdot}9571\,h_3 + 0{\cdot}4607\,p_1 + 0{\cdot}6182\,p_2 - 0{\cdot}9711\,p_3 \\
& +0{\cdot}2899\,k_3 - 0{\cdot}8875\,q_1 - 0{\cdot}7860\,q_2 - 0{\cdot}2385\,q_3
\end{aligned}$$

$$\begin{aligned}
-2{\cdot}4 = {} & \delta\epsilon - 0{\cdot}9633\,\delta x_1 + 0{\cdot}8560\,\delta x_2 - 0{\cdot}2807\,h_1 - 0{\cdot}8424\,h_2 \\
& -12{\cdot}64\,\delta n - 0{\cdot}2684\,\delta y_1 + 0{\cdot}5170\,\delta y_2 - 0{\cdot}9598\,k_1 + 0{\cdot}5388\,k_2 \\
& +0{\cdot}7536\,h_3 + 0{\cdot}5279\,p_1 + 0{\cdot}6670\,p_2 - 0{\cdot}9023\,p_3 \\
& +0{\cdot}6574\,k_3 - 0{\cdot}8493\,q_1 - 0{\cdot}7451\,q_2 - 0{\cdot}4310\,q_3
\end{aligned}$$

21. From each of these equations eliminate $\delta\epsilon$, δn, δx_1, and δy_1, by means of the equations (ϵ), (n), (x), and (y) before found, and we have the following:

$$\begin{aligned}
-142''{\cdot}0 = {} & 1{\cdot}7265\,\delta x_2 + 0{\cdot}8412\,h_1 + 1{\cdot}9521\,h_2 + 1{\cdot}3230\,h_3 \\
& -11{\cdot}3691\,\delta y_2 + 3{\cdot}6001\,k_1 - 2{\cdot}8793\,k_2 - 10{\cdot}9578\,k_3 \\
& -1{\cdot}6779\,p_1 - 1{\cdot}6400\,p_2 + 0{\cdot}2249\,p_3 \\
& +2{\cdot}6815\,q_1 + 1{\cdot}8369\,q_2 + 0{\cdot}2995\,q_3
\end{aligned}$$

$$\begin{aligned}
-105''{\cdot}2 = &-0{\cdot}4681\,\delta x_2 - 0{\cdot}7311\,h_1 - 1{\cdot}2776\,h_2 - 0{\cdot}0609\,h_3 \\
&-9{\cdot}6249\,\delta y_2 + 3{\cdot}7087\,k_1 - 2{\cdot}1926\,k_2 - 9{\cdot}5426\,k_3 \\
&-1{\cdot}7765\,p_1 - 1{\cdot}4924\,p_2 + 0{\cdot}2786\,p_3 \\
&+1{\cdot}6997\,q_1 + 1{\cdot}1014\,q_2 + 0{\cdot}7934\,q_3
\end{aligned}$$

$$\begin{aligned}
-126{\cdot}1 = &-0{\cdot}2035\,\delta x_2 - 0{\cdot}9653\,h_1 - 1{\cdot}4730\,h_2 + 0{\cdot}1937\,h_3 \\
&-9{\cdot}7719\,\delta y_2 + 3{\cdot}5895\,k_1 - 2{\cdot}5827\,k_2 - 9{\cdot}5123\,k_3 \\
&-1{\cdot}7649\,p_1 - 1{\cdot}4598\,p_2 + 0{\cdot}2133\,p_3 \\
&+1{\cdot}5629\,q_1 + 1{\cdot}0070\,q_2 + 0{\cdot}8437\,q_3
\end{aligned}$$

$$\begin{aligned}
-199{\cdot}1 = &-0{\cdot}1917\,\delta x_2 - 1{\cdot}3218\,h_1 + 1{\cdot}5284\,h_2 + 0{\cdot}0260\,h_3 \\
&-9{\cdot}8232\,\delta y_2 + 0{\cdot}8943\,k_1 - 3{\cdot}4359\,k_2 - 9{\cdot}9270\,k_3 \\
&-0{\cdot}7901\,p_1 - 0{\cdot}5885\,p_2 - 0{\cdot}3497\,p_3 \\
&+0{\cdot}2540\,q_1 + 0{\cdot}1607\,q_2 + 0{\cdot}4028\,q_3
\end{aligned}$$

$$\begin{aligned}
-174{\cdot}7 = &0{\cdot}2985\,\delta x_2 - 1{\cdot}1595\,h_1 + 1{\cdot}6072\,h_2 + 0{\cdot}5979\,h_3 \\
&-9{\cdot}5788\,\delta y_2 + 0{\cdot}7062\,k_1 - 2{\cdot}9425\,k_2 - 9{\cdot}5877\,k_3 \\
&-0{\cdot}6712\,p_1 - 0{\cdot}4970\,p_2 - 0{\cdot}3251\,p_3 \\
&+0{\cdot}1946\,q_1 + 0{\cdot}1238\,q_2 + 0{\cdot}3277\,q_3
\end{aligned}$$

$$\begin{aligned}
-166{\cdot}7 = &0{\cdot}8171\,\delta x_2 - 1{\cdot}0088\,h_1 + 1{\cdot}6018\,h_2 + 1{\cdot}1442\,h_3 \\
&-9{\cdot}1122\,\delta y_2 + 0{\cdot}5586\,k_1 - 2{\cdot}4890\,k_2 - 9{\cdot}0258\,k_3 \\
&-0{\cdot}5688\,p_1 - 0{\cdot}4203\,p_2 - 0{\cdot}2956\,p_3 \\
&+0{\cdot}1498\,q_1 + 0{\cdot}0958\,q_2 + 0{\cdot}2658\,q_3
\end{aligned}$$

$$\begin{aligned}
-114{\cdot}2 = &2{\cdot}0482\,\delta x_2 - 0{\cdot}6027\,h_1 + 1{\cdot}2894\,h_2 + 2{\cdot}2661\,h_3 \\
&-6{\cdot}6781\,\delta y_2 + 0{\cdot}2576\,k_1 - 1{\cdot}3421\,k_2 - 6{\cdot}4080\,k_3 \\
&-0{\cdot}3256\,p_1 - 0{\cdot}2384\,p_2 - 0{\cdot}1971\,p_3 \\
&+0{\cdot}0628\,q_1 + 0{\cdot}0419\,q_2 + 0{\cdot}1298\,q_3
\end{aligned}$$

$$\begin{aligned}
-72{\cdot}4 = &2{\cdot}2815\,\delta x_2 - 0{\cdot}3786\,h_1 + 0{\cdot}9257\,h_2 + 2{\cdot}3601\,h_3 \\
&-4{\cdot}4181\,\delta y_2 + 0{\cdot}1283\,k_1 - 0{\cdot}7339\,k_2 - 4{\cdot}1495\,k_3 \\
&-0{\cdot}1957\,p_1 - 0{\cdot}1428\,p_2 - 0{\cdot}1286\,p_3 \\
&+0{\cdot}0283\,q_1 + 0{\cdot}0198\,q_2 + 0{\cdot}0671\,q_3
\end{aligned}$$

$$\begin{aligned}
-42{\cdot}0 = &2{\cdot}1139\,\delta x_2 - 0{\cdot}2652\,h_1 + 0{\cdot}6985\,h_2 + 2{\cdot}1241\,h_3 \\
&-3{\cdot}1027\,\delta y_2 + 0{\cdot}0772\,k_1 - 0{\cdot}4646\,k_2 - 2{\cdot}8790\,k_3 \\
&-0{\cdot}1348\,p_1 - 0{\cdot}0984\,p_2 - 0{\cdot}0924\,p_3 \\
&+0{\cdot}0154\,q_1 + 0{\cdot}0114\,q_2 + 0{\cdot}0412\,q_3
\end{aligned}$$

25. The largest terms depending on the eccentricity of the disturbing planet occur in p_3, q_3; it will be proper, therefore, to combine the above equations in such a manner that these quantities may acquire the largest co-efficients possible. This will be done by multiplying each equation by a quantity nearly proportional to the co-efficient of each of the unknown quantities p_3 and q_3, and adding together the several results. It was thought unsafe to employ the first of the above equations, since it is derived from the single observation of Flamsteed, made in 1690, twenty-two years anterior to any other observation.

Hence the equation for finding p_3 may be formed by multiplying the above equations, taken in order, by

$$-0\cdot 8,\ -0\cdot 6,\ +1\cdot 0,\ +1\cdot 0,\ +0\cdot 9,\ +0\cdot 6,\ +0\cdot 4,\ +0\cdot 3$$

beginning with the second; and the equation for q_3 by multiplying the same equations by

$$1\cdot 0,\ 1\cdot 0,\ 0\cdot 5,\ 0\ 4,\ 0\cdot 3,\ 0\cdot 2,\ 0\cdot 1,\ 0\cdot 1,$$

Hence we obtain

$$\begin{aligned}
-474''\cdot 1 = {} & 4\cdot 114\,\delta x_2 - 2\cdot 817\,h_1 + 7\cdot 837\,h_2 + 4\cdot 528\,h_3 \\
& -20\cdot 745\,\delta y_2 - 2\cdot 789\,k_1 - 6\cdot 551\,k_2 - 20\cdot 666\,k_3 \\
& +0\cdot 193\,p_1 + 0\cdot 377\,p_2 - 1\cdot 489\,p_3 \\
& -1\cdot 660\,q_1 - 1\cdot 078\,q_2 - 0\cdot 054\,q_3
\end{aligned}$$

$$\begin{aligned}
-485\cdot 0 = {} & 0\cdot 446\,\delta x_2 - 3\cdot 308\,h_1 - 0\cdot 442\,h_2 + 1\cdot 629\,h_3 \\
& -32\cdot 961\,\delta y_2 + 8\cdot 267\,k_1 - 8\cdot 805\,k_2 - 32\cdot 546\,k_3 \\
& -4\cdot 473\,p_1 - 3\cdot 643\,p_2 + 0\cdot 037\,p_3 \\
& +3\cdot 530\,q_1 + 2\cdot 278\,q_2 + 2\cdot 086\,q_3
\end{aligned}$$

26. Eliminate δx_2 and δy_2 from these equations by means of (x) and (y) and they become

$$\begin{aligned}
(3)\qquad -476''\cdot 7 = {} & -2\cdot 930\,h_1 + 7\cdot 572\,h_2 + 4\cdot 332\,h_3 \\
& -2\cdot 751\,k_1 - 6\cdot 348\,k_2 - 20\cdot 350\,k_3 \\
& +0\cdot 155\,p_1 + 0\cdot 350\,p_2 - 1\cdot 686\,p_3 \\
& -1\cdot 653\,q_1 - 1\cdot 074\,q_2 + 0\cdot 047\,q_3
\end{aligned}$$

$$\begin{aligned}
(4)\qquad -485\cdot 9 = {} & -3\cdot 463\,h_1 - 0\cdot 805\,h_2 + 1\cdot 360\,h_3 \\
& +8\cdot 345\,k_1 - 8\cdot 391\,k_2 - 31\cdot 900\,k_3 \\
& -4\cdot 525\,p_1 - 3\cdot 679\,p_2 - 0\cdot 233\,p_3 \\
& +3\cdot 545\,q_1 + 2\cdot 286\,q_2 + 2\cdot 292\,q_3
\end{aligned}$$

These equations, with (1) and (2) of Article 22, suffice for the solution of our problem.

27. Eliminate the left hand members from equations (2), (3), (4), by means of equation (1) and we have

$$\begin{aligned}
0 = {} & 0\cdot 4819\,h_1 - 0\cdot 5950\,h_2 - 5\cdot 0570\,h_3 + 0\cdot 2063\,p_1 + 0\cdot 1475\,p_2 + 0\cdot 4300\,p_3 \\
& -0\cdot 6812\,k_1 + 3\cdot 2982\,k_2 + 29\cdot 5618\,k_3 - 0\cdot 7804\,q_1 - 0\cdot 2375\,q_2 - 0\cdot 4789\,q_3
\end{aligned}$$

$$\begin{aligned}
0 = {} & -1\cdot 2005\,h_1 + 2\cdot 4466\,h_2 - 24\cdot 0122\,h_3 + 0\cdot 9735\,p_1 + 0\cdot 9412\,p_2 - 0\cdot 8575\,p_3 \\
& -2\cdot 6633\,k_1 - 5\cdot 8825\,k_2 - 19\cdot 6219\,k_3 - 1\cdot 6362\,q_1 - 1\cdot 0648\,q_2 + 0\cdot 2791\,q_3
\end{aligned}$$

$$\begin{aligned}
0 = {} & -1\cdot 7003\,h_1 - 6\cdot 0294\,h_2 - 27\cdot 5295\,h_3 - - 3\cdot 6908\,p_1 - 3\cdot 0772\,p_2 + 0\cdot 6118\,p_3 \\
& +8\cdot 4344\,k_1 - 7\cdot 9162\,k_2 - 31\cdot 1583\,k_3 + 3\cdot 5621\,q_1 + 2\cdot 2954\,q_2 + 2\cdot 5285\,q_3
\end{aligned}$$

28. If now we put $\varepsilon - \varepsilon' = \theta$ and $\varepsilon - \varpi = \beta$, it is easily seen that

$$\frac{h_1}{m'} = -36''\cdot 99 \sin \theta, \qquad \frac{h_2}{m'} = 58''\cdot 97 \sin 2\theta$$

$$\frac{k_1}{m'} = -36\cdot 99 \cos \theta, \qquad \frac{k_2}{m'} = 58\cdot 97 \cos 2\theta$$

$$\frac{h_3}{m'} = \quad 5''\cdot 80 \sin 3\theta \quad +0\cdot 007460 \frac{p_3}{m'} + 0\cdot 008974 \frac{q_3}{m'}$$

$$\frac{k_3}{m'} = \quad 5\cdot 80 \cos 3\theta \quad -0\cdot 008974 \frac{p_3}{m'} + 0\cdot 007460 \frac{q_3}{m'}$$

$$\frac{p_1}{m'} = \quad 0\cdot 18 \sin(\theta - \beta) \quad -0\cdot 046247 \left\{\frac{p_3}{m'}\cos 2\theta - \frac{q_3}{m'}\sin 2\theta\right\}$$

$$\frac{q_1}{m'} = -\,0\cdot 18 \cos(\theta - \beta) \quad +0\cdot 046247 \left\{\frac{p_3}{m'}\sin 2\theta + \frac{q_3}{m'}\cos 2\theta\right\}$$

$$\frac{p_2}{m'} = \quad 24\cdot 91 \sin(2\theta - \beta) + 0\cdot 13055 \left\{\frac{p_3}{m'}\cos\theta - \frac{q_3}{m'}\sin\theta\right\}$$

$$\frac{q_2}{m'} = \quad 24\cdot 91 \cos(2\theta - \beta) + 0\cdot 13055 \left\{\frac{p_3}{m'}\sin\theta + \frac{q_3}{m'}\cos\theta\right\}$$

29. Substituting these expressions in the equations of Art. 27, and putting for β its value $50^\circ\ 15'\cdot 8$, we obtain, after a slight reduction,

$$\begin{aligned}
0 = &-(1\cdot 24782)\sin\theta + (1\cdot 40248)\cos\theta - (1\cdot 57155)\sin 2\theta + (2\cdot 27388)\cos 2\theta \\
&-(1\cdot 46746)\sin 3\theta + (2\cdot 23430)\cos 3\theta + (9\cdot 10380)\frac{p_3}{m'} - (9\cdot 48254)\frac{q_3}{m'} \\
&+(8\cdot 28455)\left\{\frac{p_3}{m'}\cos\theta - \frac{q_3}{m'}\sin\theta\right\} - (8\cdot 49138)\left\{\frac{p_3}{m'}\sin\theta + \frac{q_3}{m'}\cos\theta\right\} \\
&-(7\cdot 97958)\left\{\frac{p_3}{m'}\cos 2\theta - \frac{q_3}{m'}\sin 2\theta\right\} - (8\cdot 55742)\left\{\frac{p_3}{m'}\sin 2\theta + \frac{q_3}{m'}\cos 2\theta\right\}
\end{aligned}$$

$$\begin{aligned}
0 = &\ (1\cdot 65083)\sin\theta + (1\cdot 99378)\cos\theta + (2\cdot 14259)\sin 2\theta - (2\cdot 58192)\cos 2\theta \\
&-(2\cdot 14400)\sin 3\theta - (2\cdot 05631)\cos 3\theta - (9\cdot 93475)\frac{p_3}{m'} - (8\cdot 91803)\frac{q^3}{m'} \\
&+(9\cdot 08947)\left\{\frac{p_3}{m'}\cos\theta - \frac{q_3}{m'}\sin\theta\right\} - (9\cdot 14306)\left\{\frac{p_3}{m'}\sin\theta + \frac{q_3}{m'}\cos\theta\right\} \\
&-(8\cdot 65341)\left\{\frac{p_3}{m'}\cos 2\theta - \frac{q_3}{m'}\sin 2\theta\right\} - (8\cdot 87892)\left\{\frac{p_3}{m'}\sin 2\theta + \frac{q_3}{m'}\cos 2\theta\right\}
\end{aligned}$$

$$\begin{aligned}
0 = &\ (1\cdot 79213)\sin\theta - (2\cdot 49403)\cos\theta - (2\cdot 55700)\sin 2\theta - (2\cdot 56972)\cos 2\theta \\
&-(2\cdot 20337)\sin 3\theta - (2\cdot 25714)\cos 3\theta + (9\cdot 83632)\frac{p_3}{m'} + (0\cdot 31156)\frac{q_3}{m'} \\
&-(9\cdot 60395)\left\{\frac{p_3}{m'}\cos\theta - \frac{q_3}{m'}\sin\theta\right\} + (9\cdot 47665)\left\{\frac{p_3}{m'}\sin\theta + \frac{q_3}{m'}\cos\theta\right\} \\
&+(9\cdot 23220)\left\{\frac{p_3}{m'}\cos 2\theta - \frac{q_3}{m'}\sin 2\theta\right\} + (9\cdot 21679)\left\{\frac{p_3}{m'}\sin 2\theta + \frac{q_3}{m'}\cos 2\theta\right\}
\end{aligned}$$

where the numbers enclosed within parentheses denote the logarithms of the corresponding coefficients.

30. These equations may be rapidly solved by approximation. The coefficients of $\frac{p_3}{m'}$ and $\frac{q_3}{m'}$ in the first equation being small, we may find from it an approximate value of θ, the substitution of which in the second and third equations will give approximate values of $\frac{p_3}{m'}$ and $\frac{q_3}{m'}$. By means of these a more accurate value of θ may be found from the first equation, and the process being repeated will enable us to satisfy all the equations as nearly as we please.

Thus we find $\theta = -51^\circ\,30'$, $\quad \frac{p_3}{m'} = 271''{\cdot}57$, $\quad \frac{q_3}{m'} = -207''{\cdot}24$.

Now ϵ is known and $= 217^\circ\ 55'$ $\therefore$ $\epsilon' = 269^\circ\ 25'$ the mean longitude of the disturbing planet at the epoch 1810·328. The sidereal motion in 36 synodic periods of Uranus $= 55^\circ\ 12'$, Precession $= 30'$, $\therefore$ Mean Longitude at the time 1846·762, or October 6, 1846, $= 325^\circ\ 7'$.

Also, the analytical expressions for $\frac{p_3}{m'}$ and $\frac{q_3}{m'}$ are

$$\frac{p_3}{m'} = 48''{\cdot}55 \sin(3\,\theta - \beta) - 93''{\cdot}01\, e' \sin(3\,\theta - \beta')$$

$$\frac{q_3}{m'} = 48{\cdot}55 \cos(3\,\theta - \beta) - 93{\cdot}01\, e' \cos(3\,\theta - \beta')$$

where $\epsilon - \varpi' = \beta'$. Equating these to the values given above, we find $e' = 3{\cdot}2206$, $\beta' = 262^\circ\ 28'$, and $\therefore \varpi' = 315^\circ\ 27'$. Hence long. of Perihelion in 1846 $= 315^\circ\ 57'$.

Lastly, substituting the values just obtained in equation (1), we find $m' = 0{\cdot}82816$.

31. Hence the values of the mass and elements of the orbit of the disturbing planet, resulting from the first hypothesis as to the mean distance, are the following:

$$\frac{a}{a'} = 0{\cdot}5$$

	°	'
Mean Long. of the planet, October 6, 1846,	325	7
Longitude of the Perihelion - - - - -	315	57
Eccentricity of the Orbit - - - - - -	0·16103	
Mass (that of the Sun being 1) - - - -	0·0001656	

These are the results which I communicated to the Astronomer Royal in October, 1845.

32. I next entered upon a similar investigation, founded on the assumption that the mean distance was about $\frac{1}{30}$th part less than before, so that $\frac{a}{a'}$ or $\alpha = \sin 31^\circ = 0{\cdot}515$. The method employed was, in principle, exactly the same as that given before; but the numerical calculations were somewhat shortened by a few alterations in the process, which had been suggested by my previous solution.

33. Assuming then that $\alpha = \sin 31°$, the values of the quantities b, $\alpha\frac{db}{d\alpha}$, $\alpha^2\frac{d^2b}{d\alpha^2}$ will be

$$\log. b_0 = 0 \cdot 33385; \quad \log. \alpha\frac{db_0}{d\alpha} = 9 \cdot 57333; \quad \log. \alpha^2\frac{d^2b_0}{d\alpha^2} = 9 \cdot 82911$$

$$\log. b_1 = 9 \cdot 76106; \quad \log. \alpha\frac{db_1}{d\alpha} = 9 \cdot 86149; \quad \log. \alpha^2\frac{d^2b_1}{d\alpha^2} = 9 \cdot 76573$$

$$\log. b_2 = 9 \cdot 35361; \quad \log. \alpha\frac{db_2}{d\alpha} = 9 \cdot 71359; \quad \log. \alpha^2\frac{d^2b_2}{d\alpha^2} = 9 \cdot 92466$$

$$\log. b_3 = 8 \cdot 98918; \quad \log. \alpha\frac{db_3}{d\alpha} = 9 \cdot 50854; \quad \log. \alpha^2\frac{d^2b_3}{d\alpha^2} = 9 \cdot 91563$$

Hence, by means of the formulæ given before, the principal inequalities of the mean longitude of URANUS, produced by the action of a planet whose mass is $\frac{m'}{5000}$, that of the SUN being unity, and the eccentricity of whose orbit is $\frac{e'}{20}$, may be found to be the following:

$$\begin{array}{llll}
 & '' & & \\
-42 \cdot 33 & m' & \sin & \{nt-n't+\varepsilon-\varepsilon'\} \\
+76 \cdot 55 & m' & \sin 2 & \{nt-n't+\varepsilon-\varepsilon'\} \\
+\ 7 \cdot 25 & m' & \sin 3 & \{nt-n't+\varepsilon-\varepsilon'\} \\
+\ 2 \cdot 34 & m' & \sin & \{n't+\varepsilon'-\varpi\} \\
-\ 4 \cdot 74 & m'e' & \sin & \{n't+\varepsilon'-\varpi'\} \\
+41 \cdot 72 & m' & \sin & \{nt-2n't+\varepsilon-2\varepsilon'+\varpi\} \\
-16 \cdot 47 & m'e' & \sin & \{nt-2n't+\varepsilon-2\varepsilon'+\varpi'\} \\
+33 \cdot 93 & m' & \sin & \{2nt-3n't+2\varepsilon-3\varepsilon'+\varpi\} \\
-63 \cdot 41 & m'e' & \sin & \{2nt-3n't+2\varepsilon-3\varepsilon'+\varpi'\}
\end{array}$$

To these we may add the following, which are of two dimensions in terms of the eccentricities:

$$\begin{array}{llll}
+0\overset{''}{\cdot}40 & m' & \sin 3 & \{nt-n't+\varepsilon-\varepsilon'\} \\
-0 \cdot 74 & m'e' & \sin & \{3(nt-n't+\varepsilon-\varepsilon')-\varpi+\varpi'\}
\end{array}$$

34. Now, on our present assumption, $n = 13° \ 0' \cdot 6$, $n' = 4° \ 48' \cdot 5$, $n-n' = 8° \ 12' \cdot 1$, $n-2n' = 3° \ 23' \cdot 6$, $2n-3n' = 11° \ 35' \cdot 7$.

Hence the equations of condition given by the modern observations will be of the form

$$\begin{aligned}
c'' = {} & \delta\varepsilon + \delta x_1 \cos\{13° \ 0' \cdot 5\}t + \delta x_2 \cos\{26° \ 1' \cdot 0\}t \\
& + t\,\delta n + \delta y_1 \sin\{13 \ 0 \cdot 5\}t + \delta y_2 \sin\{26 \ 1 \cdot 0\}t \\
& + h_1 \cos\{8 \ 12 \cdot 1\}t + h_2 \cos\{16 \ 24 \cdot 2\}t + h_3 \cos\{24° \ 36' \cdot 3\}t \\
& + k_1 \sin\{8 \ 12 \cdot 1\}t + k_2 \sin\{16 \ 24 \cdot 2\}t + k_3 \sin\{24 \ 36 \cdot 3\}t \\
& + p_1 \cos\{4 \ 48 \cdot 5\}t + p_2 \cos\{3 \ 23 \cdot 6\}t + p_3 \cos\{11 \ 35 \cdot 7\}t \\
& + q_1 \sin\{4 \ 48 \cdot 5\}t + q_2 \sin\{3 \ 23 \cdot 6\}t + q_3 \sin\{11 \ 35 \cdot 7\}t
\end{aligned}$$

35. Treating these equations of condition in the same manner as before, the equations in the first group, derived from them, are found to be the following:

$$(\epsilon)\quad 151''\cdot48 = 21\cdot0000\,\delta\epsilon + 6\cdot0670\,\delta x_1 - 4\cdot4358\,\delta x_2 + 13\cdot9515\,h_1 + 0\cdot9471\,h_2 - 4\cdot5965\,h_3 + 18\cdot3916\,p_1 + 19\cdot6752\,p_2 + 8\cdot4184\,p_3$$

$$(x)\quad 246\cdot48 = 6\cdot0670\,\delta\epsilon + 8\cdot2821\,\delta x_1 + 4\cdot1762\,\delta x_2 + 7\cdot3540\,h_1 + 8\cdot3027\,h_2 + 5\cdot0961\,h_3 + 6\cdot5793\,p_1 + 6\cdot3319\,p_2 + 8\cdot0850\,p_3$$

$$(h_1)\quad 207\cdot58 = 13\cdot9515\,\delta\epsilon + 7\cdot3540\,\delta x_1 - 0\cdot4177\,\delta x_2 + 10\cdot9735\,h_1 + 4\cdot6775\,h_2 - 0\cdot0005\,h_3 + 12\cdot8697\,p_1 + 13\cdot4050\,p_2 + 8\cdot4781\,p_3$$

$$(h_2)\quad 245\cdot17 = 0\cdot9471\,\delta\epsilon + 8\cdot3027\,\delta x_1 + 7\cdot2362\,\delta x_2 + 4\cdot6775\,h_1 + 10\cdot0259\,h_2 + 8\cdot3220\,h_3 + 2\cdot3661\,p_1 + 1\cdot6727\,p_2 + 7\cdot3073\,p_3$$

$$(h_3)\quad 103\cdot48 = -4\cdot5965\,\delta\epsilon + 5\cdot0961\,\delta x_1 + 10\cdot5558\,\delta x_2 - 0\cdot0005\,h_1 + 8\cdot3220\,h_2 + 10\cdot9749\,h_3 - 2\cdot8935\,p_1 - 3\cdot7316\,p_2 + 3\cdot5852\,p_3$$

36. Similarly the equations in the second group, are

$$(n)\quad -171\cdot27 = 77\cdot0000\,\delta n + 9\cdot3938\,\delta y_1 - 1\cdot2183\,\delta y_2 + 8\cdot7355\,k_1 + 7\cdot6213\,k_2 - 0\cdot0590\,k_3 + 5\cdot9764\,q_1 + 4\cdot3875\,q_2 + 9\cdot6152\,q_3$$

$$(y)\quad -166\cdot33 = 93\cdot9380\,\delta n + 12\cdot7179\,\delta y_1 + 1\cdot8907\,\delta y_2 + 11\cdot0393\,k_1 + 11\cdot3717\,k_2 + 3\cdot3196\,k_3 + 7\cdot3747\,q_1 + 5\cdot3825\,q_2 + 12\cdot6816\,q_3$$

$$(k_1)\quad -181\cdot31 = 87\cdot3550\,\delta n + 11\cdot0393\,\delta y_1 - 0\cdot3758\,\delta y_2 + 10\cdot0264\,k_1 + 9\cdot2740\,k_2 + 0\cdot9476\,k_3 + 6\cdot8054\,q_1 + 4\cdot9866\,q_2 + 11\cdot1971\,q_3$$

$$(k_2)\quad -99\cdot51 = 76\cdot2130\,\delta n + 11\cdot3717\,\delta y_1 + 4\cdot4810\,\delta y_2 + 9\cdot2740\,k_1 + 10\cdot9740\,k_2 + 5\cdot6294\,k_3 + 6\cdot0523\,q_1 + 4\cdot3916\,q_2 + 11\cdot0843\,q_3$$

$$(k_3)\quad 113\cdot14 = -0\cdot5900\,\delta n + 3\cdot3196\,\delta y_1 + 10\cdot2112\,\delta y_2 + 0\cdot9476\,k_1 + 5\cdot6294\,k_2 + 10\cdot0251\,k_3 + 0\cdot1746\,q_1 + 0\cdot0454\,q_2 + 2\cdot4791\,q_3$$

37. The equations (p_2), (p_3) of the first group, and (q_2), (q_3) of the second were not formed, as our previous solution shewed that when $\delta\epsilon$, δn, δx_1, and δy_1, were eliminated, the co-efficients of the remaining unknown quantities in these equations would be extremely small. It will be preferable to combine the equations (h_1), (h_2), (h_3), and (k_1), (k_2), (k_3) before, instead of after, the elimination of $\delta\epsilon$, δn, δx_1, and δy_1, from them.

If then we change the sign of the third equation in each group, and add it to the fourth and fifth, we obtain

$$141''{\cdot}07 = -17{\cdot}6009\ \delta\epsilon + 6{\cdot}0448\ \delta x_1 + 18{\cdot}2097\ \delta x_2$$
$$-\ 6{\cdot}2965\ h_1 + 13{\cdot}6704\ h_2 + 19{\cdot}2974\ h_3$$
$$-13{\cdot}3971\ p_1 - 15{\cdot}4639\ p_2 + 2{\cdot}4144\ p_3$$

$$194{\cdot}94 = -11{\cdot}7320\ \delta n + 3{\cdot}6520\ \delta y_1 + 15{\cdot}0680\ \delta y_2$$
$$+\ 0{\cdot}1951\ k_1 + 7{\cdot}3294\ k_2 + 14{\cdot}7069\ k_3$$
$$-\ 0{\cdot}5785\ q_1 - 0{\cdot}5496\ q_2 + 2{\cdot}3663\ q_3$$

38. By means of (ϵ) and (n) of Articles 35 and 36, eliminate $\delta\epsilon$ and δn from (x) and (y), and also from the equations just found, and we have

$$(x)\quad 202''{\cdot}72 = 6{\cdot}5294\ \delta x_1 + 5{\cdot}4577\ \delta x_2 + 3{\cdot}3234\ h_1 + 8{\cdot}0291\ h_2$$
$$+\ 6{\cdot}4240\ h_3 + 1{\cdot}2659\ p_1 + 0{\cdot}6477\ p_2 + 5{\cdot}6529\ p_3$$

$$(y)\quad 42{\cdot}61 = 1{\cdot}2578\ \delta y_1 + 3{\cdot}3771\ \delta y_2 + 0{\cdot}3822\ k_1 + 2{\cdot}0739\ k_2$$
$$+\ 3{\cdot}3916\ k_3 + 0{\cdot}0836\ q_1 + 0{\cdot}0298\ q_2 + 0{\cdot}9513\ q_3$$

$$268{\cdot}02 = 11{\cdot}1297\ \delta x_1 + 14{\cdot}4919\ \delta x_2 + 5{\cdot}3967\ h_1 + 14{\cdot}4642\ h_2$$
$$+15{\cdot}4449\ h_3 + 2{\cdot}0175\ p_1 + 1{\cdot}0266\ p_2 + 9{\cdot}4702\ p_3$$

$$168{\cdot}85 = 5{\cdot}0833\ \delta y_1 + 14{\cdot}8824\ \delta y_2 + 1{\cdot}5261\ k_1 + 8{\cdot}4906\ k_2$$
$$+14{\cdot}6979\ k_3 + 0{\cdot}3320\ q_1 + 0{\cdot}1189\ q_2 + 3{\cdot}8313\ q_3$$

39. Substituting for δx_2, δy_2 their values in terms of δx_1, δy_1, we find

$$6{\cdot}5294\ \delta x_1 + 5{\cdot}4577\ \delta x_2 = 6{\cdot}5700\ \delta x_1 + 0{\cdot}0490\ \delta y_1$$
$$1{\cdot}2578\ \delta y_1 + 3{\cdot}3771\ \delta y_2 = -0{\cdot}0303\ \delta x_1 + 1{\cdot}2829\ \delta y_1$$
$$11{\cdot}1297\ \delta x_1 + 14{\cdot}4919\ \delta x_2 = 11{\cdot}2378\ \delta x_1 + 0{\cdot}1300\ \delta y_1$$
$$5{\cdot}0833\ \delta y_1 + 14{\cdot}8824\ \delta y_2 = -0{\cdot}1335\ \delta x_1 + 5{\cdot}1943\ \delta y_1$$

Hence, if we add to the two latter equations

$$-\ 1{\cdot}7106\ (x) - 0{\cdot}03607\ (y)$$

and $\quad 0{\cdot}00165\ (x) - 4{\cdot}0487\ (y)$ respectively,

δx_1 and δy_1 will be eliminated, and we shall obtain the following equations:

$$(1)\quad 80''{\cdot}28 = 0{\cdot}2883\ h_1 - 0{\cdot}7295\ h_2 - 4{\cdot}4559\ h_3$$
$$+0{\cdot}0138\ k_1 + 0{\cdot}0748\ k_2 + 0{\cdot}1223\ k_3$$
$$+0{\cdot}1479\ p_1 + 0{\cdot}0813\ p_2 + 0{\cdot}1997\ p_3$$
$$+0{\cdot}0030\ q_1 + 0{\cdot}0011\ q_2 + 0{\cdot}0343\ q_3$$

$$(2)\quad 3{\cdot}34 = -0{\cdot}0055\ h_1 - 0{\cdot}0132\ h_2 - 0{\cdot}0106\ h_3$$
$$+0{\cdot}0212\ k_1 - 0{\cdot}0939\ k_2 - 0{\cdot}9662\ k_3$$
$$-0{\cdot}0021\ p_1 - 0{\cdot}0011\ p_2 - 0{\cdot}0093\ p_3$$
$$+0{\cdot}0066\ q_1 + 0{\cdot}0017\ q_2 + 0{\cdot}0203\ q_3$$

40. Again, the equations of condition given by the Ancient Observations are

$$\begin{aligned}
62''{\cdot}6 = {} & \delta\epsilon - 0{\cdot}8776\ \delta x_1 + 0{\cdot}5402\ \delta x_2 + 0{\cdot}7923\ h_1 + 0{\cdot}2554\ h_2 \\
& - 39{\cdot}31\ \delta n - 0{\cdot}4795\ \delta y_1 + 0{\cdot}8415\ \delta y_2 + 0{\cdot}6101\ k_1 + 0{\cdot}9668\ k_2 \\
& - 0{\cdot}3875\ h_3 - 0{\cdot}9877\ p_1 - 0{\cdot}6870\ p_2 - 0{\cdot}1009\ p_3 \\
& + 0{\cdot}9219\ k_3 + 0{\cdot}1566\ q_1 - 0{\cdot}7267\ q_2 - 0{\cdot}9949\ q_3
\end{aligned}$$

$$\begin{aligned}
84{\cdot}5 = {} & \delta\epsilon + 0{\cdot}4975\ \delta x_1 - 0{\cdot}5050\ \delta x_2 - 0{\cdot}0887\ h_1 - 0{\cdot}9843\ h_2 \\
& - 32{\cdot}30\ \delta n - 0{\cdot}8675\ \delta y_1 - 0{\cdot}8631\ \delta y_2 + 0{\cdot}9961\ k_1 - 0{\cdot}1767\ k_2 \\
& + 0{\cdot}2634\ h_3 - 0{\cdot}9085\ p_1 - 0{\cdot}3355\ p_2 + 0{\cdot}9681\ p_3 \\
& - 0{\cdot}9647\ k_3 - 0{\cdot}4178\ q_1 - 0{\cdot}9420\ q_2 - 0{\cdot}2506\ q_3
\end{aligned}$$

$$\begin{aligned}
67{\cdot}2 = {} & \delta\epsilon + 0{\cdot}6732\ \delta x_1 - 0{\cdot}0935\ \delta x_2 - 0{\cdot}2243\ h_1 - 0{\cdot}8994\ h_2 \\
& - 31{\cdot}34\ \delta n - 0{\cdot}7394\ \delta y_1 - 0{\cdot}9956\ \delta y_2 + 0{\cdot}9745\ k_1 - 0{\cdot}4371\ k_2 \\
& + 0{\cdot}6277\ h_3 - 0{\cdot}8720\ p_1 - 0{\cdot}2815\ p_2 + 0{\cdot}9982\ p_3 \\
& - 0{\cdot}7785\ k_3 - 0{\cdot}4895\ q_1 - 0{\cdot}9596\ q_2 - 0{\cdot}0591\ q_3
\end{aligned}$$

$$\begin{aligned}
-51{\cdot}8 = {} & \delta\epsilon - 0{\cdot}2616\ \delta x_1 - 0{\cdot}8631\ \delta x_2 - 0{\cdot}9436\ h_1 + 0{\cdot}7809\ h_2 \\
& - 19{\cdot}59\ \delta n + 0{\cdot}9652\ \delta y_1 - 0{\cdot}5050\ \delta y_2 - 0{\cdot}3310\ k_1 + 0{\cdot}6247\ k_2 \\
& - 0{\cdot}5301\ h_3 - 0{\cdot}0731\ p_1 + 0{\cdot}3991\ p_2 - 0{\cdot}6801\ p_3 \\
& - 0{\cdot}8479\ k_3 - 0{\cdot}9973\ q_1 - 0{\cdot}9169\ q_2 + 0{\cdot}7331\ q_3
\end{aligned}$$

$$\begin{aligned}
-43{\cdot}2 = {} & \delta\epsilon - 0{\cdot}4741\ \delta x_1 - 0{\cdot}5505\ \delta x_2 - 0{\cdot}8861\ h_1 + 0{\cdot}5704\ h_2 \\
& - 18{\cdot}58\ \delta n + 0{\cdot}8805\ \delta y_1 - 0{\cdot}8348\ \delta y_2 - 0{\cdot}4634\ k_1 + 0{\cdot}8213\ k_2 \\
& - 0{\cdot}1248\ h_3 + 0{\cdot}0115\ p_1 + 0{\cdot}4532\ p_2 - 0{\cdot}8147\ p_3 \\
& - 0{\cdot}9922\ k_3 - 0{\cdot}9999\ q_1 - 0{\cdot}8914\ q_2 + 0{\cdot}5798\ q_3
\end{aligned}$$

$$\begin{aligned}
-50{\cdot}1 = {} & \delta\epsilon - 0{\cdot}6430\ \delta x_1 - 0{\cdot}1731\ \delta x_2 - 0{\cdot}8191\ h_1 + 0{\cdot}3420\ h_2 \\
& - 17{\cdot}68\ \delta n + 0{\cdot}7659\ \delta y_1 - 0{\cdot}9849\ \delta y_2 - 0{\cdot}5736\ k_1 + 0{\cdot}9397\ k_2 \\
& + 0{\cdot}2588\ h_3 + 0{\cdot}0871\ p_1 + 0{\cdot}5001\ p_2 - 0{\cdot}9063\ p_3 \\
& - 0{\cdot}9659\ k_3 - 0{\cdot}9962\ q_1 - 0{\cdot}8660\ q_2 + 0{\cdot}4225\ q_3
\end{aligned}$$

$$\begin{aligned}
-37{\cdot}8 = {} & \delta\epsilon - 0{\cdot}9492\ \delta x_1 + 0{\cdot}8021\ \delta x_2 - 0{\cdot}5743\ h_1 - 0{\cdot}3404\ h_2 \\
& - 15{\cdot}25\ \delta n + 0{\cdot}3145\ \delta y_1 - 0{\cdot}5972\ \delta y_2 - 0{\cdot}8186\ k_1 + 0{\cdot}9403\ k_2 \\
& + 0{\cdot}9652\ h_3 + 0{\cdot}2872\ p_1 + 0{\cdot}6192\ p_2 - 0{\cdot}9984\ p_3 \\
& - 0{\cdot}2613\ k_3 - 0{\cdot}9579\ q_1 - 0{\cdot}7852\ q_2 - 0{\cdot}0560\ q_3
\end{aligned}$$

$$\begin{aligned}
-20{\cdot}5 = {} & \delta\epsilon - 0{\cdot}9985\ \delta x_1 + 0{\cdot}9942\ \delta x_2 - 0{\cdot}3671\ h_1 - 0{\cdot}7304\ h_2 \\
& - 13{\cdot}60\ \delta n - 0{\cdot}0538\ \delta y_1 + 0{\cdot}1074\ \delta y_2 - 0{\cdot}9302\ k_1 + 0{\cdot}6830\ k_2 \\
& + 0{\cdot}9035\ h_3 + 0{\cdot}4164\ p_1 + 0{\cdot}6928\ p_2 - 0{\cdot}9251\ p_3 \\
& + 0{\cdot}4286\ k_3 - 0{\cdot}9092\ q_1 - 0{\cdot}7212\ q_2 - 0{\cdot}3796\ q_3
\end{aligned}$$

$$\begin{aligned}
-\ 2{\cdot}4 = {} & \delta\epsilon - 0{\cdot}9633\ \delta x_1 + 0{\cdot}8560\ \delta x_2 - 0{\cdot}2363\ h_1 - 0{\cdot}8883\ h_2 \\
& - 12{\cdot}64\ \delta n - 0{\cdot}2684\ \delta y_1 + 0{\cdot}5170\ \delta y_2 - 0{\cdot}9717\ k_1 + 0{\cdot}4593\ k_2 \\
& + 0{\cdot}6562\ h_3 + 0{\cdot}4882\ p_1 + 0{\cdot}7327\ p_2 - 0{\cdot}8345\ p_3 \\
& + 0{\cdot}7546\ k_3 - 0{\cdot}8727\ q_1 - 0{\cdot}6806\ q_2 - 0{\cdot}5511\ q_3
\end{aligned}$$

41. The equation for finding p_3 may be formed, as before, by multiplying the above equations taken in order by

$$-0\cdot8,\ -0\cdot6,\ +1\cdot0,\ +1\cdot0,\ +0\cdot9,\ +0\cdot6,\ +0\cdot4,\ +0\cdot3$$

beginning with the second; and the equation for q_3 by multiplying the same equations by

$$1\cdot0,\ 1\cdot0,\ 0\cdot5,\ 0\cdot4,\ 0\cdot3,\ 0\cdot2,\ 0\cdot1,\ 0\cdot1.$$

Thus we obtain

$$\begin{aligned}-279''\cdot64 = {} & 2\cdot80\ \delta\epsilon - 3\cdot3742\ \delta x_1 + 0\cdot0265\ \delta x_2 - 2\cdot9237\ h_1 + 2\cdot2232\ h_2 \\ & -27\cdot82\ \delta n + 3\cdot7593\ \delta y_1 - 1\cdot0986\ \delta y_2 - 3\cdot8471\ k_1 + 3\cdot6706\ k_2 \\ & + 0\cdot1281\ h_3 + 1\cdot7522\ p_1 + 2\cdot6081\ p_2 - 4\cdot9033\ p_3 \\ & - 1\cdot2295\ k_3 - 3\cdot4661\ q_1 - 2\cdot2221\ q_2 + 1\cdot5785\ q_3\end{aligned}$$

$$\begin{aligned}83\cdot56 = {} & 3\cdot60\ \delta\epsilon + 0\cdot2714\ \delta x_1 - 0\cdot9567\ \delta x_2 - 1\cdot5602\ h_1 - 1\cdot3924\ h_2 \\ & -91\cdot84\ \delta n - 0\cdot5116\ \delta y_1 - 2\cdot7976\ \delta y_2 + 1\cdot0937\ k_1 + 0\cdot6112\ k_2 \\ & + 1\cdot0027\ h_3 - 1\cdot6385\ p_1 + 0\cdot1802\ p_2 + 0\cdot6529\ p_3 \\ & - 2\cdot7879\ k_3 - 2\cdot4746\ q_1 - 3\cdot2736\ q_2 + 0\cdot3113\ q_3\end{aligned}$$

42. Eliminate $\delta\epsilon$ and δn by means of (ϵ) and (n) of Articles 35 and 36, and these equations become

$$\begin{aligned}-361''\cdot72 = {} & -4\cdot1831\ \delta x_1 + 0\cdot6179\ \delta x_2 - 4\cdot7839\ h_1 + 2\cdot0969\ h_2 \\ & + 7\cdot1533\ \delta y_1 - 1\cdot5388\ \delta y_2 - 0\cdot6909\ k_1 + 6\cdot4242\ k_2 \\ & + 0\cdot7410\ h_3 - 0\cdot7000\ p_1 - 0\cdot0153\ p_2 - 6\cdot0258\ p_3 \\ & - 1\cdot2508\ k_3 - 1\cdot3068\ q_1 - 0\cdot6369\ q_2 + 5\cdot0525\ q_3\end{aligned}$$

$$\begin{aligned}-146\cdot69 = {} & -0\cdot7686\ \delta x_1 - 0\cdot1963\ \delta x_2 - 3\cdot9519\ h_1 - 1\cdot5548\ h_2 \\ & + 10\cdot6926\ \delta y_1 - 4\cdot2508\ \delta y_2 + 11\cdot5128\ k_1 + 9\cdot7013\ k_2 \\ & + 1\cdot7907\ h_3 - 4\cdot7913\ p_1 - 3\cdot1927\ p_2 - 0\cdot7902\ p_3 \\ & - 2\cdot8583\ k_3 + 4\cdot6536\ q_1 + 1\cdot9595\ q_2 + 11\cdot7796\ q_3\end{aligned}$$

43. Substituting for δx_2, δy_2, their values in terms of δx_1, δy_1, we find

$$-4\cdot1831\ \delta x_1 + 7\cdot1533\ \delta y_1 + 0\cdot6179\ \delta x_2 - 1\cdot5388\ \delta y_2 = -4\cdot1647\ \delta x_1 + 7\cdot1473\ \delta y_1$$

$$-0\cdot7686\ \delta x_1 + 10\cdot6926\ \delta y_1 - 0\cdot1963\ \delta x_2 - 4\cdot2508\ \delta y_2 = -0\cdot7319\ \delta x_1 + 10\cdot6591\ \delta y_1$$

Hence, if to the equations just found, we add

$$+0\cdot60808\ (x) - 5\cdot5942\ (y)$$

and $\qquad +0\cdot07306\ (x) - 8\cdot3110\ (y)$ respectively,

δx_1 and δy_1 will be eliminated, and we shall obtain the following equations:

$$\begin{aligned}(3) \qquad -476''\cdot84 = {} & -2\cdot7630\ h_1 + 6\cdot9793\ h_2 + 4\cdot6473\ h_3 \\ & -2\cdot8290\ k_1 - 5\cdot1777\ k_2 - 20\cdot2242\ k_3 \\ & +0\cdot0698\ p_1 + 0\cdot3785\ p_2 - 2\cdot5884\ p_3 \\ & -1\cdot7748\ q_1 - 0\cdot8036\ q_2 - 0\cdot2693\ q_3\end{aligned}$$

$$(4) \qquad -486''\cdot03 = -3\cdot7091\,h_1 - 0\cdot9682\,h_2 + 2\cdot2600\,h_3$$
$$+8\cdot3364\,k_1 - 7\cdot5348\,k_2 - 31\cdot0457\,k_3$$
$$-4\cdot6988\,p_1 - 3\cdot1454\,p_2 - 0\cdot3772\,p_3$$
$$+3\cdot9584\,q_1 + 1\cdot7118\,q_2 + 3\cdot8734\,q_3$$

44. Eliminate the left hand members from equations (2), (3), and (4) of Article 39 and 43, by means of equation (1), and we have

$$0 = 0\cdot4200\,h_1 - 0\cdot4114\,h_2 - 4\cdot2014\,h_3 + 0\cdot1980\,p_1 + 0\cdot1069\,p_2 + 0\cdot4236\,p_3$$
$$-0\cdot4964\,k_1 + 2\cdot3306\,k_2 + 23\cdot3213\,k_3 - 0\cdot1567\,q_1 - 0\cdot0409\,q_2 - 0\cdot4531\,q_3$$
$$0 = -1\cdot0507\,h_1 + 2\cdot6465\,h_2 - 21\cdot8182\,h_3 + 0\cdot9482\,p_1 + 0\cdot8614\,p_2 - 1\cdot4023\,p_3$$
$$-2\cdot7471\,k_1 - 4\cdot7334\,k_2 - 19\cdot4976\,k_3 - 1\cdot7569\,q_1 - 0\cdot7972\,q_2 - 0\cdot0655\,q_3$$
$$0 = -1\cdot9638\,h_1 - 5\cdot3845\,h_2 - 24\cdot7155\,h_3 - 3\cdot8034\,p_1 - 2\cdot6532\,p_2 + 0\cdot8317\,p_3$$
$$+8\cdot4199\,k_1 - 7\cdot0819\,k_2 - 30\cdot3051\,k_3 + 3\cdot9767\,q_1 + 1\cdot7183\,q_2 + 4\cdot0811\,q_3$$

45. If, as before, we put $\varepsilon - \varepsilon' = \theta$ and $\varepsilon - \varpi = \beta$, it may be seen that

$$\frac{h^1}{m'} = -42''\cdot33 \sin\theta, \qquad \frac{h_2}{m'} = 76''\cdot55 \sin 2\theta$$

$$\frac{k_1}{m'} = -42\cdot33 \cos\theta, \qquad \frac{k_2}{m'} = 76\cdot55 \cos 2\theta$$

$$\frac{h_3}{m'} = 7''\cdot25 \sin 3\theta + 0\cdot007460\,\frac{p_3}{m'} + 0\cdot008974\,\frac{q_3}{m'}$$

$$\frac{k_3}{m'} = 7\cdot25 \cos 3\theta - 0\cdot008974\,\frac{p_3}{m'} + 0\cdot007460\,\frac{q_3}{m'}$$

$$\frac{p_1}{m'} = 0\cdot20 \sin(\theta - \beta) - 0\cdot074738 \left\{\frac{p_3}{m'}\cos 2\theta - \frac{q_3}{m'}\sin 2\theta\right\}$$

$$\frac{q_1}{m'} = -0\cdot20 \cos(\theta - \beta) + 0\cdot074738 \left\{\frac{p_3}{m'}\sin 2\theta + \frac{q_3}{m'}\cos 2\theta\right\}$$

$$\frac{p_2}{m'} = 32\cdot91 \sin(2\theta - \beta) + 0\cdot259765 \left\{\frac{p_3}{m'}\cos\theta - \frac{q_3}{m'}\sin\theta\right\}$$

$$\frac{q_2}{m'} = 32\cdot91 \cos(2\theta - \beta) + 0\cdot259765 \left\{\frac{p_3}{m'}\sin\theta + \frac{q_3}{m'}\cos\theta\right\}$$

46. Substituting these expressions in the above equations, and putting for β its value $50°\ 15'\cdot8$, we obtain

$$0 = -(1\cdot24872)\sin\theta + (1\cdot32231)\cos\theta - (1\cdot48110)\sin 2\theta + (2\cdot24265)\cos 2\theta$$
$$-(1\cdot48373)\sin 3\theta + (2\cdot22809)\cos 3\theta + (9\cdot26254)\frac{p_3}{m'} - (9\cdot50079)\frac{q_3}{m'}$$
$$+(8\cdot44376)\left\{\frac{p_3}{m'}\cos\theta - \frac{q_3}{m'}\sin\theta\right\} - (8\cdot02630)\left\{\frac{p_3}{m'}\sin\theta + \frac{q_3}{m'}\cos\theta\right\}$$
$$-(8\cdot17031)\left\{\frac{p_3}{m'}\cos 2\theta - \frac{q_3}{m'}\sin 2\theta\right\} - (8\cdot06861)\left\{\frac{p_3}{m'}\sin 2\theta + \frac{q_3}{m'}\cos 2\theta\right\}$$

$$0 = (1\cdot65190)\sin\theta + (2\cdot06584)\cos\theta + (2\cdot30220)\sin 2\theta - (2\cdot60306)\cos 2\theta$$
$$-(2\cdot19916)\sin 3\theta - (2\cdot15032)\cos 3\theta - (0\cdot14305)\frac{p_3}{m'} - (9\cdot60933)\frac{q_3}{m'}$$
$$+(9\cdot34981)\left\{\frac{p_3}{m'}\cos\theta - \frac{q_3}{m'}\sin\theta\right\} - (9\cdot31615)\left\{\frac{p_3}{m'}\sin\theta + \frac{q_3}{m'}\cos\theta\right\}$$
$$-(8\cdot85046)\left\{\frac{p_3}{m'}\cos 2\theta - \frac{q_3}{m'}\sin 2\theta\right\} - (9\cdot11828)\left\{\frac{p^3}{m'}\sin 2\theta + \frac{q_3}{m'}\cos 2\theta\right\}$$

$$0 = (1\cdot91407)\sin\theta - (2\cdot55189)\cos\theta - (2\cdot62790)\sin 2\theta - (2\cdot64230)\cos 2\theta$$
$$-(2\cdot25331)\sin 3\theta - (2\cdot34185)\cos 3\theta + (9\cdot96344)\frac{p_3}{m'} + (0\cdot56029)\frac{q_3}{m'}$$
$$-(9\cdot83835)\left\{\frac{p_3}{m'}\cos\theta - \frac{q_3}{m'}\sin\theta\right\} + (9\cdot64968)\left\{\frac{p_3}{m'}\sin\theta + \frac{q_3}{m'}\cos\theta\right\}$$
$$+(9\cdot45371)\left\{\frac{p_3}{m'}\cos 2\theta - \frac{q_3}{m'}\sin 2\theta\right\} + (9\cdot47306)\left\{\frac{p_3}{m'}\sin 2\theta + \frac{q_3}{m'}\cos 2\theta\right\}$$

where the numbers enclosed within parentheses denote the logarithms of the corresponding co-efficients, as before.

47. From these equations, we find, by the same method as before

$$\theta = -46^\circ\, 55', \quad \frac{p_3}{m'} = 138''\cdot92, \quad \frac{q_3}{m'} = -109''\cdot83$$

Hence, since $\epsilon = 217^\circ\, 55'$, $\epsilon' = 264^\circ\, 50'$, the mean longitude of the disturbing planet at the epoch 1810·328. The sidereal motion in 36 synodic periods of Uranus $= 57^\circ\, 42'$, Precession $= 30'$. $\therefore$ Mean Longitude at the time 1846·762, or October 6, 1846, $= 323^\circ\, 2'$.

Also, the expressions for $\frac{p_3}{m'}$ and $\frac{q_3}{m'}$ are

$$\frac{p_3}{m'} = 33''\cdot93\sin(3\theta - \beta) - 63''\cdot41\, e'\sin(3\theta - \beta')$$

$$\frac{q_3}{m'} = 33\cdot93\cos(3\theta - \beta) - 63\cdot41\, e'\cos(3\theta - \beta')$$

where $\epsilon - \varpi' = \beta'$.

Equating these to the values given above, we find $e' = 2\cdot4123$, $\beta' = 279^\circ\, 11'$, and $\therefore \varpi' = 298^\circ\, 41'$. Hence longitude of the perihelion in 1846 $= 299^\circ\, 11'$.

Lastly, substituting the values just obtained in equation (1) of Article 39, we find $m' = 0\cdot75017$.

48. Hence the values of the mass and elements of the orbit of the disturbing planet, resulting from the second hypothesis as to the mean distance, are the following:

$$\frac{a}{a'} = 0\cdot515$$

Mean Longitude of the Planet, October 6, 1846,	323° 2′
Longitude of the Perihelion - - - - - -	299 11
Eccentricity of the Orbit - - - - - - - -	0·120615
Mass (that of the Sun being 1) - - - -	0·00015003

49. From the values of m', θ, $\frac{p_3}{m'}$ and $\frac{q_3}{m'}$ found above, the values of the quantities h, k, p, and q, corresponding to each hypothesis, are immediately determined. Thus we find

1st *Hypothesis.* $\frac{a}{a'} = 0 \cdot 5$		2nd *Hypothesis.* $\frac{a}{a'} = 0 \cdot 515$	
$h_1 = 23''\cdot98$	$k_1 = -19''\cdot07$	$h_1 = 23''\cdot19$	$k_1 = -21''\cdot69$
$h_2 = -47\cdot58$	$k_2 = -11\cdot00$	$h_2 = -57\cdot30$	$k_2 = -3\cdot83$
$h_3 = -1\cdot93$	$k_3 = -7\cdot64$	$h_3 = -3\cdot40$	$k_3 = -5\cdot76$
$p_1 = 9\cdot93$	$q_1 = -8\cdot31$	$p_1 = 6\cdot52$	$q_1 = -7\cdot34$
$p_2 = -8\cdot54$	$q_2 = -55\cdot36$	$p_2 = -11\cdot62$	$q_2 = -54\cdot39$
$p_3 = 224\cdot90$	$q_3 = -171\cdot63$	$p_3 = 104\cdot21$	$q_3 = -82\cdot39$

50. And by substituting these values in the equations (ϵ), (n), (x), and (y), we obtain

1st *Hypothesis.* $\frac{a}{a'} = 0 \cdot 5$		2nd *Hypothesis.* $\frac{a}{a'} = 0 \cdot 515$	
$\delta\epsilon = -49''\cdot77$	$\delta n = -0''\cdot702$	$\delta\epsilon = -43''\cdot23$	$\delta n = -0''\cdot5417$
$\delta x_1 = -130\cdot69$	$\delta y_1 = 222\cdot38$	$\delta x_1 = 1\cdot77$	$\delta y_1 = 123\cdot98$
$\delta x_2 = 1\cdot02$	$\delta y_2 = 2\cdot83$	$\delta x_2 = 1\cdot13$	$\delta y_2 = 0\cdot91$

and the corresponding corrections of the elliptic elements will be

1st *Hypothesis.*	2nd *Hypothesis.*
$\frac{\delta a}{a} = 0\cdot00000999$	$\frac{\delta a}{a} = 0\cdot00000771$
$\delta e = 20''\cdot83$	$\delta e = 40''\cdot31$
$e\delta\varpi = 127\cdot27$	$e\delta\varpi = 47\cdot10$

It will be seen that the corrections of the eccentricity and longitude of perihelion vary very rapidly with a change in the assumed mean distance.

51. If these quantitities be substituted in the expressions before given, we obtain the following theoretical corrections of the mean longitude, each of these corrections being divided into two parts, of which the first is due to the changes in the elements of the orbit of Uranus, and the second to the action of the disturbing planet.

Hypothesis I.

Ancient Observations.

Year.			
	″	″	″
1712	−288 ·0	+365 ·8	=+77 ·8
1715	−283 ·1	+357 ·1	=+74 ·0
1750	+210 ·5	−260 ·7	=−50 ·2
1753	+218 ·1	−267 ·0	=−48 ·9
1756	+214 ·0	−260 ·0	=−46 ·0
1764	+154 ·0	−186 ·7	=−32 ·7
1769	+ 79 ·6	−100 ·7	=−21 ·1
1771	+ 27 ·6	− 41 ·8	=−14 ·2

Modern Observations.

Year.			
	″	″	″
1780	−126 ·12	+129 ·27	=+ 3 ·15
1783	−180 ·28	+188 ·70	=+ 8 ·42
1786	−227 ·66	+240 ·36	=+12 ·70
1789	−265 ·70	+281 ·63	=+15 ·93
1792	−292 ·25	+310 ·38	=+18 ·13
1795	−305 ·84	+325 ·27	=+19 ·43
1798	−305 ·67	+325 ·72	=+20 ·05
1801	−291 ·77	+312 ·05	=+20 ·28
1804	−264 ·95	+285 ·38	=+20 ·43
1807	−226 ·78	+247 ·51	=+20 ·73
1810	−179 ·43	+200 ·76	=+21 ·33
1813	−125 ·59	+147 ·72	=+22 ·13
1816	− 68 ·21	+ 91 ·02	=+22 ·81
1819	− 10 ·40	+ 33 ·18	=+22 ·78
1822	+ 44 ·84	− 23 ·64	=+21 ·20
1825	+ 94 ·69	− 77 ·64	=+17 ·05
1828	+136 ·73	−127 ·48	=+ 9 ·25
1831	+168 ·94	−172 ·17	=− 3 ·23
1834	+189 ·85	−211 ·04	=−21 ·19
1837	+198 ·51	−243 ·59	=−45 ·08
1840	+194 ·54	−269 ·36	=−74 ·82

Hypothesis II.

Ancient Observations.

Year.			
	″	″	″
1712	−133 ·7	+211 ·9	=+78 ·2
1715	−117 ·7	+191 ·5	=+73 ·8
1750	+ 85 ·2	−134 ·4	=−49 ·2
1753	+ 73 ·8	−122 ·2	=−48 ·4
1756	+ 59 ·1	−105 ·2	=−46 ·1
1764	+ 2 ·7	− 36 ·4	=−33 ·7
1769	− 43 ·1	+ 20 ·8	=−22 ·3
1771	− 69 ·9	+ 54 ·7	=−15 ·2

Modern Observations.

Year.			
	″	″	″
1780	−133 ·10	+135 ·98	=+ 2 ·88
1783	−149 ·47	+157 ·87	=+ 8 ·40
1786	−160 ·15	+172 ·99	=+12 ·84
1789	−164 ·52	+180 ·64	=+16 ·12
1792	−162 ·30	+180 ·58	=+18 ·28
1795	−153 ·59	+173 ·07	=+19 ·48
1798	−138 ·87	+158 ·86	=+19 ·99
1801	−118 ·95	+139 ·08	=+20 ·13
1804	− 94 ·96	+115 ·21	=+20 ·25
1807	− 68 ·25	+ 88 ·85	=+20 ·60
1810	− 40 ·33	+ 61 ·61	=+21 ·28
1813	− 12 ·72	+ 34 ·91	=+22 ·19
1816	+ 13 ·08	+ 9 ·88	=+22 ·96
1819	+ 35 ·71	− 12 ·74	=+22 ·97
1822	+ 54 ·04	− 32 ·68	=+21 ·36
1825	+ 67 ·18	− 50 ·08	=+17 ·10
1828	+ 74 ·52	− 65 ·37	=+ 9 ·15
1831	+ 75 ·74	− 79 ·21	=− 3 ·47
1834	+ 70 ·85	− 92 ·31	=−21 ·46
1837	+ 60 ·08	−105 ·25	=−45 ·17
1840	+ 43 ·98	−118 ·38	=−74 ·40

52. Comparing these with the corrections of mean longitude derived from observation, we find the remaining differences to be the following:

Ancient Observations.

Year.	Observation—Theory. Hypoth. I.	Hypoth. II.
	″	″
1712	+ 6·7	+ 6·3
1715	− 6·8	− 6·6
1750	− 1·6	− 2·6
1753	+ 5·7	+ 5·2
1756	− 4·1	− 4·0
1761	− 5·1	− 4·1
1769	+ 0·6	+ 1·8
1771	+11·8	+12·8

Modern Observations.

Year.	Observation—Theory. Hypoth. I.	Hypoth. II
	″	″
1780	+0·27	+0·54
1783	−0·23	−0·21
1786	−0·96	−1·10
1789	+1·82	+1·63
1792	−0·91	−1·06
1795	+0·09	+0·04
1798	−0·99	−0·93
1801	−0·04	+0·11
1804	+1·76	+1·94
1807	−0·21	−0·08
1810	+0·56	+0·61
1813	−0·94	−1·00
1816	−0·31	−0·46
1819	−2·00	−2·19
1822	+0·30	+0·14
1825	+1·92	+1·87
1828	+2·25	+2·35
1831	−1·06	−0·82
1834	−1·44	−1·17
1837	−1·62	−1·53
1840	+1·73	+1·31

The largest difference in the above table, *viz.*, that for 1771, is deduced from a single observation; whereas the difference immediately preceding it, which is deduced from the mean of several, is very small.

53. The results of the two theories agree very closely with each other, and with observation, till we come to the later years of the series; and it is to be observed that the difference between the theories becomes sensible at precisely the point where they both show symptoms of diverging from the observations, the errors of the second hypothesis, however, being less than those of the other.

Recent observations show that the errors of the theory soon become very sensible, though decidedly less for the second hypothesis than for the first. The following are the differences of mean longitude as deduced from theory and observation, for the oppositions of 1843, 1844, and 1845:

Year.	Observation—Theory. Hypoth. I.	Hypoth. II.
	″	″
1843	+ 7·11	+ 5·77
1844	+ 8·79	+ 7·05
1845	+12·40	+10·18

For the observations of the last two years, I am indebted to the kindness of the Astronomer Royal. The three years nearly agree in showing that the errors of the first hypothesis are to those of the second in the ratio of 5 to 4, from which I inferred, in a letter to the Astronomer Royal, dated September 2, 1846, that the assumption of $\frac{a}{a'} = \sin 35° = 0 \cdot 574$, would probably satisfy all the observations very nearly.

54. The results which I have deduced from Professor Challis's observations of the planet, strongly confirm the inference that the mean distance should be considerably diminished. It is of course impossible to determine precisely, without actual calculation, the alteration in longitude which would be produced by such a diminution in the distance. By comparing the values of θ given by the two hypotheses, it may be seen, however, that if we took successively smaller and smaller values for the mean distance, the values found for the mean longitude in 1810 would probably go on diminishing, while at the same time the mean motion from 1810 to 1846 would rapidly increase, so that the corresponding values of the mean longitude at the present time would probably soon arrive at a minimum, and afterwards begin again to increase. This I believe to be the reason why the longitude found on the supposition of too large a value for the mean distance agrees so nearly with observation. In consequence of not making sufficient allowance for the increase in the mean motion, I hastily inferred, in my letter to the Astronomer Royal mentioned above, that the effect of a diminution in the mean distance would be to diminish the mean longitude.

55. I have already mentioned that I thought it unsafe to employ Flamsteed's observation of 1690 in forming the equations of condition, as the interval between it and all the others is so large. The difference between it and the theory appears to be very considerable, and greater for the second hypothesis than for the first, the errors being $+44'' \cdot 5$ and $+50'' \cdot 0$ respectively. These errors would probably be increased by diminishing the mean distance. It would be desirable that Flamsteed's manuscripts should be examined with reference to this point.

56. The corrections of the Tabular Radius Vector of Uranus may be easily deduced from those of the mean longitude by means of the following formula:

$$\begin{aligned}\frac{\delta r}{r} = {} & \frac{1}{r}\frac{dr}{d\varepsilon}\delta\zeta - \frac{1}{2n}\frac{d}{dt}\delta\zeta + \frac{1}{4}\frac{\delta a}{a} - \frac{1}{2}\frac{e\,\delta e}{1-e^2} - \frac{1}{6}m'a^2\frac{dA_0}{da} \\ & + \frac{m'}{2}\Sigma C_i \cos i\{nt-n't+\varepsilon-\varepsilon'\} \\ & + m'e\,\Sigma D_i \cos\{i(nt-n't+\varepsilon-\varepsilon')-nt-\varepsilon+\varpi\} \\ & + m'e'\Sigma E_i \cos\{i(nt-n't+\varepsilon-\varepsilon')-nt-\varepsilon+\varpi'\}\end{aligned}$$

where $\delta\zeta$ denotes the whole correction of the mean longitude at the time t,

$$\frac{1}{r}\frac{dr}{d\varepsilon} = e\sin\{nt+\varepsilon-\varpi\} + \frac{3e^2}{2}\sin 2\{nt+\varepsilon-\varpi\} \text{ nearly,}$$

$$C_i = \frac{1}{2}\frac{n}{n-n'}aA_i$$

$$D_i = -\frac{1}{4}\,\frac{in}{i(n-n')-n}\left\{2\,ia\,A_i + a^2\frac{dA_i}{da}\right\}$$

$$E_i = \frac{1}{4}\,\frac{(i-1)\,n}{i(n-n')-n}\left\{(2\,i-1)\,a\,A_{i-1} + a^2\frac{dA_{i-1}}{da}\right\}$$

i assuming all integral values positive and negative not including zero.

57. By substituting in this formula the values of m', δa, δe, &c., already obtained, and putting $a = 19 \cdot 191$, we find the following results corresponding to the two assumed values of the mean distance.

HYPOTHESIS I.

$$\begin{aligned}\frac{a}{r}\,\delta r = \frac{a}{r}\frac{dr}{d\varepsilon}\,\delta\zeta - \frac{a}{2}\frac{d\,\delta\zeta}{ndt} & - 0 \cdot 000089 \\ & + 0 \cdot 000069 \cos \{nt - n't + \varepsilon - \varepsilon'\} \\ & + 0 \cdot 000259 \cos 2\{nt - n't + \varepsilon - \varepsilon'\} \\ & + 0 \cdot 000109 \cos 3\{nt - n't + \varepsilon - \varepsilon'\} \\ & + 0 \cdot 000016 \cos \{n't + \varepsilon' - \varpi\} \\ & - 0 \cdot 000168 \cos \{nt - 2n't + \varepsilon - 2\varepsilon' + \varpi\} \\ & + 0 \cdot 000078 \cos \{nt - 2n't + \varepsilon - 2\varepsilon' + \varpi'\} \\ & - 0 \cdot 000049 \cos \{2nt - 3n't + 2\varepsilon - 3\varepsilon' + \varpi\} \\ & + 0 \cdot 000209 \cos \{2nt - 3n't + 2\varepsilon - 3\varepsilon' + \varpi'\}\end{aligned}$$

HYPOTHESIS II.

$$\begin{aligned}\frac{a}{r}\,\delta r = \frac{a}{r}\frac{dr}{d\varepsilon}\,\delta\zeta - \frac{a}{2}\frac{d\,\delta\zeta}{ndt} & - 0 \cdot 000144 \\ & + 0 \cdot 000073 \cos \{nt - n't + \varepsilon - \varepsilon'\} \\ & + 0 \cdot 000266 \cos 2\{nt - n't + \varepsilon - \varepsilon'\} \\ & + 0 \cdot 000115 \cos 3\{nt - n't + \varepsilon - \varepsilon'\} \\ & + 0 \cdot 000016 \cos \{n't + \varepsilon' - \varpi\} \\ & - 0 \cdot 000188 \cos \{nt - 2n't + \varepsilon - 2\varepsilon' + \varpi\} \\ & + 0 \cdot 000068 \cos \{nt - 2n't + \varepsilon - 2\varepsilon' + \varpi'\} \\ & - 0 \cdot 000053 \cos \{2nt - 3n't + 2\varepsilon - 3\varepsilon' + \varpi\} \\ & + 0 \cdot 000165 \cos \{2nt - 3n't + 2\varepsilon - 3\varepsilon' + \varpi'\}\end{aligned}$$

58. The values of $\delta\zeta$ and $\frac{d\,\delta\zeta}{dt}$ for several late years, are the following:

HYPOTHESIS I.

Year.	$\delta\zeta$	$\frac{d\,\delta\zeta}{dt}$
	″	″
1834	− 21·19	−20·93
1840	− 74·82	−32·34
1846	−148·65	−39·94

HYPOTHESIS II.

Year.	$\delta\zeta$	$\frac{d\,\delta\zeta}{dt}$
	″	″
1834	− 21·46	−20·85
1840	− 74·40	−31·62
1846	−145·91	−38·30

Hence, by means of the above formulæ, we find the corrections of the tabular radius vector, to be

Year.	Hypothesis I.	Hypothesis II.
1834	+0·00505	+0·00492
1840	+0·00722	+0·00696
1846	+0·00868	+0·00825

59. By far the most important part of these corrections arises from the term $-\frac{1}{2} r \frac{d\delta\zeta}{ndt}$, and may therefore be immediately deduced from a comparison of the observed angular motion of Uranus with that given by the Tables. In fact, the corrections given by this term alone for the epochs above-mentioned, are

Year.	Hypothesis I.	Hypothesis II.
1834	+0·00447	+0·00445
1840	+0·00694	+0·00678
1846	+0·00853	+0·00818

which, as we see, differ very little from the complete values just found. The correction for 1834, very nearly agrees with that which Mr. Airy has deduced from observation in the *Astronomische Nachrichten.* The corrections for subsequent years are rather larger than those given by the Greenwich Observations, the results of the second hypothesis being, as in the case of the longitude, nearer the truth than those of the first.

60. I made some attempts, by discussing the observations of latitude, to find approximate values of the longitude of the node and inclination of the orbit of the disturbing planet, but the results were not satisfactory. The perturbations of the latitude are in fact exceedingly small, and during the comparatively short period of three-fourths of a revolution, are nearly confounded with the effects of a constant alteration in the inclination and the position of the node of URANUS, so that very small errors in the observations may entirely vitiate the result.

61. The perturbations of Saturn produced by the new planet, though small, will still be sensible, and it would be interesting to inquire whether, if they were taken into account, the values of the masses of Jupiter and Uranus found from their action on Saturn would be more consistent with those determined by other means, than they appear to be at present. The reduction of the Greenwich Planetary Observations renders such an inquiry comparatively easy, and it is to be hoped that English astronomers will not be the last to avail themselves of the treasures of observation thus laid open to the world.

St. John's College, Cambridge,
November 12, 1846.

ON THE CORRECTION OF A LONGITUDE

DETERMINED APPROXIMATELY BY THE OBSERVATION OF A LUNAR DISTANCE.

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In the usual method of determining the Longitude by the observation of a Lunar Distance, the measurement of the arc between the Moon and the Sun or Star, is accompanied by observations of the altitudes of the extreme points of the arc. These auxiliary observations enable the observer to reduce the measured distance to the distance as seen from the Earth's centre; whence, by means of results of theoretical calculation furnished by the Nautical Almanac, the Greenwich Mean Solar Time of observation, and, by consequence, the Longitude of the place of observation, may be inferred. The peculiar advantage of the method is, that it does not require the Longitude to be previously known even *approximately*. It is not, however, capable of a great degree of accuracy. The sources of error are, first, the error of observation of the Lunar Distance, and, to some amount, the errors of the observed altitudes; next, the errors in those data of the observer's calculation which relate to the locality where the observation was taken; and lastly, the errors of the results of theoretical calculation in the Nautical Almanac. The initial determination of the Longitude by a Lunar Distance is, therefore, to be regarded only as a first approximation, which may be subsequently employed either in deducing from the same observations a more correct value, or as auxiliary to more accurate methods, such as the observation of an occultation of a Fixed Star or Planet by the Moon, or of the beginning or end of a Solar Eclipse, the calculating of which requires an approximate knowledge of the Longitude.

In order to deduce the Longitude from a Lunar Distance with as much accuracy as the kind of observation will admit of, it is necessary to go through a calculation supplementary to that which conducted to the first approximation. An obvious method of conducting this calculation is, first, to use the observed altitudes merely for clearing the Lunar Distance of refraction, for which purpose they will in general be sufficiently accurate; and then to compute the *apparent* Lunar Distance for the place of observation and the approximately known Greenwich Mean Time, in such manner as to take account of small unknown corrections which the data of the computation may require, including among them the correction of the assumed Longitude. An expression will thus be obtained for the true apparent Lunar Distance, which, being equated to the observed Lunar Distance corrected for refraction, furnishes a *provisional* equation of condition between the unknown corrections, by means of which the correction of the assumed Longitude is determined, when by any independent means the values of the other corrections have been ascertained.

If an occultation of a Fixed Star or a Planet by the Moon, or the beginning or end of a Solar Eclipse, happen to be observed at a place whose Longitude has been approximately found by a Lunar Distance, this first determination gives the means of calculating the Occultation or the Eclipse, and thus obtaining a correction of the approximate Longitude. The calculation may be made precisely in the same manner as the above mentioned supplementary calculation of a Lunar Distance, the only difference being that it admits of some simplification in consequence of the small distance of the Star, or the Sun, from the Moon's centre. An equation of condition between the unknown corrections is obtained as in that case, possessing greater value than the equation of condition for the Lunar Distance, on account of the greater degree of accuracy which the observation of the Occultation or Eclipse is capable of.

The main object of this communication is to point out an exact and expeditious method of obtaining the provisional equation just spoken of. The corrections of *all* the assumed data will be taken account of (which, as far as I am aware, has not hitherto been done), and formulæ convenient for this purpose, not requiring long or complicated calculation, will be investigated. An example will be added for the sake of illustration.

Let it be supposed that the Moon's Geocentric Right Ascension (R'), Geocentric North Polar Distance (λ'), Equatorial Horizontal Parallax (P'), and Geocentric Semidiameter (S'), at the assumed Greenwich Mean Time, have been obtained by interpolating to second differences from the Nautical Almanac.

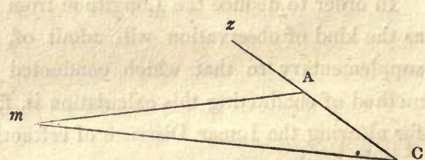
Let there be known by observation, the Right Ascension of the zenith, or sidereal time of observation, at the place (ζ); and the Lunar Distance (Δ), that is, the arc between the Moon's Limb and the Limb of the Sun or a Planet, or between the Moon's Limb and a Star, which arc is supposed to be cleared of refraction.

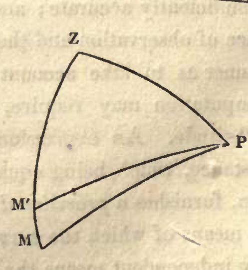
Also let l = the assumed Geocentric Co-latitude, that is, the Astronomical Co-latitude increased by the angle of the vertex.

$\theta' = R' - \zeta$ = the Geocentric Hour Angle of the Moon's centre, supposed to be *East*.

P = the Horizontal Parallax at the place.

In the annexed figures, A, C, m, are respectively the positions in space of the place of the observer, the Earth's centre, and the Moon's centre, at the time of observation. P is the North Pole of the heavens, Z and M′ are the intersections on the celestial sphere of the prolongations of C A and C m, and M on the arc Z M′ produced is the intersection of the sphere by a line through C parallel to A m.





Let $AC = \rho$, the Earth's Equatorial radius being unity, and let $Cm = r$. Then we have,

$$r \sin P' = 1 \qquad , r \sin P = \rho$$

$$\text{Hence, } \sin P = \rho \sin P' \quad \text{- - - - - - - - - - - - - - - - - - -} \quad (1)$$

Also if the angle $z A m = z$, and the angle $A m C = p$, the plane triangle $m A C$ gives,

$$\frac{\sin p}{\sin z} = \frac{\rho}{r}$$

$$\text{Hence } \sin p = \sin P \sin z \quad \text{- - - - - - - - - - - - - - - - - - -} \quad (2)$$

By the spherical triangle $P M M'$,

$$\frac{\sin MPM'}{\sin MM'} = \frac{\sin PMM'}{\sin PM'}$$

And by the spherical triangle $P Z M$,

$$\frac{\sin ZM}{\sin ZPM} = \frac{\sin PZ}{\sin PMM'}$$

Hence, by multiplying together the corresponding sides of the two equations,

$$\frac{\sin ZM \sin MPM'}{\sin MM' \sin ZPM} = \frac{\sin PZ}{\sin PM'}$$

Now the arc $ZM = z$, $MM' = p$, $PM' = \lambda'$, $PZ = l$, the angle, $ZPM' = \theta'$, and if $\theta =$ the apparent hour angle ZPM, the angle $MPM' = \theta - \theta'$. Substituting these symbols in the above equation, we have

$$\frac{\sin z \sin(\theta - \theta')}{\sin p \sin \theta} = \frac{\sin l}{\sin \lambda'}$$

Hence, by substituting for $\sin p$ from (2),

$$\sin(\theta - \theta') = \frac{\sin P \sin l}{\sin \lambda'} \sin \theta \quad \text{- - - - - - - - - - - - - - -} \quad (3)$$

$$\text{Let } G = \sin P \sin l \operatorname{cosec} \lambda' \quad \text{- - - - - - - - - - - - - - - -} \quad (4)$$

Then $\sin(\theta - \theta') = G \sin \theta = G \sin \overline{(\theta - \theta' + \theta')}$

$$\text{Consequently, } \tan(\theta - \theta') = \frac{G \sin \theta'}{1 - G \cos \theta'} \quad \text{- - - - - - - - - - - -} \quad (5)$$

The formulæ (4) and (5) serve for calculating $\theta - \theta'$, and, by consequence, the apparent hour angle θ. The calculation by the latter formula is facilitated by the use of a Table of Subtraction-Logarithms.

Formulæ for calculating the Moon's apparent North Polar Distance may be investigated as follows.

Let $\lambda = PM$, the Moon's apparent North Polar Distance,

and $z' = ZM'$, the Moon's Geocentric Zenith Distance.

Then by the spherical triangle ZPM',

$$\cos \lambda' = \cos z' \cos l + \sin z' \sin l \cos PZM'.$$

And by the spherical triangle ZPM,

$$\cos\lambda = \cos z \cos l + \sin z \sin l \cos \mathrm{PZM}.$$

Hence, dividing these equations respectively, by $\sin z'$ and $\sin z$, and subtracting the latter from the former,

$$\frac{\cos\lambda'}{\sin z'} - \frac{\cos\lambda}{\sin z} = \cos l\,(\cot z' - \cot z)$$

$$= \frac{\cos l \sin(z-z')}{\sin z \sin z'}$$

$$= \frac{\cos l \sin p}{\sin z \sin z'}$$

Substituting now for $\sin p$ from (2) and multiplying by $\sin z'$,

$$\cos\lambda' - \cos\lambda \frac{\sin z'}{\sin z} = \sin \mathrm{P} \cos l$$

$$\text{But} \quad \frac{\sin z'}{\sin\theta'} = \frac{\sin\lambda'}{\sin \mathrm{PZM'}}$$

$$\text{and} \quad \frac{\sin z}{\sin\theta} = \frac{\sin\lambda}{\sin \mathrm{PZM}}$$

$$\text{Consequently,} \quad \frac{\sin z'}{\sin z} = \frac{\sin\theta' \sin\lambda'}{\sin\theta \sin\lambda} \quad \text{- - - - - - - - - - -} \quad (6)$$

Hence, by substituting in the foregoing equation,

$$\cos\lambda' - \cot\lambda \sin\lambda' \sin\theta' \operatorname{cosec}\theta = \sin \mathrm{P} \cos l.$$

This equation gives for determining λ,

$$\cot\lambda = (\cot\lambda' - \sin \mathrm{P} \cos l \operatorname{cosec}\lambda') \sin\theta \operatorname{cosec}\theta' \quad \text{- - - - -} \quad (7)$$

To adapt this expression to logarithmic computation, let

$$\tan\phi = \sin \mathrm{P} \cos l \operatorname{cosec}\lambda' \quad \text{- - - - - - - - - - - - - - - - - -} \quad (8)$$

so that ϕ is a small angle, positive or negative according as l is less or greater than $90°$, that is, according as the place of observation is North or South of the Equator. Thus

$$\cot\lambda = \frac{\cos(\lambda' + \phi)\sin\theta}{\cos\phi \sin\lambda' \sin\theta'} \quad \text{- - - - - - - - - - - - - - - -} \quad (9)$$

The formulæ (8) and (9) are applicable for calculating λ, at whatever position on the earth's surface the observation is made.

If S be the Moon's apparent Semidiameter, we have (by the figure),

$$\frac{\mathrm{S}}{\mathrm{S'}} = \frac{\mathrm{C}m}{\mathrm{A}m} = \frac{\sin z}{\sin z'},$$

$$\text{Hence by (6),} \quad \mathrm{S} = \mathrm{S'}.\frac{\sin\theta \sin\lambda}{\sin\theta' \sin\lambda'} \quad \text{- - - - - - - - - - -} \quad (10)$$

This equation serves for calculating the Moon's apparent Semidiameter, θ and λ having been previously found.

The Moon's apparent Right Ascension (R) is equal to $\zeta + \theta$, the hour angle being supposed to be *East*.

Let the apparent North Polar Distance of the Sun, or Planet, or Star, be γ, and the difference between its apparent Right Ascension and that of the Moon be η. Then the apparent distance (c) of the Moon's centre from the centre of the Sun or Planet, or from the Star, is given by the equation,

$$\cos c = \cos \lambda \cos \gamma + \sin \lambda \sin \gamma \cos \eta \quad \text{- - - - - - - - - - -} \quad (11)$$

In the case of an Occultation or Solar Eclipse it will be advisable to make use of a modification of this formula on account of the small value of c. Since

$$\cos c = 1 - 2 \sin^2 \frac{c}{2} \qquad \text{and } \cos \eta = 1 - 2 \sin^2 \frac{\eta}{2}$$

it will be seen that

$$1 - 2 \sin^2 \frac{c}{2} = \cos (\lambda - \gamma) - 2 \sin \lambda \sin \gamma \sin^2 \frac{\eta}{2}$$

$$= 1 - 2 \sin^2 \frac{\lambda - \gamma}{2} - 2 \sin \lambda \sin \gamma \sin^2 \frac{\eta}{2}$$

Hence, $$\sin^2 \frac{c}{2} = \sin^2 \frac{\lambda - \gamma}{2} + \sin \lambda \sin \gamma \sin^2 \frac{\eta}{2}$$

If therefore d = the difference between λ and γ, and if c, d, and η, be each expressed in seconds of space, by substituting the small arcs for their sines, we have

$$c^2 = d^2 + \sin \lambda \sin \gamma\, \eta^2$$

Hence $$c = d \left(1 + \frac{\sin \lambda \sin \gamma\, \eta^2}{d^2}\right)^{\frac{1}{2}} \quad \text{- - - - - - - - - - - - -} \quad (12)$$

which formula is convenient for calculation if a Table of Addition-Logarithms be at hand. Otherwise a subsidiary angle χ may be used such that

$$\tan \chi = \frac{\eta}{d} \sqrt{\sin \lambda \sin \gamma}$$

Then, $c = d \sec \chi$

Now let s be the Tabular apparent Semidiameter of the Sun or Planet, and suppose that the distance between the *nearest* limbs was measured. Then the computed value of the Lunar Distance is

$$c - \mathrm{S} - s$$

Let the true value be,

$$c - \mathrm{S} - s + \delta (c - \mathrm{S} - s)$$

Then, assuming the observation to be correct, we have the final equation,

$$c - \mathrm{S} - s + \delta (c - \mathrm{S} - s) = \Delta$$

Or, if ε be the excess of the observed above the computed Lunar Distance, the equation of condition is,

$$\delta c - \delta \mathrm{S} - \delta s = \varepsilon \quad \text{- - - - - - - - - - - - - - -} \quad (13)$$

In the observation of an Occultation, or of the beginning or end of a Solar Eclipse, $\Delta = 0$, and refraction has no effect. Hence for these cases the equation of condition is,

$$\delta c - \delta S - \delta s = S + s - c \quad \text{- - - - - - - - - - -} \quad (14)$$

The variations δS and δs are the corrections to be added to the values of the apparent Semidiameters of the Moon and of the Sun or Planet adopted from the Nautical Almanac, in order to reduce them to the true values. The amounts of these corrections for given distances from the Earth's centre, I shall suppose to be known by repeated and exact measurements. The remainder of this investigation will be mainly employed in obtaining an expression for the correction δc, in terms of small corrections of the elements of the calculation which conducted to the value of c.

For this purpose it will be necessary to obtain expressions for the correction ($\delta \theta$) of the Moon's apparent Hour Angle and the correction ($\delta \lambda$) of the Moon's apparent North Polar Distance. Resuming the equation (3), *viz.*—

$$\sin(\theta - \theta') = \frac{\sin P \sin l}{\sin \lambda'} \sin \theta$$

it will be seen that θ is a function of θ', λ', l and P. Hence to the first order of small quantities,

$$\delta \theta = \frac{d\theta}{d\theta'} \delta \theta' + \frac{d\theta}{d\lambda'} \delta \lambda' + \frac{d\theta}{dl} \delta l + \frac{d\theta}{dP} \delta P,$$

where $\delta \theta'$, $\delta \lambda'$, δl, and δP, are respectively the corrections of the assumed values of θ', λ', l, and P; and their multipliers are the partial differential coefficients of θ with respect to the same quantities. For obtaining these partial differential coefficients it will be convenient to use, instead of equation (3), its logarithmic equation, *viz.*—

$$\text{Log} \sin(\theta - \theta') = \text{Log} \sin \theta - \text{Log} \sin \lambda' + \text{Log} \sin l + \text{Log} \sin P$$

Hence, differentiating with respect to θ',

$$\cot(\theta - \theta') \left(\frac{d\theta}{d\theta'} - 1 \right) = \cot \theta \frac{d\theta}{d\theta'},$$

$$\therefore \frac{d\theta}{d\theta'} = \frac{\cot(\theta - \theta')}{\cot(\theta - \theta') - \cot \theta} = \frac{\sin \theta \cos(\theta - \theta')}{\sin \theta'} \quad \text{- - - - - -} \quad (15)$$

Differentiating with respect to λ',

$$\cot(\theta - \theta') \frac{d\theta}{d\lambda'} = \cot \theta \frac{d\theta}{d\lambda'} - \cot \lambda'$$

$$\therefore \frac{d\theta}{d\lambda'} = -\frac{\cot \lambda'}{\cot(\theta - \theta') - \cot \theta} = -\frac{\cot \lambda' \sin \theta \sin(\theta - \theta')}{\sin \theta'} \quad \text{- - -} \quad (16)$$

Differentiating with respect to l, since l and $-\lambda'$ are similarly involved in the logarithmic equation, we have by (16),

$$\frac{d\theta}{dl} = \frac{\cot l \sin \theta \sin(\theta - \theta')}{\sin \theta'} \quad \text{- - - - - - - - - - - - - - - - - -} \quad (17)$$

So by differentiating with respect to P,

$$\frac{d\theta}{dP} = \frac{\cot P \sin\theta \sin(\theta - \theta')}{\sin\theta'} \text{ - - - - - - - - - - - - - - - } (18)$$

Again, resuming the equation (7), *viz.*—

$$\cot\lambda = (\cot\lambda' - \sin P \cos l \operatorname{cosec}\lambda') \sin\theta \operatorname{cosec}\theta',$$

which, since θ is a function of θ', λ', l, and P, shows that λ is a function of the same quantities, we have to the first order of small variations,

$$\delta\lambda = \frac{d\lambda}{d\theta'}\delta\theta' + \frac{d\lambda}{d\lambda'}\delta\lambda' + \frac{d\lambda}{dl}\delta l + \frac{d\lambda}{dP}\delta P.$$

To obtain conveniently the partial differential coefficients, the logarithmic equation of (7) will be made use of, *viz.*—

$$\text{Log}\cot\lambda = \text{Log}(\cos\lambda' - \sin P\cos l) + \text{Log}\sin\theta - \text{Log}\sin\theta' - \text{Log}\sin\lambda'.$$

Differentiating this equation with respect to θ',

$$-\sec\lambda\operatorname{cosec}\lambda\frac{d\lambda}{d\theta'} = \cot\theta\frac{d\theta}{d\theta'} - \cot\theta'$$

And by (15) $\quad \dfrac{d\theta}{d\theta'} = \sin\theta\operatorname{cosec}\theta'\cos(\theta - \theta')$

Hence $\quad \dfrac{d\lambda}{d\theta'} = \sin\theta\operatorname{cosec}\theta'\sin\lambda\cos\lambda\sin(\theta - \theta')$ - - - - (19)

Differentiating with respect to λ',

$$-\sec\lambda\operatorname{cosec}\lambda\frac{d\lambda}{d\lambda'} = -\frac{\sin\lambda'}{\cos\lambda' - \sin P\cos l} - \cot\lambda' + \cot\theta\frac{d\theta}{d\lambda'}$$

But by (7), $\dfrac{1}{\cos\lambda' - \sin P\cos l} = \tan\lambda\sin\theta\operatorname{cosec}\theta'\operatorname{cosec}\lambda$

And by (16), $\dfrac{d\theta}{d\lambda'} = -\cot\lambda'\sin\theta\operatorname{cosec}\theta'\sin(\theta - \theta')$

Substituting these values it will readily be found that—

$$\frac{d\lambda}{d\lambda'} = \frac{\sin\theta\sin\lambda}{\sin\theta'\sin\lambda'}\{\sin\lambda\sin\lambda' + \cos\lambda\cos\lambda'\cos(\theta - \theta')\} \text{ - - - - } (20)$$

Differentiating with respect to l,

$$-\sec\lambda\operatorname{cosec}\lambda\frac{d\lambda}{dl} = \frac{\sin P\sin l}{\cos\lambda' - \sin P\cos l} + \cot\theta\frac{d\theta}{dl}$$

But by (7), $\dfrac{1}{\cos\lambda' - \sin P\cos l} = \tan\lambda\sin\theta\operatorname{cosec}\theta'\operatorname{cosec}\lambda'$

And by (17), $\quad \dfrac{d\theta}{dl} = \cot l\sin\theta\operatorname{cosec}\theta'\sin(\theta - \theta')$

Hence $\quad \dfrac{d\lambda}{dl} = -\dfrac{\sin P\sin\theta\sin\lambda}{\sin\theta'\sin\lambda'}(\sin\lambda\sin l + \cos\lambda\cos l\cos\theta)$ - - - (21)

Lastly, differentiating with respect to P,

$$-\sec\lambda\,\text{cosec}\,\lambda\,\frac{d\,\lambda}{d\,P}=-\frac{\cos l\cos P}{\cos\lambda'-\sin P\cos l}+\cot\theta\,\frac{d\,\theta}{d\,P}$$

But as before, $\dfrac{1}{\cos\lambda'-\sin P\cos l}=\tan\lambda\sin\theta\,\text{cosec}\,\theta'\,\text{cosec}\,\lambda'$

And by (18), $\dfrac{d\,\theta}{d\,P}=\cot P\sin\theta\,\text{cosec}\,\theta'\sin(\theta-\theta')$

$=\cos P\sin^2\theta\,\text{cosec}\,\theta'\sin l\,\text{cosec}\,\lambda'$ by (3).

Hence $\dfrac{d\,\lambda}{d\,P}=\dfrac{\cos P\sin\theta\sin\lambda}{\sin\theta'\sin\lambda'}(\sin\lambda\cos l-\cos\lambda\sin l\cos\theta)$ - - - - (22)

We have next to obtain expressions for the small variations $\delta\,\theta'$, $\delta\,\lambda'$, $\delta\,l$, and δ P. Since by supposition the Hour Angle is *East*,

$$\theta' = R' - \zeta$$

$$\text{Hence } \delta\,\theta' = \delta\,R' - \delta\,\zeta$$

Suppose now that the true Longitude of the place of observation is equal to the assumed Longitude $+\ \tau$, an *East* Longitude being reckoned *negative.* Also let the true Right Ascension of the Zenith be $\zeta + t$. Then the correction of the assumed Greenwich Mean Time of observation is $\mu\ (t+\tau)$, μ being the factor [9,99881] which converts an interval of Sidereal Time into an interval of Mean Solar Time. Hence if t and τ be expressed in seconds, and if α be the increment of the Moon's Geocentric R. A. at the time of observation in one second of Mean Time, the calculated Geocentric R. A. requires the correction $+\ \mu\,\alpha\,(t+\tau)$ on account of the error of the assumed Greenwich Mean Time. Let α be expressed in seconds of arc, and let the correction of the Tabular R. A. of the Moon at the same time be x in seconds of arc. Then

$$\delta\,R' = \mu\,\alpha\,(t+\tau)+x$$

The correction ($\delta\,\zeta$) of the Right Ascension of the Zenith expressed in arc is 15 t. Hence

$$\delta\,\theta' = \mu\,\alpha\,(t+\tau)-15\,t+x$$

So if β be the increment of the Moon's N. P. D. at the time of observation in one second of Mean Time, and y be the correction of the Moon's Tabular N. P. D.,

$$\delta\,\lambda' = \mu\,\beta\,(t+\tau)+y$$

The values of α and β are readily obtained after the calculation of the interpolated values of R$'$ and λ'. For by the usual formula of interpolation to second differences,

$$R' = a_1 + b_1\frac{h}{H} + c_1\frac{h^2}{H^2} \qquad \lambda' = a_2 + b_2\frac{h}{H} + c_2\frac{h^2}{H^2}$$

h being the algebraic excess of the time of observation above the nearest Epoch, and H being the common interval (1^h) between the Epochs, both expressed in seconds. Hence, R$'$ being *in time*,

$$\frac{d\,\mathrm{R}'}{d\,h} = \frac{b_1}{\mathrm{H}} + \frac{2\,c_1\,h}{\mathrm{H}^2} = \frac{\alpha}{15} \quad \text{- - - - - - - - - - - - - - - - -} \quad (23)$$

$$\frac{d\,\lambda'}{d\,h} = \frac{b_2}{\mathrm{H}} + \frac{2\,c_2\,h}{\mathrm{H}^2} = \beta \quad \text{- - - - - - - - - - - - - - - - -} \quad (24)$$

The correction $\delta\, l$ is the sum of the correction of the assumed astronomical co-latitude, and the correction of the angle of the vertex. Let us suppose that $\delta\, l = \nu$ in seconds.

The error of P depends on the error of the Tabular Equatorial Parallax, the error of the assumed distance of the place of observation from the Earth's centre, and the error of the assumed Greenwich Mean Time of observation. By varying the symbols in equation (1) we have,

$$\cos \mathrm{P}\, \delta\, \mathrm{P} = \delta\, \rho \sin \mathrm{P}' + \rho \cos \mathrm{P}'\, \delta\, \mathrm{P}',$$

or, omitting small quantities of an order that may be neglected,

$$\delta\, \mathrm{P} = \mathrm{P}'\, \delta\, \rho + \rho\, \delta\, \mathrm{P}'.$$

If the true Equatorial Horizontal Parallax of the Moon for a given distance from the Earth's centre has been ascertained to be $\mathrm{P}_1\,(1 + 0{,}001\, m_1)$, P_1 being the corresponding value in the Nautical Almanac, the true Equatorial Horizontal Parallax at the time of observation may be taken to be $\mathrm{P}'\,(1 + 0{,}001\, m_1)$. Also if ι be the increment of the Equatorial Horizontal Parallax at the time of observation in one second of Mean Time, the correction of P' for error of the assumed Greenwich Mean Time is $+ \mu\, \iota\, (t + \tau)$. Hence

$$\delta\, \mathrm{P}' = 0{,}001\ \mathrm{P}'\, m_1 + \mu\, \iota\, (t + \tau).$$

Let the true distance of the place of observation from the Earth's centre be $\rho\,(1 + 0{,}001\, m_2)$, so that $\delta\, \rho = 0{,}001\, m_2\, \rho$. Then

$$\delta\, \mathrm{P} = 0{,}001\, m_2\, \rho\, \mathrm{P}' + 0{,}001\, m_1\, \rho\, \mathrm{P}' + \mu\, \iota\, \rho\, (t + \tau).$$

Hence, putting m for $m_1 + m_2$ and supposing that $\rho = 1$ in the third term, which is very small,

$$\delta\, \mathrm{P} = 0{,}001\, m\, \mathrm{P} + \mu\, \iota\, (t + \tau).$$

If the interpolated value of P' be

$$a_3 + b_3 \frac{h'}{\mathrm{H}'} + c_3 \frac{h'^2}{\mathrm{H}'^2}$$

and h', H' be expressed in seconds, then with sufficient approximation,

$$\frac{d\,\mathrm{P}'}{d\,h'} = \frac{b_3}{\mathrm{H}'} = \iota \quad \text{- - - - - - - - - - - - - - - -} \quad (25)$$

For the purpose of facilitating the logarithmic computation by the eight formulæ (15—22), and for the sake of brevity of expression, the following substitutions will now be made:

$$L = \frac{\sin\theta}{\sin\theta'} \qquad Q = \frac{L\sin\lambda}{\sin\lambda'} \qquad M = Q\cos\lambda \qquad N = Q\sin\lambda$$

$$A = \frac{d\theta}{d\theta'} \qquad B = \frac{d\theta}{d\lambda'} \qquad C = \frac{d\theta}{dl} \qquad D = \frac{P}{1000}\cdot\frac{d\theta}{dP}$$

$$A' = \frac{d\lambda}{d\theta'} \qquad B' = \frac{d\lambda}{d\lambda'} \qquad C' = \frac{d\lambda}{dl} \qquad D' = \frac{P}{1000}\cdot\frac{d\lambda}{dP}$$

By these substitutions the formulæ are changed to the following:

$$A = L\cos(\theta-\theta') \quad (26)$$
$$B = -L\cot\lambda'\sin(\theta-\theta') \quad (27)$$
$$C = L\cot l\sin(\theta-\theta') \quad (28)$$
$$D = 0{,}001\ P\,L\cot P\sin(\theta-\theta') \quad (29)$$
$$A' = M\sin\lambda'\sin(\theta-\theta') \quad (30)$$
$$B' = N\sin\lambda' + M\cos\lambda'\cos(\theta-\theta') \quad (31)$$
$$C' = -N\sin P\sin l - M\sin P\cos l\cos\theta \quad (32)$$
$$D' = 0{,}001\,P\,N\cos P\cos l - 0{,}001\ P\,M\cos P\sin l\cos\theta \quad (33)$$

Most of the logarithms required for computing by these formulæ, will have been already obtained in the calculation of θ and λ.

Now since $\delta R = \delta\zeta + \delta\theta$, by referring to the values which have been found for $\delta\theta$, $\delta\zeta$, $\delta\theta'$, $\delta\lambda'$, δl, δP, it will be seen that

$$\begin{aligned}\delta R = 15t &+ A(\mu\alpha(t+\tau) - 15t + x)\\ &+ B(\mu\beta(t+\tau)+y)\\ &+ C\nu\\ &+ D\left(m + \frac{1000\,\mu\iota}{P}(t+\tau)\right),\end{aligned}$$

which equation takes the form,

$$\delta R = at + b\tau + Ax + By + C\nu + Dm,$$

$$\text{if } b = \mu(A\alpha + B\beta) + \frac{1000\,D\mu\iota}{P} \quad (34)$$

$$\text{and } a = \mu(A\alpha+B\beta) - 15(A-1) + \frac{1000\,D\mu\iota}{P} = b - 15(A-1) \quad (35)$$

So the expression for $\delta\lambda$ is,

$$\begin{aligned}\delta\lambda = &\ A'(\mu\alpha(t+\tau) - 15t + x)\\ &+ B'(\mu\beta(t+\tau)+y)\\ &+ C'\nu\\ &+ D'\left(m + \frac{1000\,\mu\iota}{P}(t+\tau)\right),\end{aligned}$$

which takes the form,

$$\delta\lambda = a't + b'\tau + A'x + B'y + C'\nu + D'm$$

$$\text{if } b' = \mu(A'\alpha + B'\beta) + \frac{1000\,D'\mu\iota}{P} \quad (36)$$

$$\text{and } a' = \mu(A'\alpha + B'\beta) - 15A' + \frac{1000\,D'\mu\iota}{P} = b' - 15A' \quad (37)$$

By making all the symbols vary in equation (11) we obtain,

$$\begin{aligned}\sin c\,\delta c = & (\sin\lambda\cos\gamma - \cos\lambda\sin\gamma\cos\eta)\,\delta\lambda \\ & + (\cos\lambda\sin\gamma - \sin\lambda\cos\gamma\cos\eta)\,\delta\gamma \\ & + \sin\lambda\sin\gamma\sin\eta\,\delta\eta\end{aligned}$$

Hence, as may be readily shewn,

$$\begin{aligned}\delta c = & \left(\sin(\gamma+\lambda)\sin^2\frac{\eta}{2} + \sin(\lambda-\gamma)\cos^2\frac{\eta}{2}\right)\frac{\delta\lambda}{\sin c} \\ & + \left(\sin(\gamma+\lambda)\sin^2\frac{\eta}{2} - \sin(\lambda-\gamma)\cos^2\frac{\eta}{2}\right)\frac{\delta\gamma}{\sin c} \\ & + \sin\lambda\sin\gamma\sin\eta\,\frac{\delta\eta}{\sin c}\end{aligned}$$

Suppose the correction of the assumed R. A. of the Sun, Planet, or Star, to be e in seconds of space, and the correction of the assumed N. P. D. to be f. Then $\delta\gamma = f$, and $\delta\eta = \delta R - e$, or $e - \delta R$, according as the Moon is more *Eastward* or *Westward* than the other body. Putting d for the difference between the arcs λ and γ, let the three quantities N_1, N_2, N_3, be calculated by the formulæ

$$N_1 = \frac{\sin d}{\sin c}\cos^2\frac{\eta}{2} \quad - - \quad (38) \qquad N_2 = \frac{\sin(\gamma+\lambda)}{\sin c}\sin^2\frac{\eta}{2} \quad - - - - \quad (39)$$

$$N_3 = \frac{\sin\lambda\sin\gamma\sin\eta}{\sin c} \quad - - - - - - - - - - - - - - \quad (40)$$

In the case of an Occultation, or Solar Eclipse, on account of the small values of c, d, and η, the following formulæ may with sufficient accuracy be used instead of the foregoing.

$$N_1 = \frac{d}{c} \quad - - - - - - \quad (41) \qquad N_2 = \frac{\eta^2}{4\,c\,r''}\sin(\gamma+\lambda) \quad - - - - \quad (42)$$

$$N_3 = \frac{\eta}{c}\sin\lambda\sin\gamma \quad - - - - - - - - - - - - - - - \quad (43)$$

r'' being the number of seconds in an arc equal to radius, and c, d, η, being all expressed in seconds of space.

Now let $T = N_2 \pm N_1$ and $U = N_2 \mp N_1$, the *upper* or *lower* sign being taken according as λ is *greater* or *less* than γ; and let $W = \pm N_3$, the *upper* or *lower* sign being taken according as the Moon's apparent place is more *Eastward* or *Westward* than that of the other body. Then

$$\delta c = W(\delta R - e) + T\,\delta\lambda + Uf$$

The error of the Moon's calculated apparent Semidiameter (S) depends on the error of the Tabular Geocentric Semidiameter, and the error of the assumed Greenwich mean time of observation. It has been shewn that

$$S = S'\,\frac{\sin\theta\sin\lambda}{\sin\theta'\sin\lambda'}, \text{ or } S = S'$$

$$\text{Hence } \delta S = Q \,\delta S' + S' \delta Q$$
$$= Q \,\delta S' + S. \frac{\delta Q}{Q}$$

If the true Geocentric Semidiameter for a given distance from the Earth's centre be known from exact measurements to be $S_1 (1 + 0{,}001 n)$, S_1 being the corresponding value in the NAUTICAL ALMANAC, the true value at the time of observation may be taken to be $S' (1 + 0{,}001\, n)$. Also if κ be the increment of the Geocentric Semidiameter at the time of observation in one second of mean time, the correction of S′ for error of the assumed Greenwich mean time is $+ \mu \kappa (t + \tau)$. Hence

$$\delta S' = 0{,}001\, S' n + \mu \kappa (t + \tau)$$

$$\text{and } Q \,\delta S' = 0{,}001\, S n + \mu \kappa (t + \tau), \text{ since } Q = 1 \text{ nearly.}$$

The equation $Q = \dfrac{\sin \theta \sin \lambda}{\sin \theta' \sin \lambda'}$ gives,

$$\frac{\delta Q}{Q} = \cot \theta \,\delta \theta - \cot \theta' \,\delta \theta' + \cot \lambda \,\delta \lambda - \cot \lambda' \,\delta \lambda'.$$

By considering only the parts of $\delta \theta$ and $\delta \lambda$ which depend on $\delta \theta'$ and $\delta \lambda'$, which is allowable, the following result to the first order of small quantities may be arrived at.

$$\frac{r'' \delta Q}{Q} = - \sin^2 \lambda \sin (\theta - \theta') \,\delta \theta' - \sin (\lambda - \lambda') \,\delta \lambda'$$

It will in no case be necessary to take account of the term involving $\delta \lambda'$, (which term when multiplied by $\frac{S}{r''}$ becomes exceedingly small), and it will suffice to put $- 15 t$ for $\delta \theta'$. Thus

$$S. \frac{\delta Q}{Q} = \sin^2 \lambda \sin (\theta - \theta') \frac{15\, S}{r''} t$$

$$\text{Hence, if } \varpi = \frac{15\, S}{r''} \sin^2 \lambda \sin (\theta - \theta') \text{ - - - - - - - - - - } (44)$$

$$\text{we have } \delta S = 0{,}001\, S n + \mu \kappa (t + \tau) + \varpi t$$

If the interpolated value of S′ be

$$a_4 + b_4 \frac{h'}{H'} + c_4 \frac{h'^2}{H'^2}$$

and h', H′ be expressed in seconds, then very nearly,

$$\frac{d\, S'}{d\, h'} = \frac{b_4}{H'} = \kappa$$

We are now prepared to substitute in the equation of condition (13), the values of δc and δS which it was proposed to investigate.

The substitution gives,

$$\varepsilon + \delta s = W (\delta R - e) + T \,\delta \lambda + U f - 0{,}001\, S n - \mu \kappa (t + \tau) - \varpi t,$$

or, putting for δR and $\delta \lambda$ their values obtained above,

$$\begin{aligned}\epsilon + \delta s = {} & \mathrm{W}\,(a\,t + b\,\tau + \mathrm{A}\,x + \mathrm{B}\,y + \mathrm{C}\,\nu + \mathrm{D}\,m)\\ & + \mathrm{T}\,(a'\,t + b'\,\tau + \mathrm{A}'x + \mathrm{B}'y + \mathrm{C}'\,\nu + \mathrm{D}'\,m)\\ & - (\mu\,\kappa + \varpi)\,t - \mu\,\kappa\,\tau - \mathrm{W}\,e + \mathrm{U}\,f - 0{,}001\,\mathrm{S}\,n\end{aligned}$$

Consequently, if the values of [1], [2], [3], [4], [5], [6], be obtained by the formulæ,

$$[1] = a\mathrm{W} + a'\mathrm{T} - \mu\,\kappa - \varpi \qquad [2] = b\,\mathrm{W} + b'\,\mathrm{T} - \mu\kappa \qquad [3] = \mathrm{A\,W} + \mathrm{A'\,T}$$
$$[4] = \mathrm{B\,W} + \mathrm{B'\,T} \qquad [5] = \mathrm{C\,W} + \mathrm{C'\,T} \qquad [6] = \mathrm{D\,W} + \mathrm{D'\,T}$$

the final equation, completely calculated is,

$$\epsilon + \delta s = [1]\,t + [2]\,\tau + [3]\,x - \mathrm{W}\,e + [4]\,y + \mathrm{U}f + [5]\,\nu + [6]\,m - 0{,}001\,\mathrm{S}\,n.$$

For the sake of simplicity the Hour Angle has all along been supposed to be *East.* For Hour Angle *West*, $\theta' = \zeta - \mathrm{R}'$, the calculations of θ and λ are the same as for Hour Angle East, $\mathrm{R} = \zeta - \theta$, and the signs of B, C, D, A′, and ϖ, are changed by the change of sign of θ and θ'. No other alterations are required for adapting the formulæ to this case.

The foregoing investigation has been conducted so as to take fully into account any error in the elements of the calculation that can possibly affect the calculated result of the observation. This degree of exactness may be required in the case of an Occultation, especially the disappearance or reappearance of a star at the Moon's dark limb, which is an observation admitting of great precision: but in the ordinary case of a Lunar Distance much of the calculation may be omitted, as relating to quantities too minute to be worth taking into account.

To facilitate the use of the method above investigated, I propose now to collect the formulæ of calculation, and arrange them in the order in which they are to be employed, and at the same time to point out those parts of the calculation which may be omitted in the case of a Lunar Distance. The formulæ, according to their relation to each other, will be contained in separate articles numbered I, II, III, &c., for the sake of future reference.

I. The initial calculations are the interpolations of R′, λ′, P′, S′, to second differences from the Nautical Almanac. The formulæ are,

$$\mathrm{R}' = a_1 + b_1\frac{h}{\mathrm{H}} + c_1\frac{h^2}{\mathrm{H}^2} \qquad \lambda' = a_2 + b_2\frac{h}{\mathrm{H}} + c_2\frac{h^2}{\mathrm{H}^2}$$

$$\mathrm{P}' = a_3 + b_3\frac{h'}{\mathrm{H}'} + c_3\frac{h'^2}{\mathrm{H}'^2} \qquad \mathrm{S}' = a_4 + b_4\frac{h'}{\mathrm{H}'} + c_4\frac{h'^2}{\mathrm{H}'^2}$$

in which the symbols have the usual significations, which it is unnecessary here to state. R′ is supposed to be expressed in time, and h, h', H, H′, are all expressed in seconds.

$$a = \frac{15\,b_1}{\mathrm{H}} + \frac{30\,c_1\,h}{\mathrm{H}^2} \qquad \beta = \frac{b_2}{\mathrm{H}} + \frac{2\,c_2 h}{\mathrm{H}^2}$$

$$\iota = \frac{b_3}{\mathrm{H}'} \qquad \kappa = \frac{b_4}{\mathrm{H}'}$$

For a Lunar Distance the calculation of the second terms of α and β, and the calculation of ι and κ, may be omitted.

II. Formulæ for calculating the Moon's apparent Hour Angle (θ)

The Hour Angle (θ') is the difference between R′ and ζ expressed in arc, and is *East* or *West* according as R′ is *greater* or *less* than ζ. In the application of this rule it is sometimes necessary to add 360° to one of these arcs.

The Astronomical Co-latitude is supposed to be reckoned from the North Pole to 180°, and l is equal to this Co-latitude increased or diminished by the angle of the vertex according as the place of observation is North or South of the Equator.

The angle of the vertex and the Earth-radius ρ, may be taken from Tables of the values of these quantities calculated for different latitudes on an assumed ellipticity. *See* Appendix to the Nautical Almanac for 1836, pp. 57 and 58.

$$\sin P = \rho \sin P'$$

$$G = \frac{\sin P \sin l}{\sin \lambda'}$$

$$\tan(\theta - \theta') = \frac{G \sin \theta'}{1 - G \cos \theta'}$$

III. Formulæ for calculating the Moon's apparent N. P. D. (λ).

$$\tan \phi = \frac{\sin P \cos l}{\sin \lambda'} = G \cot l$$

$$L = \frac{\sin \theta}{\sin \theta'}$$

$$\cot \lambda = \frac{L \cos(\lambda' + \phi)}{\cos \phi \sin \lambda'}$$

IV. Formulæ for calculating the Moon's apparent Semidiameter (S).

$$Q = \frac{L \sin \lambda}{\sin \lambda'}$$

$$S = Q\, S'$$

V. Formulæ for calculating the apparent Distance between the centres of the two bodies (c).

The Moon's apparent R. A. (R) is $\zeta + \theta$ or $\zeta - \theta$, according as the Hour Angle is *East* or *West*. The difference between R and the apparent R. A. of the other body is η.

$$\cos c = \cos \lambda \cos \gamma + \sin \lambda \sin \gamma \cos \eta.$$

Transformations of this formula convenient for logarithmic computation are well known. Tables of Addition and Subtraction-Logarithms might be used to calculate by it in its present form.

In the case of an Occultation, or Solar Eclipse, d being the difference between λ and γ, and c, d, η, being expressed in seconds of space,

$$e = d\left(1 + \frac{\eta^2 \sin \lambda \sin \gamma}{d}\right)^{\frac{1}{2}}.$$

Or, making use of a subsidiary angle,

$$\tan \chi = \frac{\eta}{d} \sqrt{\sin \lambda \sin \gamma}$$

$$c = d \sec \chi.$$

VI. Formulæ for calculating the co-efficients of t, τ, x, y, ν, m, in the expression for δ R, viz.,

$$\delta \mathrm{R} = at + b\tau + \mathrm{A}x + \mathrm{B}y + \mathrm{C}\nu + \mathrm{D}m.$$

It will be advisable to commence this part of the calculation by finding log sin $(\theta - \theta')$ by the formulæ

$$\operatorname{Log} q = \pm \log \tan (\theta - \theta')$$

$$\operatorname{Log} \sin (\theta - \theta') = \log q + \log \cos (\theta - \theta'),$$

Log tan $(\theta - \theta')$ being taken as deduced by the formulæ of Art. II., and the *upper* or *lower* sign being attached according as the hour angle is *East* or *West.* By using the resulting value of log sin $(\theta - \theta')$ in all the subsequent calculations, no farther attention to the double sign is necessary, the same formulæ applying whether the Hour Angle be *East* or *West.*

As it is known that the correction δ P of the Horizontal Parallax will always be a small quantity, it will be sufficiently accurate to take $\frac{\mathrm{P}}{r''} \cot \mathrm{P} = 1$ in the value of D.

$$\mathrm{A} = \mathrm{L} \cos (\theta - \theta')$$

$$\mathrm{B} = -\mathrm{L} \cot \lambda' \sin (\theta - \theta')$$

$$\mathrm{C} = \mathrm{L} \cot l \sin (\theta - \theta')$$

$$\mathrm{D} = 0{,}001\, r'' \mathrm{L} \sin (\theta - \theta') = [2{,}31443]\, \mathrm{L} \sin (\theta - \theta')$$

$$b = \mu \mathrm{A} \alpha + \mu \mathrm{B} \beta + \frac{1000}{\mathrm{P}} \mu \mathrm{D} \iota$$

$$a = b - 15\,(\mathrm{A} - 1).$$

In the expression for b, P must be in seconds. For a Lunar Distance, the term containing ι is to be omitted, and, generally with sufficient accuracy, A = 1, B = 0, $a = \mu \alpha = b$.

VII. Formulæ for calculating the coefficients of t, τ, x, y, ν, m, in the expression for $\delta \lambda$, viz.,

$$\delta \lambda = a't + b'\tau + \mathrm{A}'x + \mathrm{B}'y + \mathrm{C}'\nu + \mathrm{D}'m.$$

For the reason above given we may put cos P = 1 in the expression for D′.

$$\mathrm{M} = \mathrm{Q} \cos \lambda$$

$$\mathrm{A}' = \mathrm{M} \sin \lambda' \sin (\theta - \theta')$$

$$\mathrm{N} = \mathrm{Q} \sin \lambda$$

$$\mathrm{B}' = \mathrm{N} \sin \lambda' + \mathrm{M} \cos \lambda' \cos (\theta - \theta') = n_1 + n_2$$

$$\mathrm{C}' = -\sin \mathrm{P}\,(\mathrm{N} \sin l + \mathrm{M} \cos l \cos \theta) = -\sin \mathrm{P}\,(n'_1 + n'_2)$$

$$\mathrm{D}' = 0{,}001\, \mathrm{P}\,(\mathrm{N} \cos l - \mathrm{M} \sin l \cos \theta) = 0{,}001\, \mathrm{P}\,(n''_1 + n''_2)$$

$$b' = \mu \mathrm{A}' \alpha + \mu \mathrm{B}' \beta + \frac{1000}{\mathrm{P}} \mu \mathrm{D}' \iota$$

$$a' = b' - 15\, \mathrm{A}'.$$

P, as before, is to be expressed in seconds. For a Lunar Distance, the term containing ι is to be omitted, and, generally with sufficient approximation, $A' = 0$, $B' = 1$, $a' = \mu\,\beta = b'$.

VIII. Formulæ for calculating T, U, W, and ϖ.

For a Lunar Distance,

$$N_1 = \frac{\sin d \cos^2 \frac{\eta}{2}}{\sin c}$$

$$N_2 = \frac{\sin (\gamma + \lambda) \sin^2 \frac{\eta}{2}}{\sin c}$$

$$N_3 = \frac{\sin \lambda \sin \gamma \sin \eta}{\sin c}$$

For an Occultation, or Solar Eclipse,

$$N_1 = \frac{d}{c}$$

$$N_2 = \frac{\eta^2}{4\,c\,r''} \sin (\gamma + \lambda) = [4{,}08351] \frac{\eta^2}{c} \sin (\gamma + \lambda)$$

$$N_3 = \frac{\eta}{c} \sin \lambda \sin \gamma$$

c, d, and η, being in seconds of space.

$$\left.\begin{array}{l} T = N_2 + N_1 \text{ or } N_2 - N_1 \\ U = N_2 - N_1 \text{ or } N_2 + N_1 \end{array}\right\} \text{according as the Moon's Apparent N. P. D. is } \textit{greater} \text{ or } \textit{less} \text{ than that of the other body.}$$

$W = + N_3$ or $- N_3$, according as the Moon is more *Eastward* or more *Westward* than the other body.

$$\varpi = \frac{15\,S}{r''} \sin^2\lambda \sin (\theta - \theta') = [5{,}8617]\; S \sin^2\lambda \sin (\theta - \theta').$$

For a Lunar Distance, the calculation of ϖ may be omitted.

IX. Formulæ for calculating the co-efficients of t, τ, x, y, ν, m, in the Final Equation, viz.,

$$\varepsilon + \delta s = [1]\, t + [2]\, \tau + [3]\, x - W e + [4]\, y + U f + [5]\, \nu + [6]\, m - 0{,}001\, S\, n.$$

$$\begin{aligned} [1] &= a\,W + a'\,T - \mu\,\kappa - \varpi \\ [2] &= b\,W + b'\,T - \mu\,\kappa \\ [3] &= A\,W + A'\,T \\ [4] &= B\,W + B'\,T \\ [5] &= C\,W + C'\,T \\ [6] &= D\,W + D'\,T \end{aligned}$$

For a Lunar Distance, $\mu\,\kappa$ and ϖ are to be neglected.

In general, $\varepsilon = \Delta + S + s - c$. For the beginning or end of a Solar Eclipse, $\varepsilon = S + s - c$; for the occultation of a Planet, $\varepsilon = S \pm s - c$, according as the contact was *exterior* or *interior*; for the occultation of a Star, $\varepsilon = S - c$.

The correction (δs) of the Tabular Semi-diameter of the Sun or the Planet may be left out of consideration in an observation of a Lunar Distance, as being small compared to the probable error of the observation.

The small quantities ι, κ, ϖ, which have been taken into account to render the investigation as complete as possible, can scarcely in any observation produce an appreciable effect on the final result, and, if extreme accuracy be not required, may be safely neglected.

Example.

The example I have selected to illustrate the use of the foregoing formulæ is an Occultation of a Fixed Star, the calculation of which will sufficiently indicate the course to be followed in any other instance.

1845, September 14, I observed the disappearance of κ Aquarii at the Moon's Dark Limb with the Northumberland Telescope at the Cambridge Observatory.

Sidereal Time of observation - - - - - - - - - $19^h\ 10^m\ 51^s{,}27$ (ζ)

Assumed Longitude of the Cambridge Observatory - - - - $-23^s{,}54$

Greenwich Mean Solar Time of observation - - 1845, Sept. 14, $7^h\ 35^m\ 55^s{,}94$.

1. Calculation of R′, λ', P′, S′, α, β, ι, κ, by the formulæ in Article I., from data in the Nautical Almanac.

For the interpolation of R′ and λ', the middle epoch is 8^h, and $H = 3600^s$. Hence $h = -\ 1444^s{,}06$.

For the interpolation of P′ and S′, the middle Epoch is Midnight, and $H' = 43200^s$. Hence $h' = -\ 15844^s{,}06$.

b_1 = half the sum of the two first differences = $+\ 135''{,}62$

c_1 = half the second difference - - - - - - = $-\ 0''{,}07$

$b_2 = -\ 735''{,}8$ $\quad b_3 = -\ 12''{,}35$ $\quad b_4 = -\ 3''{,}35$

$c_2 = -\ 0''{,}6$ $\quad c_3 = -\ 1''{,}25$ $\quad c_4 = -\ 0''{,}35$

It is unnecessary to exhibit the details of the interpolation. The results are,

$R' = 22^h\ 27^m\ 48^s{,}23$ $\quad P' = 59'\ 25''{,}56$

$\lambda' = 94°\ 28'\ 21''{,}86$ $\quad S' = 16'\ 11''{,}58 = 971''{,}58$.

The following is the calculation of α, β, ι, and κ:—

$\text{Log}\,\dfrac{15}{H}$ - - - - - - - - -	$+\ 7{,}61979$	A. C. Log H - - - - - -	$+\ 6{,}44370$
Log b_1 - - - - - - - - -	$+\ 2{,}13232$	Log b_2 - - - - - - - -	$-\ 2{,}86676$
Sum = $\text{Log}\,\dfrac{15\,b_1}{H}$ - - - -	$+\ 9{,}75211$	Sum = $\text{Log}\,\dfrac{b_2}{H}$ - - - -	$-\ 9{,}31046$
$\text{Log}\,\dfrac{30}{H^2}$ - - - - - - - - -	$+\ 4{,}3645$	$\text{Log}\,\dfrac{2}{H^2}$ - - - - - - - -	$+\ 3{,}1884$
Log h - - - - - - - - -	$-\ 3{,}1596$	Log h - - - - - - - - -	$-\ 3{,}1596$
Log c_1 - - - - - - - - -	$-\ 8{,}8451$	Log c_2 - - - - - - - - -	$-\ 9{,}7782$
Sum = $\text{Log}\,\dfrac{30\,c_1\,h}{H^2}$ - - -	$+\ 6{,}3692$	Sum = $\text{Log}\,\dfrac{2\,c_2\,h}{H^2}$ - - -	$+\ 6{,}1262$

$\therefore \frac{15\,b_1}{H}$	$=$	$+\ 0{,}56508''$
$\frac{30\,c_1\,h}{H^2}$	$=$	$+\ 0{,}00023$
Sum $= \alpha =$		$+\ 0{,}56531$

$\therefore \frac{b_2}{H}$	$=$	$-\ 0{,}20439''$
$\frac{2\,c_2\,h}{H^2}$	$=$	$+\ 0{,}00013$
Sum $= \beta =$		$-\ 0{,}20426$

A. C. Log H′ - -	$+$ 5,3645
Log b_3 - - - - - -	$-$ 1,0917
Sum = Log ι - - -	$-$ 6,4562

A. C. Log H′ - -	$+$ 5,3645
Log b_4 - - - - - -	$-$ 0,5250
Sum = Log κ - - -	$-$ 5,8895

It is unnecessary to take out the values of ι and κ.

2. Calculation of θ by the formulæ in Article II.

	°	′	″
Right Ascension of the Zenith in arc (ζ) - -	287	42	49,05
Geocentric R. A. of the Moon in arc (R′) - -	336	57	3,45
Difference = the Hour Angle (θ') - - - - -	49	14	14,40

The Hour Angle is *East*.

	°	′	″
Assumed Astronomical Co-latitude - - - - - -	37	47	8,37
Assumed Angle of the vertex - - - - - - - -	0	11	12,00
Sum = Assumed Geocentric Co-latitude (l) -	37	58	20,37

Assumed Log ρ - - -	$+$ 9,9990916
Log sin P′ - - - - -	$+$ 8,2376810
Sum = Log sin P- - -	$+$ 8,2367726
Log sin λ'- - - - - - -	$+$ 9,9986753
A. C. Log sin λ' - - -	$+$ 0,0013247
Log sin l - - - - - -	$+$ 9,7890733
Log sin P - - - - - - -	$+$ 8,2367726
Sum = Log G- - - -	$+$ 8,0271706
Log cos θ' - - - - - -	$+$ 9,8148647
Sum = Log G cos θ' -	$+$ 7,8420353

A. C. Log G cos θ' - -	$+$ 2,1579647
A. C. Log (1 − G cos θ')	$+$ 0,0030292*
Log G - - - - - - - -	$+$ 8,0271706
Log sin θ' - - - - - - -	$+$ 9,8793371
Sum = Log tan ($\theta - \theta'$)	$+$ 7,9095369

	°	′	″
$\therefore \theta - \theta' =$	0	27	54,76
and $\theta' =$	49	14	14,40
Sum $= \theta =$	49	42	9,16

Also P = 59′ 18″,11 = 3558″,11

3. Calculation of λ and S by the formulæ in Articles III. and IV.

Log G - - - - - - -	$+$ 8,0271706
Log cot l - - - - - -	$+$ 0,1076227
Sum = Log tan ϕ - -	$+$ 8,1347933

	°	′	″
$\therefore \phi =$	0	46	53,14
And $\lambda' =$	94	28	21,86
Sum $= \lambda' + \phi =$	95	15	15,00

* Taken out of a Table of Subtraction-Logarithms, the argument being the Logarithm immediately above.

Log sin θ - - - - -	$+$ 9,8823521
A. C. Log sin θ' - -	$+$ 0,1206629
Sum = Log L - - -	$+$ 0,0030150
Log sec ϕ - - - - -	$+$ 0,0000404
A. C. Log sin λ' - - -	$+$ 0,0013247
Log cos $(\lambda' + \phi)$ - - -	$-$ 8,9617724
Sum = Log cot λ - - -	$-$ 8,9661525

$\therefore\ \lambda = 95^\circ\ 17'\ 5'',82$

Log sin λ - - - - - -	$+$ 9,9981498
A. C. Log sin λ' - -	$+$ 0,0013247
Log L - - - - - - - -	$+$ 0,0030150
Sum = Log Q - - -	$+$ 0,0024895
Log S$'$ - - - - - - -	$+$ 2,9874786
Sum = Log S - - -	$+$ 2,9899681

$\therefore\ S = 977'',17$

4. Calculation of c by the approximate formulæ in Art. V.

		°	′	″
Assumed apparent N. P. D. of Star (γ)	=	95	1	2,30
λ	=	95	17	5,82
Difference (d)	=		16	3,52 = 963″,52

		°	′	″
ζ	=	287	42	49,05
θ	=	49	42	9,16
Hour Angle being *East*, R = $\zeta + \theta$	=	337	24	58,21
Assumed apparent R. A. of Star in arc	=	337	26	59,55
Difference (η)	=		2	1,34 = 121″,34

Log d - - - - - -	$+$ 2,9838607
A. C. Log d - - -	$+$ 7,0161393
Log η - - - - - -	$+$ 2,0840040
Sum = Log $\frac{\eta}{d}$ - -	$+$ 9,1001433
2 Log $\frac{\eta}{d}$ - - - -	$+$ 8,2002866
Log sin λ - - - -	$+$ 9,9981498
Log sin γ - - - -	$+$ 9,9983327
Sum = Log tan$^2 \chi$	$+$ 8,1967691

A. C. Log tan$^2 \chi$ - -	$+$ 1,8032309
Log sec$^2 \chi$ - - - -	$+$ 0,0067789*
Log sec χ - - - -	$+$ 0,0033894
Log d - - - - - - - -	$+$ 2,9838607
Sum = Log c - - - -	$+$ 2,9872501

$\therefore\ c = 971'',07$

and $S = 977,17$

$\therefore\ S - c = +6,10 = \varepsilon$

5. Calculation of A, B, C, D, b, a, by the formulæ in Art. VI.

Hour Angle being *East*, Log q = + Log tan $(\theta - \theta')$.

Log q - - - - - - - - -	$+$ 7,90954
Log cos $(\theta - \theta')$ - - - -	$+$ 9,99999
Sum = Log sin $(\theta - \theta')$ -	$+$ 7,90953
Log L - - - - - - - - - -	$+$ 0,00302
Log cos $(\theta - \theta')$ - - - -	$+$ 9,99999
Sum = Log A - - - - - -	$+$ 0,00301

Log L - - - - - - - - - -	$+$ 0,00302
Log sin $(\theta - \theta')$ - - - - -	$+$ 7,90953
$-$ Log cot λ' - - - - - -	$+$ 8,89333
Sum = Log B - - - - - -	$+$ 6,80588

* Taken out of a Table of Addition-Logarithms, the argument being the Logarithm immediately above.

Log L + Log sin $(\theta - \theta')$	+ 7,91255
Log cot l - - - - - - - -	+ 0,10762
Sum = Log C - - - - - -	+ 8,02017
Log L sin $(\theta - \theta')$ - - -	+ 7,91255
Constant Log - - - - -	+ 2,31443
Sum = Log D - - - - - -	+ 0,22698
Log μ - - - - - - - - -	+ 9,99881
Log α - - - - - - - - -	+ 9,75229
Sum = Log $\mu\,\alpha$ - - - - -	+ 9,75110
Log A - - - - - - - - - -	+ 0,00301
Sum = Log μ A α - - - -	+ 9,75411
Log μ - - - - - - - - -	+ 9,99881
Log β - - - - - - - - - -	− 9,31018
Sum = Log $\mu\,\beta$ - - - - -	− 9,30899
Log B - - - - - - - - - -	+ 6,80588
Sum = Log μ B β - - - -	− 6,11487

Log 0,001 P - - - - - -	+ 0,55122
A. C. Log 0,001 P - - -	+ 9,4488
Log μ - - - - - - - - - -	+ 9,9988
Log ι - - - - - - - - - - -	− 6,4562
Sum = Log $\frac{1000\,\mu\,\iota}{P}$ - - -	− 5,9038
Log D - - - - - - - - - -	+ 0,2270
Sum = Log $\frac{1000\,D\,\mu\,\iota}{P}$ - -	− 6,1308
$\therefore$ μ A α =	+ 0,56769
μ B β =	− 0,00013
$\frac{1000\,D\,\mu\,\iota}{P}$ =	− 0,00014
Sum = b =	+ 0,56742
also A =	+ 1,00696
$\therefore$ − 10 (A − 1) =	− 0,06960
and − 5 (A − 1) =	− 0,03480
Sum = − 15 (A − 1) =	− 0,10440
and b =	+ 0,56742
Sum = a =	+ 0,46302

The numbers answering to Log B, Log C, and Log D will not be required.

6. Calculation of A′, B′, C′, D′, b', a', by the formulæ in Art. VII.

Log Q - - - - - - - -	+ 0,00249
Log cos λ - - - - - - -	− 8,96431
Sum = Log M - - -	− 8,96680
Log sin λ' - - - - - -	+ 9,99868
Log sin $(\theta - \theta')$ - - -	+ 7,90953
Sum = Log A′ - - -	− 6,87501
Log $\mu\,\alpha$ - - - - - - -	+ 9,75110
Sum = Log μ A′ α - -	− 6,62611
Log Q - - - - - - - -	+ 0,00249
Log sin λ - - - - - - -	+ 9,99815
Sum = Log N - - - -	+ 0,00064
Log sin λ' - - - - - -	+ 9,99868
Sum = Log n_1 - - - -	+ 9,99932

Log M - - - - - - - - -	− 8,96680
Log cos λ' - - - - - -	− 8,89201
Log cos $(\theta - \theta')$ - - -	+ 9,99999
Sum = Log n_2 - - - -	+ 7,85880
n_1 - - - =	+ 0,99844
n_2 - - - =	+ 0,00722
$n_1 + n_2$ = B′ =	+ 1,00566
Log B′ - - - - - - - -	+ 0,00245
Log $\mu\,\beta$ - - - - - - - -	− 9,30899
Sum = Log μ B′ β - -	− 9,31144

Log N - - - - - - -	$+$ 0,00064
Log sin l - - - - - -	$+$ 9,78907
Sum = Log n'_1 - - -	$+$ 9,78971
Log cot l - - - - - -	$+$ 0,10762
Sum = Log n''_1 - - -	$+$ 9,89733
Log M - - - - - - -	$-$ 8,96680
Log cos l - - - - - -	$+$ 9,89670
Log cos θ - - - - - -	$+$ 9,81074
Sum = Log n'_2 - - -	$-$ 8,67424
$-$ Log tan l - - - - -	$-$ 9,89238
Sum = Log n''_2 - - -	$+$ 8,56662
n'_1 =	$+$ 0,61618
n'_2 =	$-$ 0,04723
$n'_1 + n'_2$ =	$+$ 0,56895
Log $(n'_1 + n'_2)$ - - - -	$+$ 9,75507
$-$ Log sin P - - - - -	$-$ 8,23677
Sum = Log C′ - - - - -	$-$ 7,99184

The numbers answering to Log C′ and Log D′ will not be required.

n''_1 =	$+$ 0,78946
n''_2 =	$+$ 0,03686
$n''_1 + n''_2$ =	$+$ 0,82632
Log $(n''_1 + n''_2)$ - - -	$+$ 9,91715
Log 0,001 P - - - - - -	$+$ 0,55122
Sum = Log D′ - - - -	$+$ 0,46837
Log $\frac{1000\,\mu\,\iota}{\text{P}}$ - - - - - -	$-$ 5,9038
Sum $\doteq$ Log $\frac{1000\,\text{D}'\mu\,\iota}{\text{P}}$	$-$ 6,3722
$\therefore \frac{1000\,\text{D}'\mu\,\iota}{\text{P}}$ =	$-$ 0,00024
and A′$\mu\alpha$ =	$-$ 0,00042
B′$\mu\beta$ =	$-$ 0,20485
Sum = b' =	$-$ 0,20551
Also $-$ 10 A′ =	$+$ 0,00750
$-$ 5 A′ =	$+$ 0,00375
$\therefore -$ 15 A′ =	$+$ 0,01125
and b' =	$-$ 0,20551
Sum = a' =	$-$ 0,19426

7. Calculation of T, U, W, ϖ by the approximate formulæ in Art. VIII.

The Moon has greater N. P. D., and is more Westward, than the Star.

A. C. Log c -	$+$7,01275
Log d - - - -	$+$2,98386
Sum = Log N_1	$+$9,99661
γ =	95° 1′ 2″
λ =	95 17 6
$\gamma + \lambda$ =	190 18 8
Log sin $(\gamma + \lambda)$	$-$9,25247
2 Log η - - -	$+$4,16801
A. C. Log c -	$+$7,01275
Constant Log -	$+$4,08351
Sum = Log N_2	$-$4,51674

Log η - - - -	$+$2,08400
A. C. Log c -	$+$7,01275
Log sin λ - -	$+$9,99815
Log sin γ - -	$+$9,99833
Sum = Log N_3	$+$9,09323
N_2 =	$-$0,00000
N_1 =	$+$0,99222
$T = N_2 + N_1$ =	$+$0,99222
$U = N_2 - N_1$ =	$-$0,99222
W = $-N_3$ =	$-$0,12395

Constant Log	$+$5,8617
2 Log sin λ -	$+$9,9963
Log sin$(\theta - \theta')$	$+$7,9095
Log S - - -	$+$2,9900
Sum = Log ϖ	$+$6,7575
Log κ - - -	$-$5,8895
Log μ - - -	$+$9,9988
Sum = Log $\mu\,\kappa$	$-$5,8883
$\mu\,\kappa$ =	$-$0,00008
ϖ =	$+$0,00057
$\therefore \mu\,\kappa + \varpi$ =	$+$0,00049

8. Calculation of [1], [2], [3], [4], [5], [6] by the formulæ in Art. IX.

Log a	+9,66560	Log b	+9,75390	Log A	+0,00301
Log W	−9,09323	Log W	−9,09323	Log W	−9,09323
Log a W	−8,75883	Log b W	−8,84713	Log A W	−9,09624
Log a'	−9,28838	Log b'	−9,31283	Log A′	−6,87501
Log T	+9,99661	Log T	+9,99661	Log T	+9,99661
Log a' T	−9,28499	Log b' T	−9,30944	Log A′ T	−6,87162
a' T =	−0,19275	b' T =	−0,20391		
a W =	−0,05739	b W =	−0,07033	A′ T =	−0,00074
$-(\mu\kappa+\varpi)$ =	−0,00049	$-\mu\kappa$ =	+0,00008	A W =	−0,12481
Sum = [1] =	−0,25063	Sum = [2] =	−0,27416	Sum = [3] =	−0,12555

Log B	+6,80588	Log C	+8,02017	Log D	+0,22698
Log W	−9,09323	Log W	−9,09323	Log W	−9,09323
Log B W	−5,89911	Log C W	−7,11340	Log D W	−9,32021
Log B′	+0,00245	Log C′	−7,99184	Log D′	+0,46837
Log T	+9,99661	Log T	+9,99661	Log T	+9,99661
Log B′ T	+9,99906	Log C′ T	−7,98845	Log D′ T	+0,46498
B′ T =	+0,99784	C′ T =	−0,00974	D′ T =	+2,91729
B W =	−0,00008	C W =	−0,00130	D W =	−0,20903
Sum = [4] =	+0,99776	Sum = [5] =	−0,01104	Sum = [6] =	+2,70826

Final Equation:

$$+\,6'',10 = -\,0,2506\,t - 0,2742\,\tau - 0,1256\,x + 0,1240\,e + 0,9978\,y - 0,9922\,f$$
$$-\,0,0110\,\nu + 2,7083\,m - 0,9772\,n$$

To ascertain the correction of the Longitude given by this equation, I shall now substitute for the other small corrections the most probable values that I have been able to obtain.

By the result of observations of the Moon on the meridian at Greenwich on the same day (Sept. 14), $x = -\,20'',70$ and $y = +\,9'',19$.

Taking the place of κ Aquarii from the Twelve-year Greenwich Catalogue, $e = -\,0'',90$ and $f = +\,2'',65$.

Adopting $+\,2'',61$ for the correction of the Moon's Semi-diameter as determined by Greenwich Observations (See Introduction to vol. for 1847, p. ci.) the resulting value of n is 2,773.

Adopting Henderson's correction of Burckhardt's Constant of Parallax, *viz.* $+\,1'',30$, (Memoirs of the Royal Astronomical Society, vol. x., p. 294) the value of m_2 is $+\,0,380$.

The ratio of the Earth's axes assumed by Henderson is that of 299 to 300, while the ratio employed in the foregoing calculation is that of 297 to 298. The correction from this to the former ratio gives $m_1 = + 0{,}014$. Hence $m_1 + m_2 = + 0{,}394$, which is the value of m.

The alteration of the ellipticity requires the angle of the vertex to be corrected by $- 4'',47$, which is the value of ν, the astronomical co-latitude being supposed to require no correction.

As the observation was considered to be very exact, it will be supposed that $t = 0$.

By substituting these values the equation becomes,

$$+ 6'',10 = - 0'',2742\,\tau + 2'',600 - 0'',112 + 9'',170 - 2'',629 + 0'',049 + 1'',067 - 2'',707$$

$$\text{or, } 0'',2742\,\tau = + 1'',34.$$

Hence the correction of the Longitude by this observation is $+ 4^s,89$. I have no ground for thinking that the Longitude of the Cambridge Observatory requires a correction to any such amount. The instance is evidently unfavourable to the determination of the Longitude on account of the small value of the multiplier of τ. Slight errors in the adopted places of the Moon and the Star might account for the above result without supposing any error in the assumed Longitude. The result, however, serves to show the advantage of taking account of small corrections which the data of calculation may require, whether the calculation applies to an Occultation or a Lunar Distance; for on the supposition that none of the data except the assumed Longitude required correction, we should have had,

$$- 0'',2742\,\tau = + 6'',10,$$

$$\text{or the correction of the Longitude} = - 22^s,25.$$

But the assumed Longitude is really not one second in error; consequently, as the observation was probably not in error more than a small fraction of a second, nearly the whole of the above result is attributable to errors in the elements of the calculation, and will serve to indicate the effect of such errors independently of the degree of accuracy of which the observation was capable. In fact, as we have seen, by correcting those errors as nearly as was practicable, the result was reduced to $+ 4^s,89$. It may be remarked that this instance is confirmatory of the correction applied to the Moon's Semi-diameter.

CAMBRIDGE OBSERVATORY,
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